CS181

Applied Logic and Automated Reasoning

Spring 2023

Instructor: Lucas Bang

Quick Introduction: About Me

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- Grew up in Las Vegas

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- UNLV: Math and CS

UNLV

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- UNLV: Math and CS
- In parallel:

UNLV MS Computer Science

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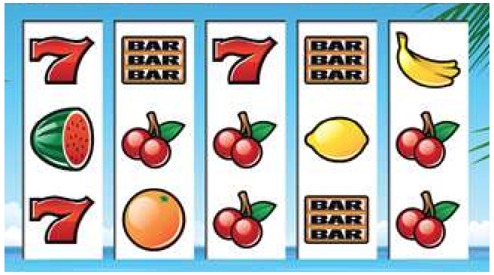

- UNLV: Math and CS
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  Test and Verification Engineer in Casino Gaming

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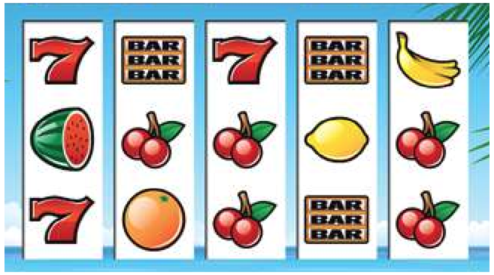
$E($  $) < 0$

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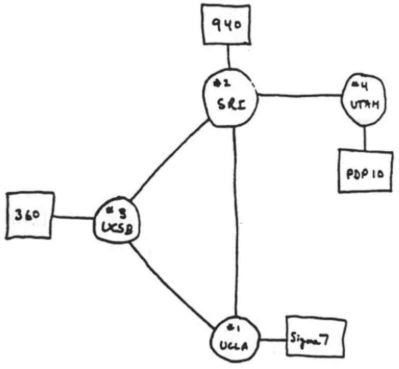

THE ARPA NETWORK

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4 NODES

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  Test and Verification Engineer in Casino Gaming

- PhD, UC Santa Barbara
- HMC Prof. since fall 2018

CS181u Website

Course information
Syllabus
Lecture slides
Assignments
Important links

www.cs.hmc.edu/∼bang/cs181u

Informal Poll

Piazza
Slack
Discord
Email
Something else?

What is this class about?

By the end of this class, I hope you have an appreciation for both how logic can be used to solve complex problems and how logic fits in the broader context of culture, history, and society.

But, what even is logic (or… what are logic**s**)?

What is **formal** logic?

What are some limitations of logic?

If controversies were to arise, there would be no more need of disputation between two philosophers than between two calculators. For it would suffice for them to take their pencils in their hands and to sit down at the abacus, and say to each other (and if they so wish also to a friend called to help): Let us calculate.
-Liebniz

It is a profoundly erroneous truism, repeated by all copy books and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them.
- Whitehead

The thematic center of this volume is the relationship between our informal assumptions about concepts such as difference, identity, and generality and our efforts to produce precise formal representations of these concepts.
- Hass and Falmange

The rest of today's class:

Example of modeling a system with logic.

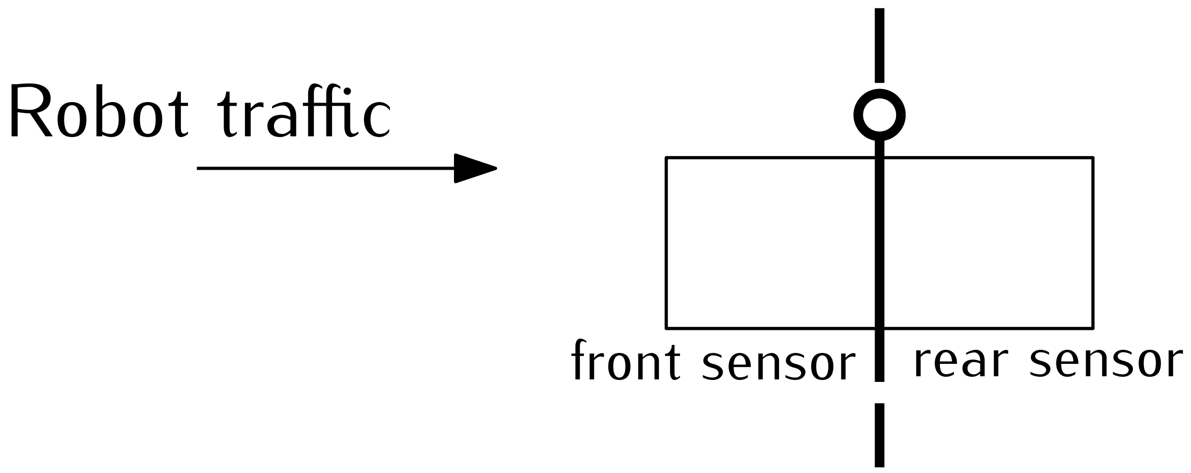
What logical properties might we care about?

Properties in temporal logic.

Using NuSMV to check properties.

Course technology and HW preview.

Example: an automatic door controller.



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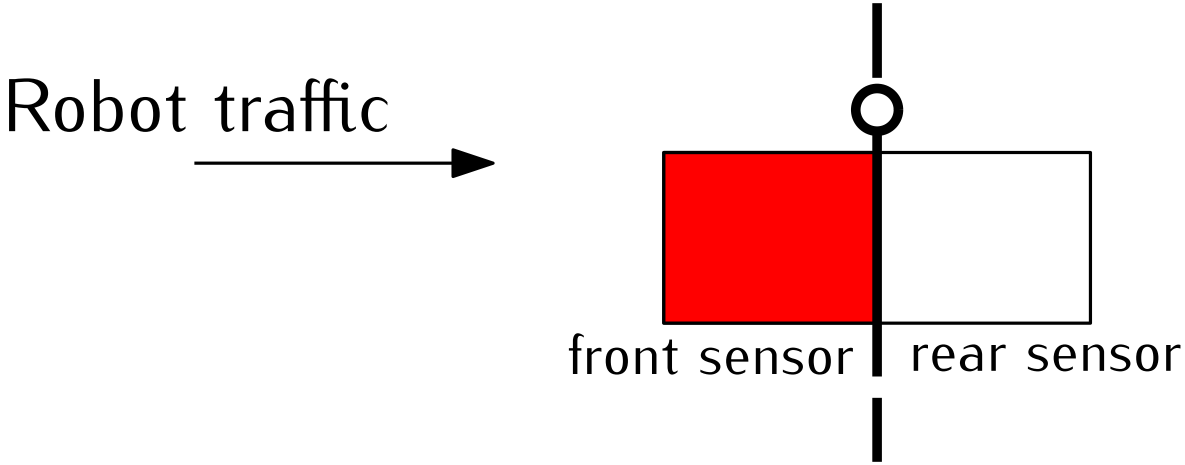
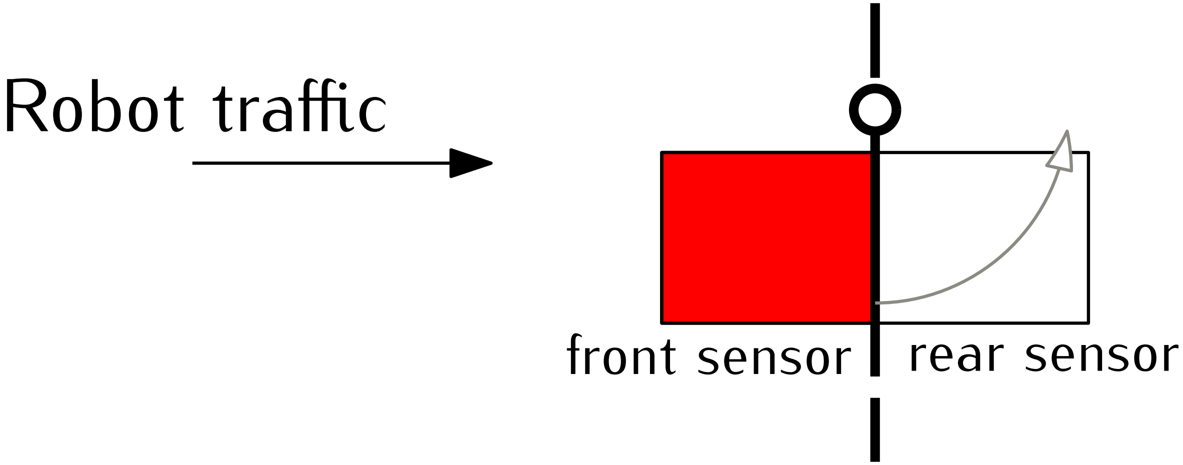


