Assignment 9
Regular and Non-Regular Languages
Due Monday 14 April 2014 at 9:00 p.m.

1. [10 points] (Sipser 1.16b) Use the subset construction to convert the following NFA to an equivalent DFA.

```
1 -> (1, 2)
<table>
<thead>
<tr>
<th>a</th>
</tr>
</thead>
</table>
1     | a, b |
|      |
3     |      |
|      |
2 ->  |      |
|  ε  |
```

2. [20 points] (Sipser 1.19b) Construct a DFA that accepts the language represented by the regular expression ((00)*11 U 01)*.

3. [10 points] (Sipser 1.21) Derive a regular expression for the language of the DFA below using the method described in class (on Wed. 4/9).

```
1 -> (1, 2)
|  a, b |
|       |
1     |      |
|  a   |
2     |  a   |
| b    |
3     |      |
| b    |
```

4. [10 points] Given an algorithm for solving the following problem: Given two regular expressions over the same alphabet, is there at least one string common to both of their languages.

5. [15 points] (Sipser, 1.53)

Let $\Sigma = \{0, 1, +, =\}$. Show that the language $ADD$ over $\Sigma$ is not regular.

$ADD = \{x=y+z | x, y,\text{ and } z\text{ are binary numerals and } x\text{ is the sum of } y+z\}$.

(use the Myhill-Nerode theorem or the pumping lemma)
6. **[15 points]** If the following language is regular, derive a regular expression for it. If it is not regular, prove it.

\[ \{ x \in \{0, 1\}^* \mid \text{For every prefix } y \text{ of } x \mid \#_0(y) - \#_1(y) \mid \leq 1 \} \]

Here \( | | \) represents absolute value and \( \#_\sigma(y) \) means the number of occurrences of \( \sigma \) in \( y \).

7. **[20 points]** Show that the language

\[ PRIMES = \{ 1^p \mid p \text{ is prime} \} = \{ 11, 111, 1111, 11111, \ldots \} \]

is not regular.