Most problems are adapted from exercises and problems in Sipser 3rd edition, chapter 1. There are also lecture slides posted on the course website.

A language over an alphabet $\Sigma$ is any set of finite strings of symbols in $\Sigma$. The language accepted by a DFA (Deterministic Finite-State Acceptor) $M$ is the set of all strings that take $M$ from its initial state to some accepting state. A language is called regular iff it is accepted by some DFA.

1.6 Give state diagrams of DFAs accepting the following languages:

  f. [10 points] $\{w \in \{0, 1\}^* \mid w$ doesn't contain the substring 110$\}$

  j. [10 points] $\{w \in \{0, 1\}^* \mid w$ contains at least two 0s and at most one 1$\}$

1.37 [15 points] Let $C_n = \{x \mid x$ is a binary numeral that is a multiple of $n, \text{most-significant bit first}\}$. Show that for each $n \geq 1$, the language $C_n$ is regular. You may adopt the convention that the empty string $\varepsilon$ is in every $C_n$.

1.37 annex [5 points] Illustrate with a DFA for $C_6 = \{\varepsilon, 0, 110, 1100, 10010, 11000, 11110, \ldots\}$.

1.31 [15 points] For any string $w = w_1w_2 \cdots w_n$, the reverse of $w$, written $w^R$, is the string in reverse order, $w_n \cdots w_2w_1$. For any language $A$, define the reverse of $A$ as $A^R = \{w^R \mid w \in A\}$. In other words, $A^R$ is the set of reverses of strings in $A$. Show that if $A$ is accepted by some DFA, so is $A^R$. For this you may want to use the concept of an NFA (Non-Deterministic Finite-State Acceptor).

1.31 annex [5 points] Show an example of a DFA accepting $A^R$ where $A$ is the language of the DFA in 1.6f above, by using your construction in 1.31.

1.32 [20 points] Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$ 

$\Sigma_3$ contains all size 3 columns of 0s and 1s. A string of symbols in $\Sigma_3$ can be viewed as three rows of 0s and 1s. Consider each row to be a binary numeral MSB first, and let $B = \{ w \in \Sigma_3^* \mid \text{the bottom row is the sum of the first two rows} \}$
For example,

\[
\begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
1 \\
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0 \\
\end{bmatrix}
\in B, \quad \text{but} \quad \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
1 \\
\end{bmatrix}
\notin B.
\]

meaning that, in binary:

\[011 + 001 = 100, \text{ but } 01 + 00 = 11\]

Show that B is regular by presenting the state diagram of a DFA for it. (Hint: Working with \(B^R\) is easier, because addition is done LSB first. You may assume the result of Problem 1.31.)

1.57 [15 points] If A is any language, let firsthalves(A) be the set of all first halves of strings in A so that

\[\text{firsthalves}(A) = \{x| \text{ for some } y, |x| = |y| \text{ and } xy \in A\}.\]

Show that if A is regular, then so is firsthalves(A)

1.57 annex [5 points] Illustrate the use of your construction in 1.57 to derive a DFA for firsthalves(A), where A is the language of the DFA in 1.6j.