A robot can activate the front sensor.

Example: an automatic door controller.



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Activating the front sensor opens the door.

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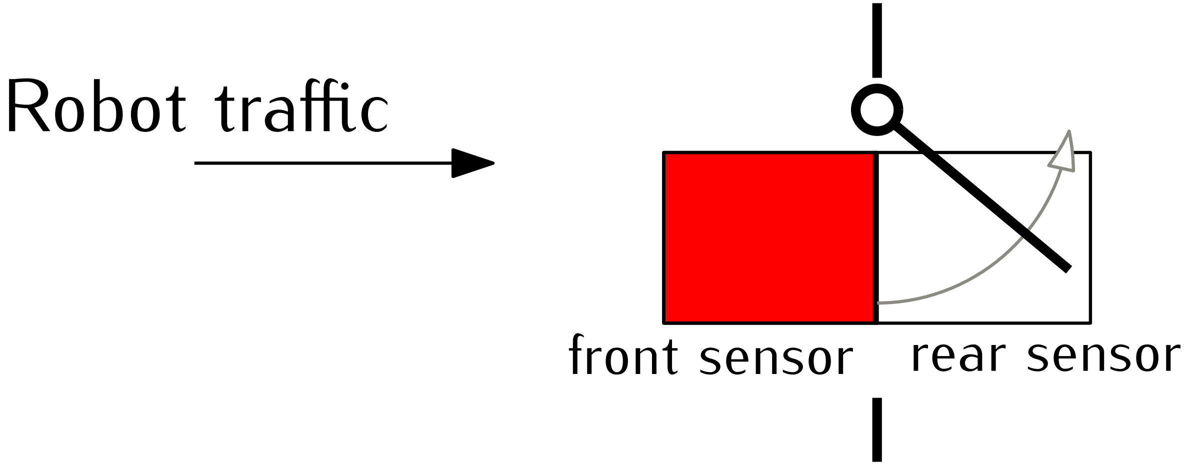

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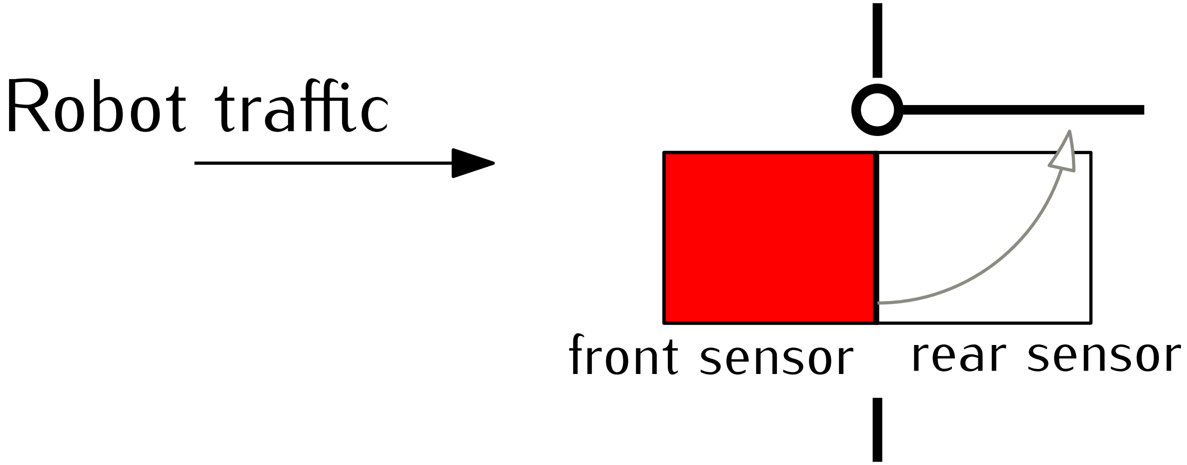

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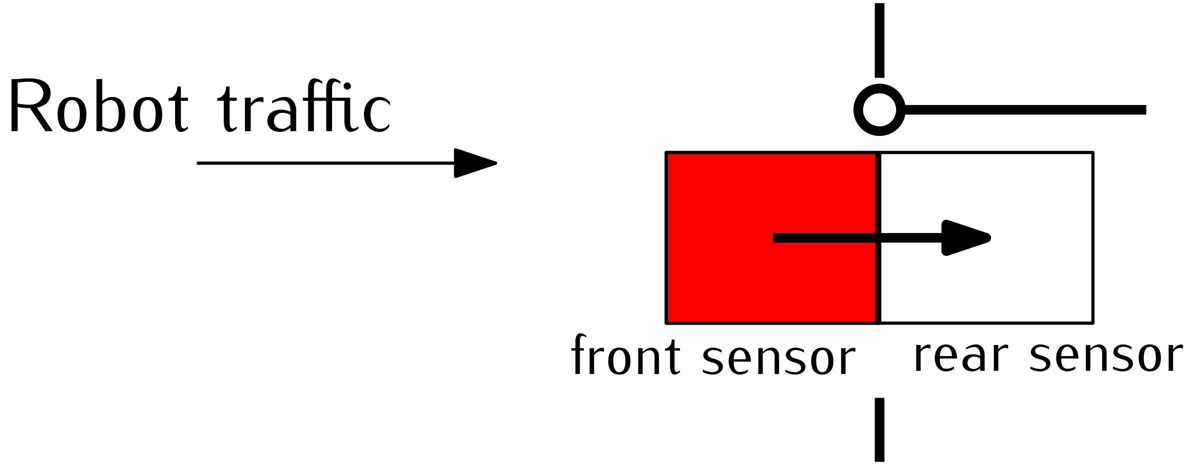

