Extra Credit: Once you have worked a problem, you can get the same number of points as bonus by re-working it using JAPE. Include a screenshot of your proof in order to get these points. The Japeish version of the program is included for your convenience in some cases, however, you may need to change some of the characters to the JAPE character set.

1. [5 points] Using WP reasoning, prove the partial correctness of the following program, with respect to the indicated assumption and expectation.
   Assumption: \( x > 1 \)
   Expectation: \( y > 0 \land x > y \)
   Program:
   
   ```
   a := 1;
y := x;
y := y - a;
   ```

   Japeish version:
   ```
   WHERE DISTINCT a, y, x \( \vdash \) \( \{ x > 1 \} \) (a := 1; y := x; y := y - a) \{ y > 0 \land x > y \}
   ```

2. [20 points] Using WP reasoning, and finding a loop invariant and variant, prove the total correctness of program Multi. This will entail showing the 3 partial-correctness VCs and two termination VCs. (Hint: The invariant is a conjunction of two relational expressions, one of which is needed for termination.)
   
   Assumption: \( y > 0 \)
   Expectation: \( z = x \times y \)
   Program Multi:
   
   ```
   a := 0;
z := 0;
while a ≠ y
do
   z := z + x;
a := a + 1;
od
   ```

   Japeish version:
   ```
   WHERE DISTINCT a, y, x, z \( \vdash \) \( \{ y > 0 \} \)
   (a := 0; z := 0) \{ \ldots your invariant \ldots \} while a ≠ y do z := z + x; a := a + 1 od \{ z = x \times y \}
   ```

   Because of the inserted invariant, the first rule to use is Ntuple, rather than Sequence.
3. [25 points] The program Div below is supposed to compute the quotient of integers x by y, defined to be the unique integer d such that there exists some integer r, the remainder, with r < y and x = d*y + r. For example, if x = 15 and y = 6, then d = 2 because 15 = 2*6 + 3, where r = 3.
Prove the total correctness of Div.

Assumption: y > 0
Expectation: x = d*y + r \land r < y

Program: Div:

\[
\begin{align*}
&\text{r := x; } \\
&\text{d := 0; } \\
&\text{while } r \geq y \text{ do} \\
&\quad \text{r := r } - \text{ y; } \\
&\quad \text{d := d } + \text{ 1; } \\
&\quad \text{od}
\end{align*}
\]

(Create your own Japeish version, using previous problems as a guide.)

4. [50 points] Prove the total correctness of the program Pow below, with respect to the assumption and expectation indicated.

Assumption: n = N \land N > 0 \land b > 0
Expectation: r = b^N

Program Pow:

\[
\begin{align*}
&(r, s) := (1, b); \\
&\text{while } n > 0 \text{ do} \\
&\quad \text{if( mod(n, 2) == 1 )} \\
&\quad \quad \{ \\
&\quad \quad \quad r := r \ast s; \\
&\quad \quad \} \\
&\quad (n, s) := (n/2, s\ast s); \\
&\quad \text{od}
\end{align*}
\]

Here / represents integer (truncating) division and mod is the modulus function. For example, 5/2 = 2, 4/2 = 2, mod(5, 3) = 2, mod(6, 3) = 0, etc.