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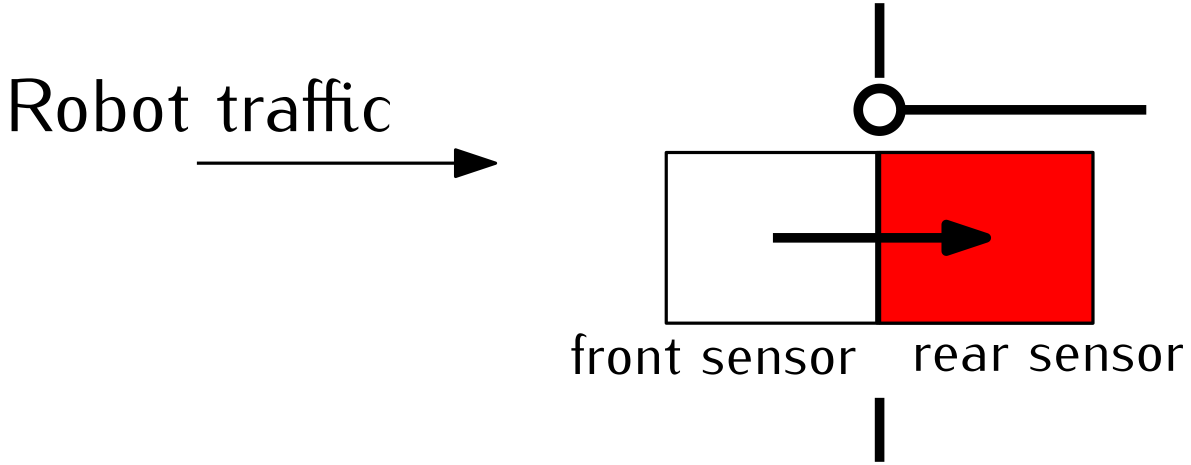
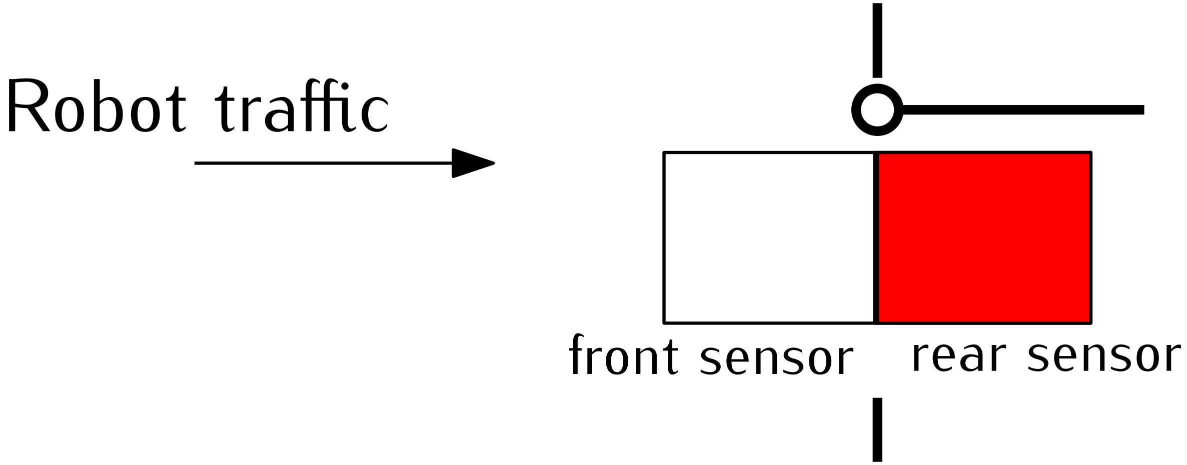


A robot can activate the front sensor.
Activating the front sensor opens the door.
Robot moves through door, activating rear sensor.

Example: an automatic door controller.



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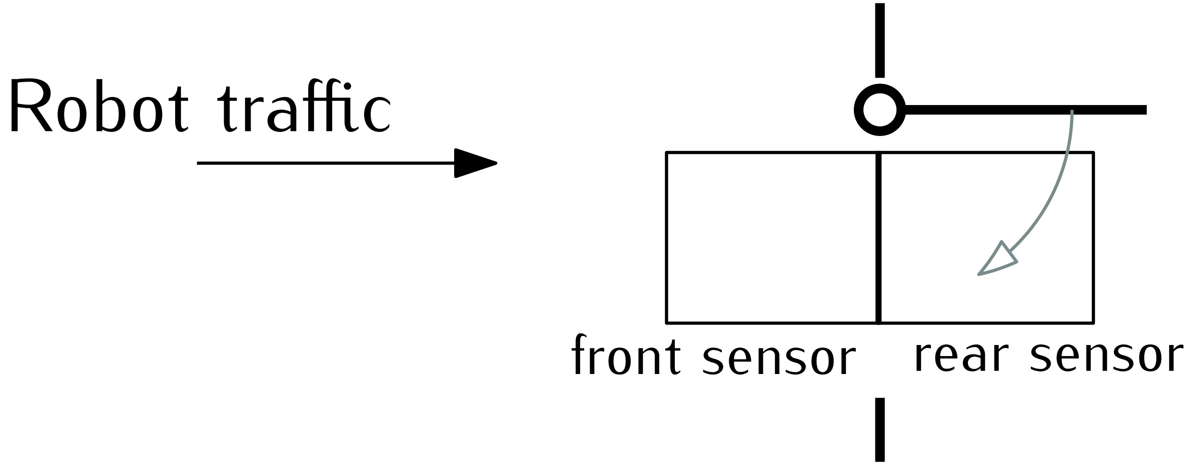
Activating the front sensor opens the door.
Robot moves through door, activating rear sensor.
The door stays open until the robot moves “out of the way”.

Example: an automatic door controller.



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The door closes.

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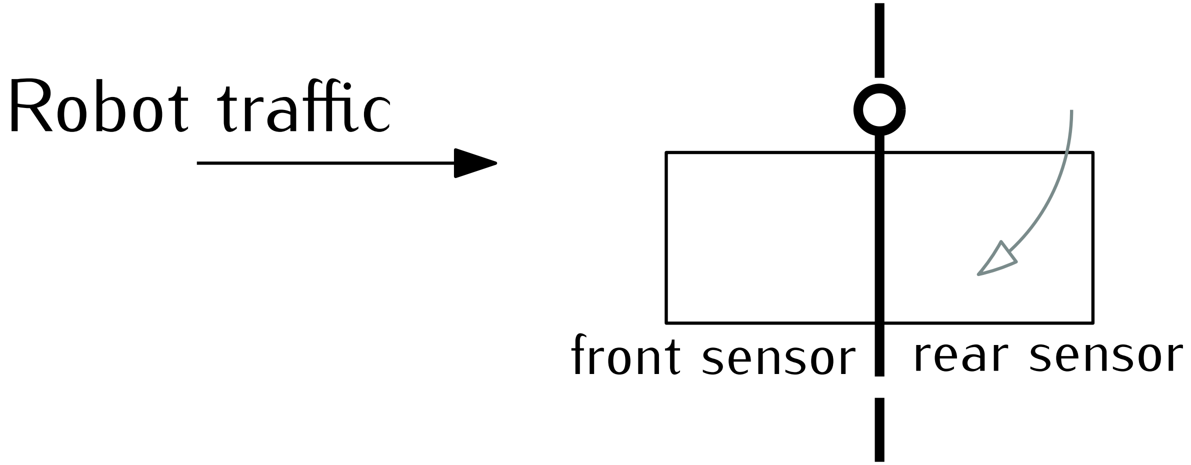

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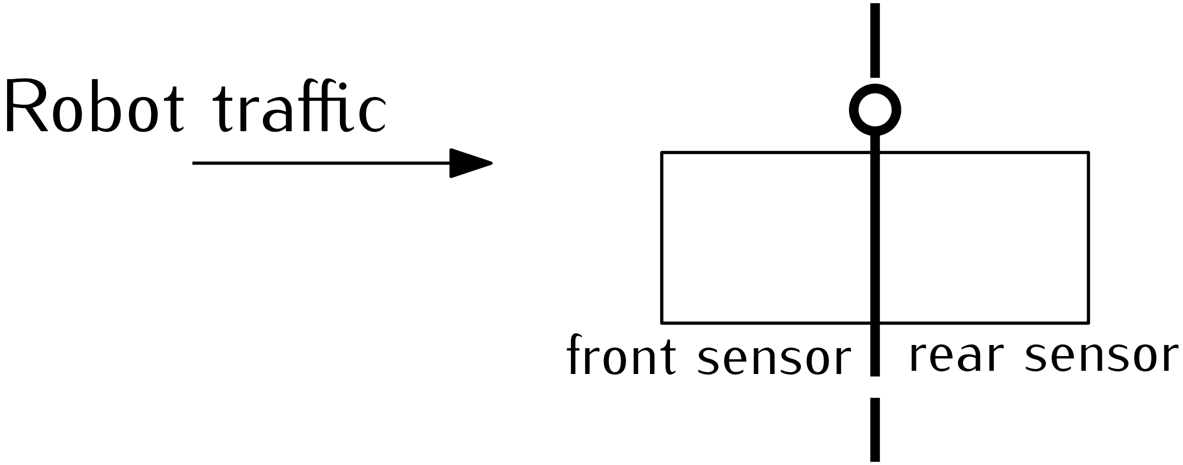
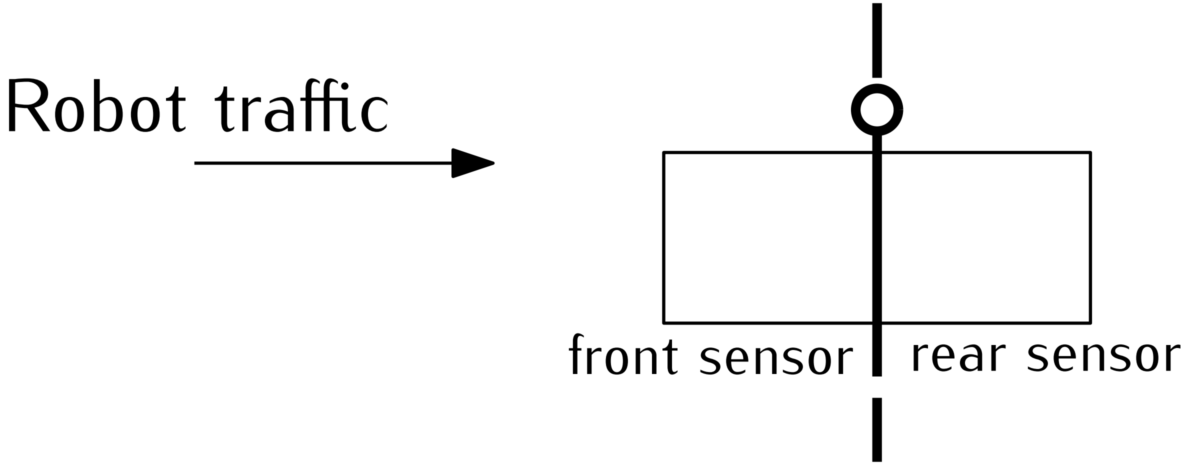


Let's encode our intuition with a transition diagram.

Example: an automatic door controller.



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closed
open

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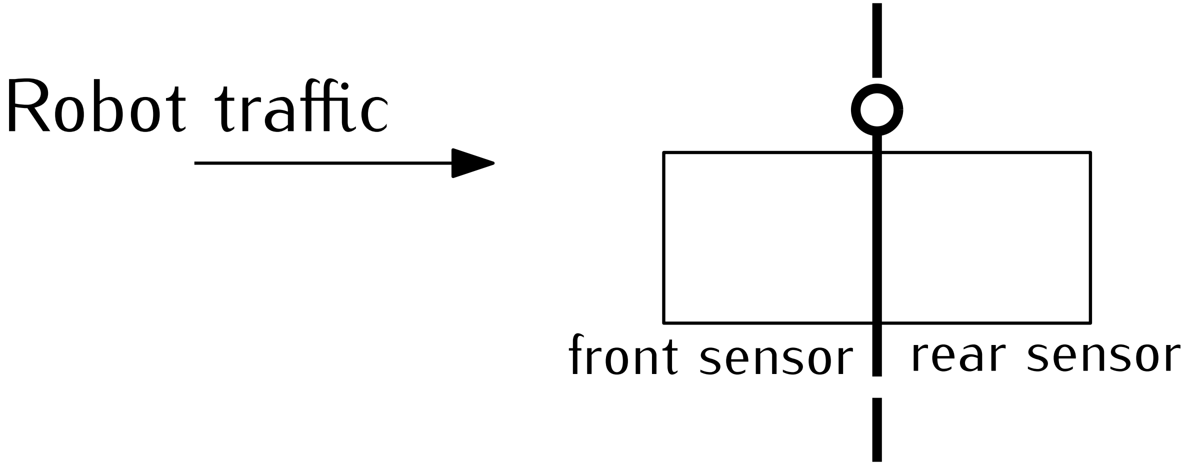

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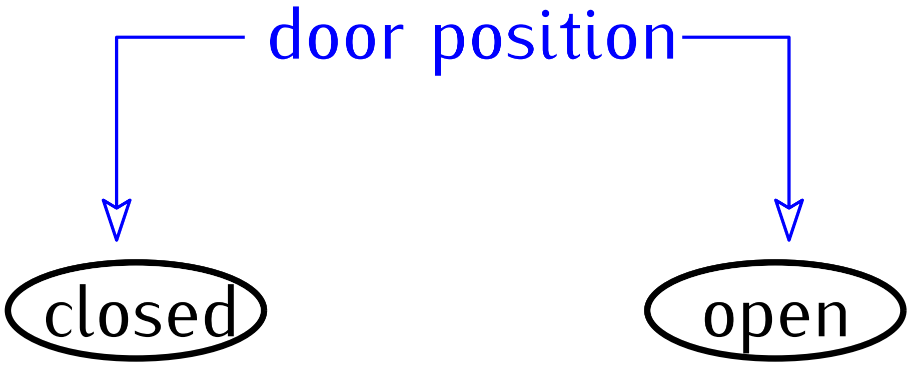

door position

closed

open

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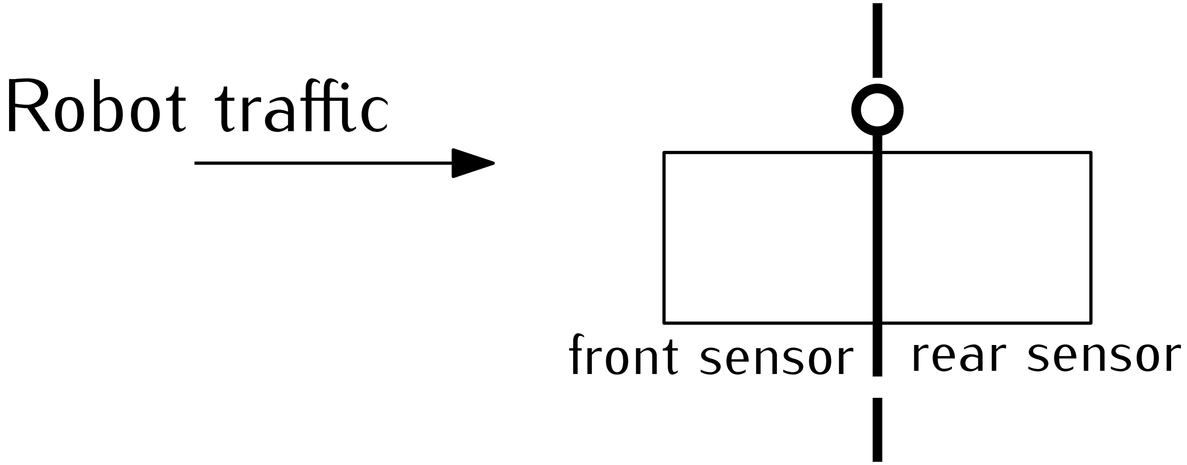

Robot traffic

front sensor

rear sensor

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open

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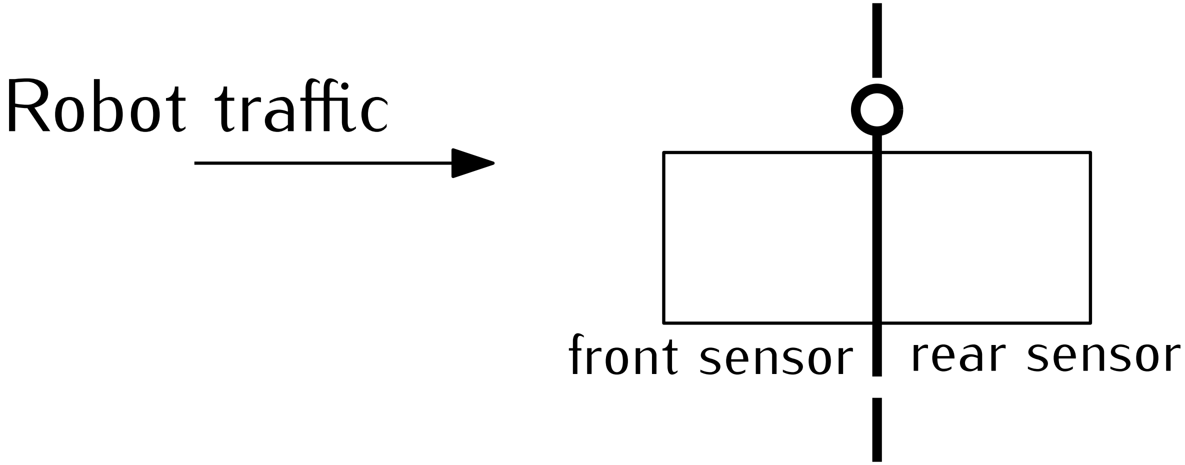


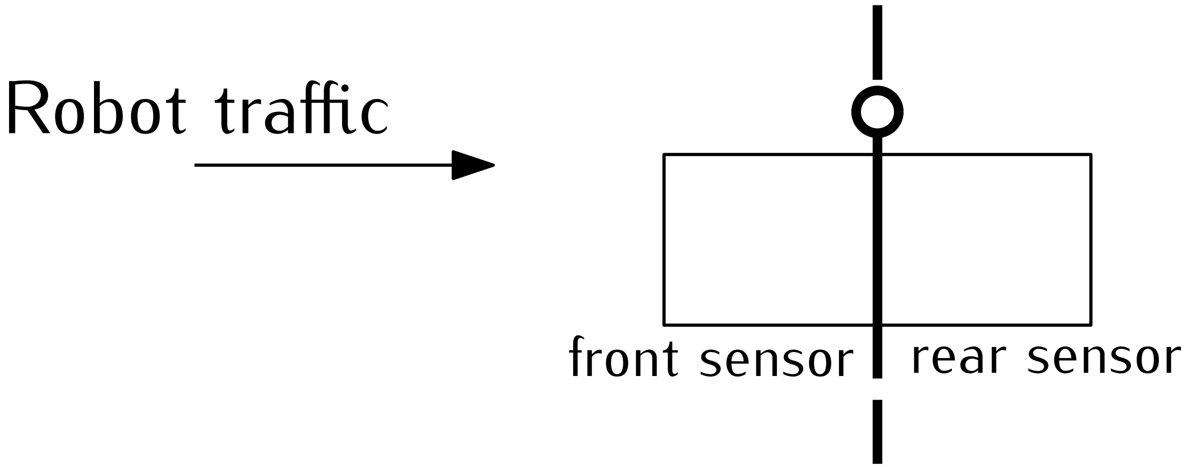
Let's encode our intuition with a transition diagram.

$f \equiv$ front sensor pad active

$r \equiv$ rear sensor pad active

closed
open

Example: an automatic door controller.



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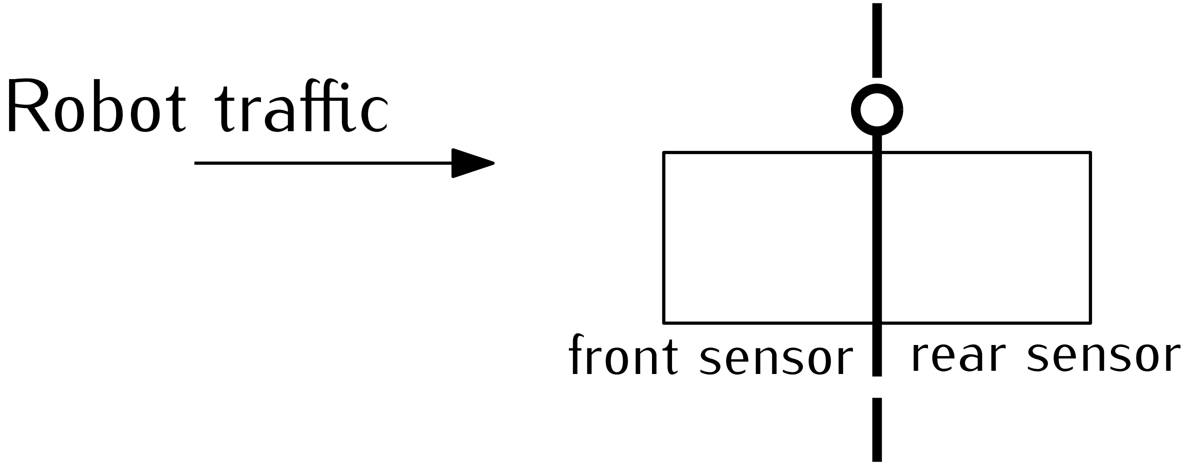

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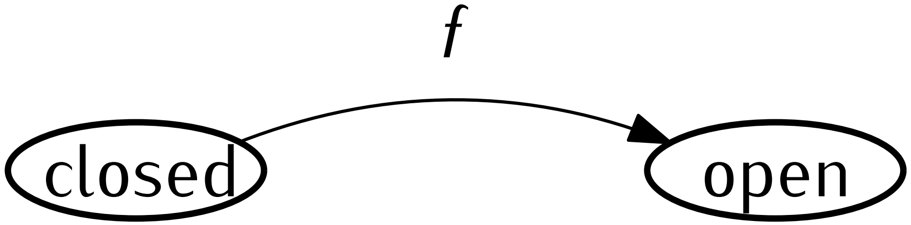

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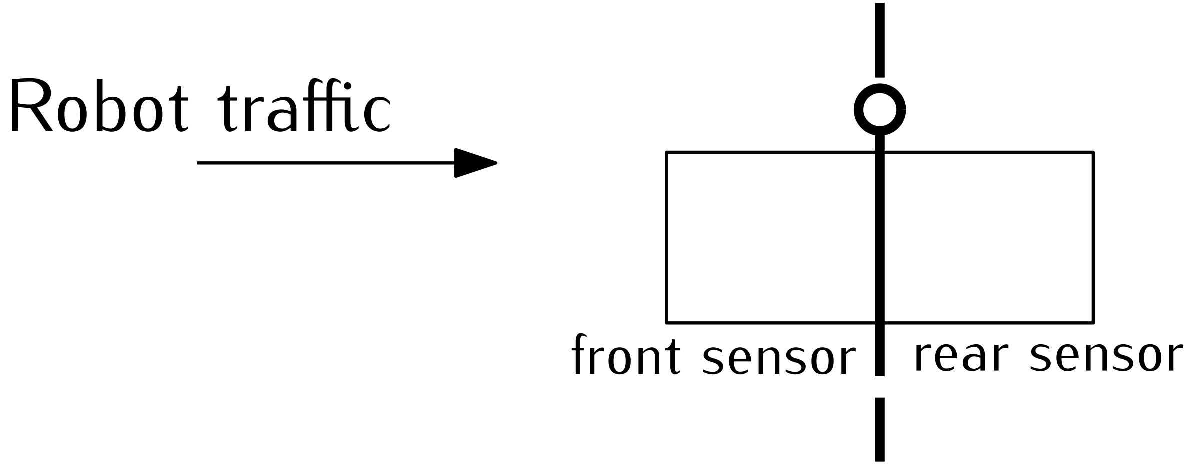

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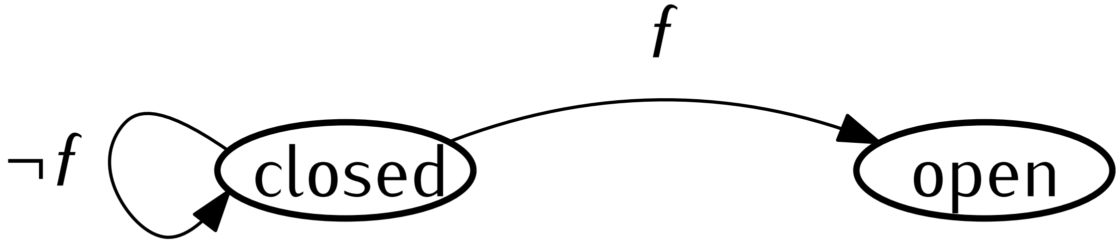

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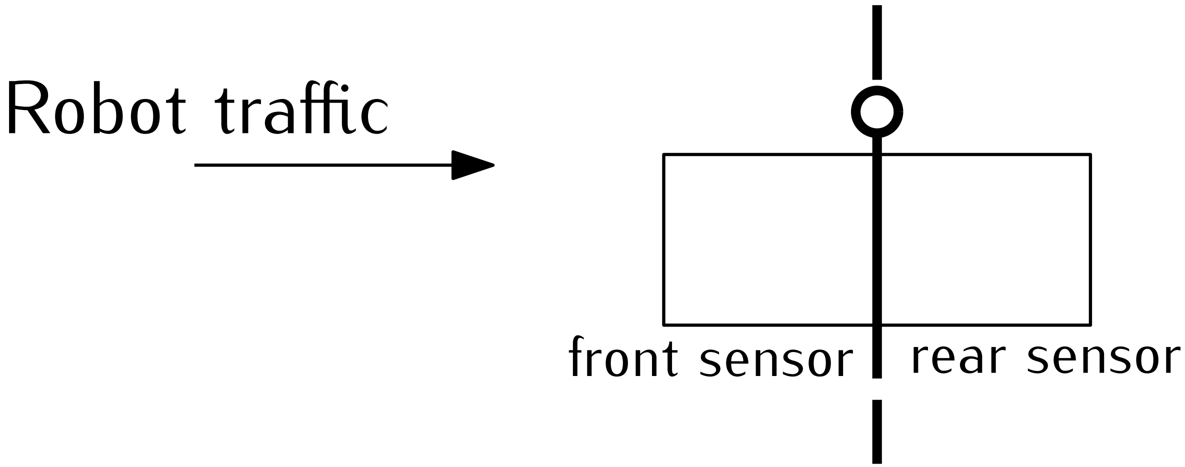

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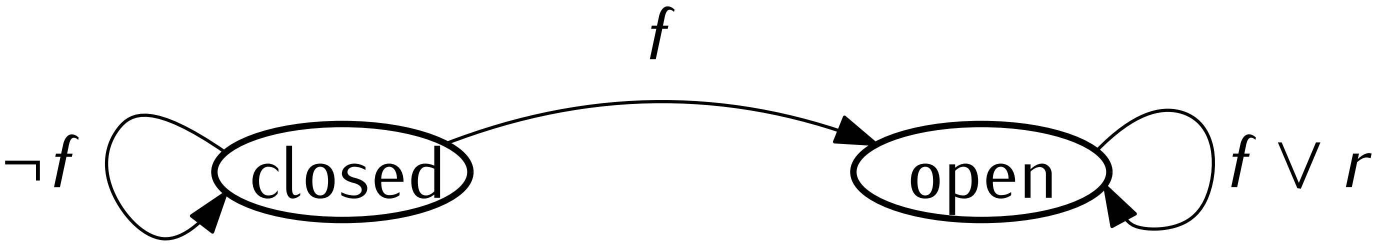

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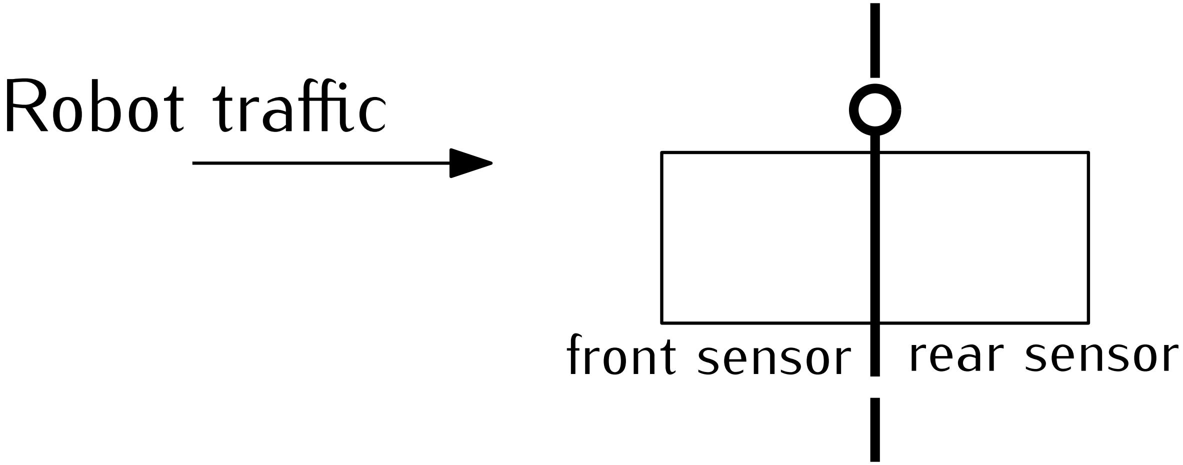

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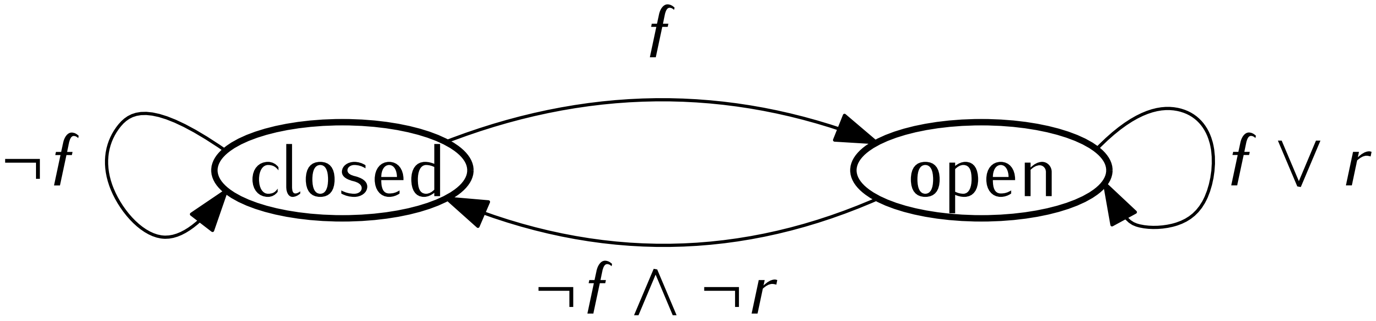

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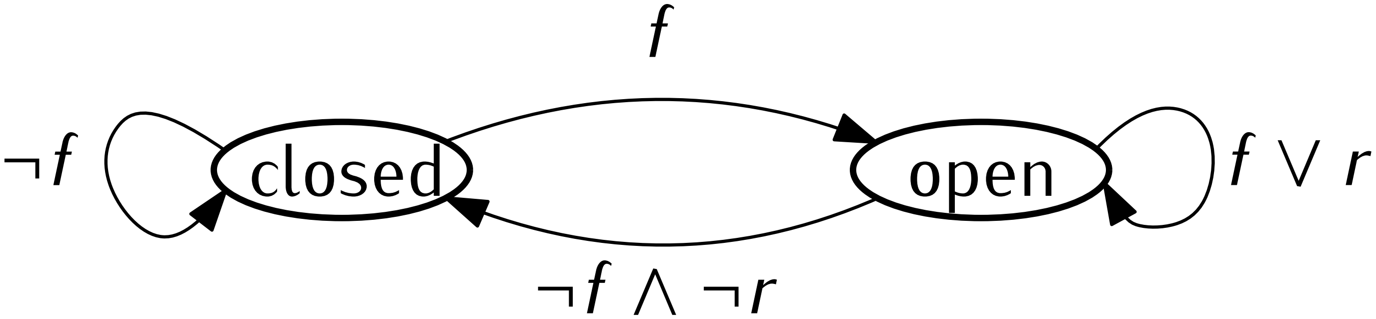

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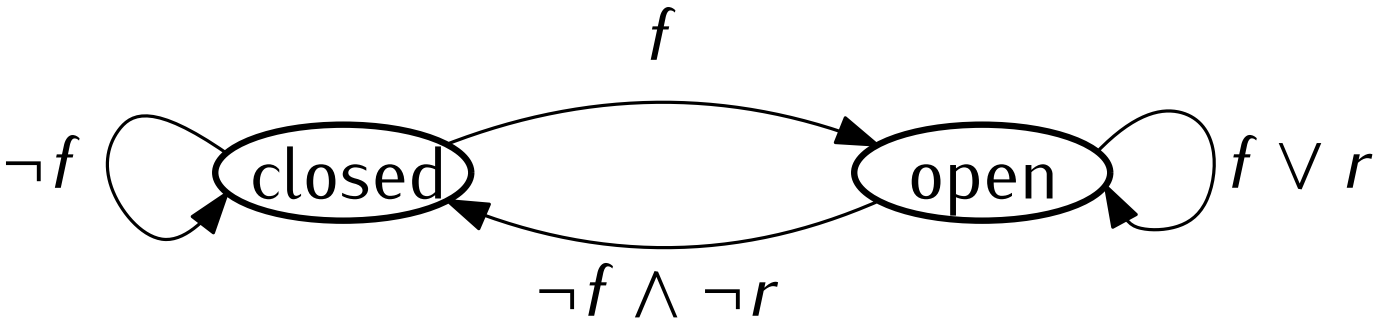
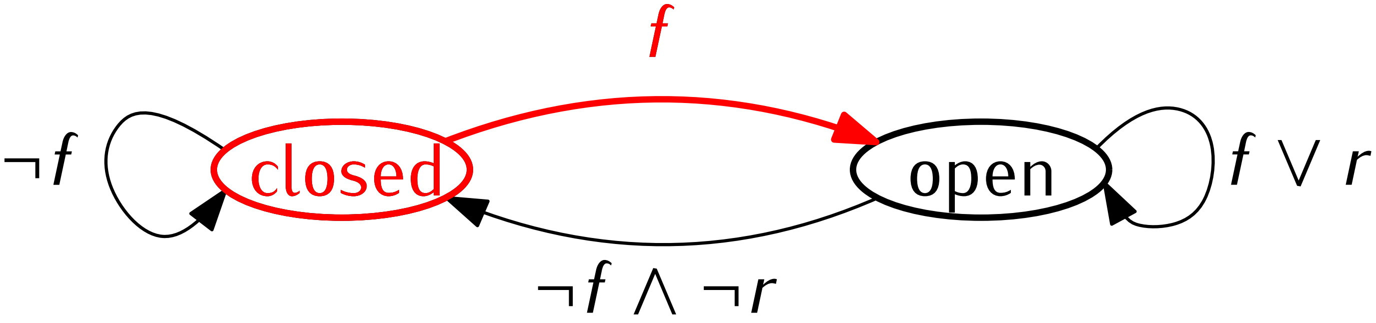


We can encode the same information in a transition table using propositional logic.

condition	next(door)

Example: an automatic door controller.



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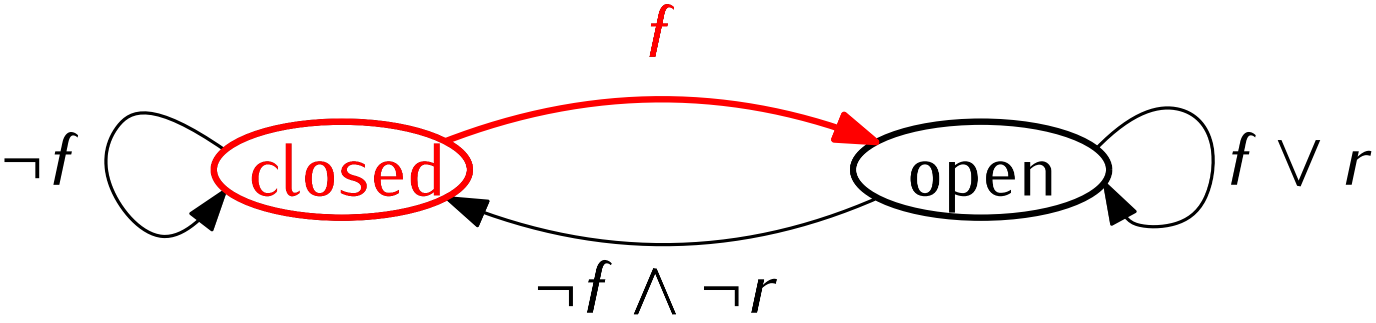

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condition	next(door)
$(door = closed) \wedge f$	$open$
$(door = closed) \wedge \neg f$	$closed$
$(door = open) \wedge (f \vee r)$	$open$
$(door = open) \wedge (\neg f \wedge \neg r)$	$closed$

Example: an automatic door controller.

We can further encode this in NuSMV (Symbolic Model Verifier).

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ASSIGN
  init(door)  := closed;
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Variable declarations

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Initialization

Example: an automatic door controller.

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Transition relation
(compare with table, previous slide)

Example: an automatic door controller.

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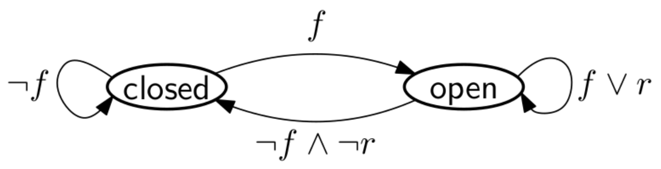
Review: The Modeling Process

Natural language description

If the robot activates front sensor then the door . . .

Unambiguous model expressed in math and logic



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A program that we can use to run or simulate our model

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What logical properties might we care about?

One important type of property

Liveness: eventually something “good” happens.

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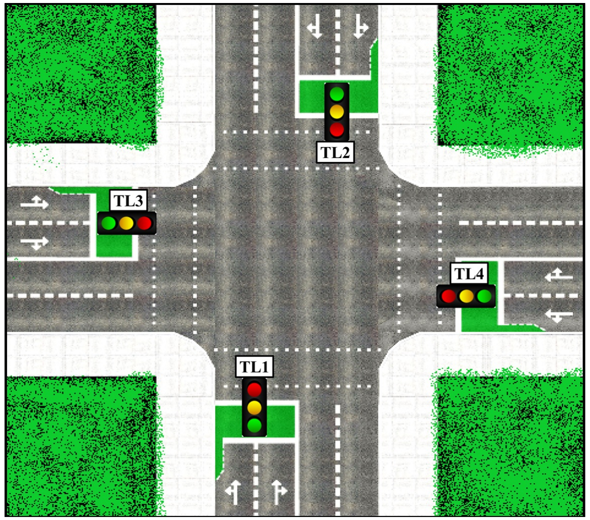

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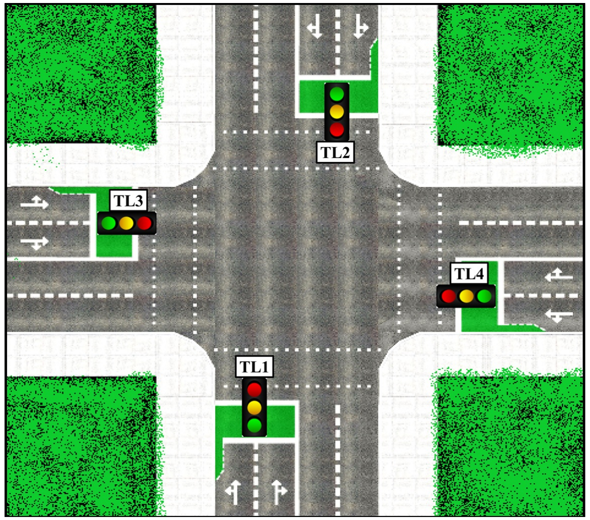

Liveness for traffic lights:
eventually one of the lights is green.

Another important type of property

Safety: a bad thing never happens.

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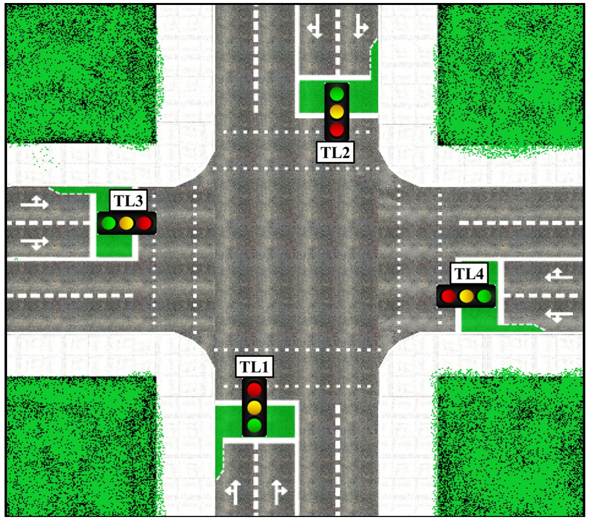

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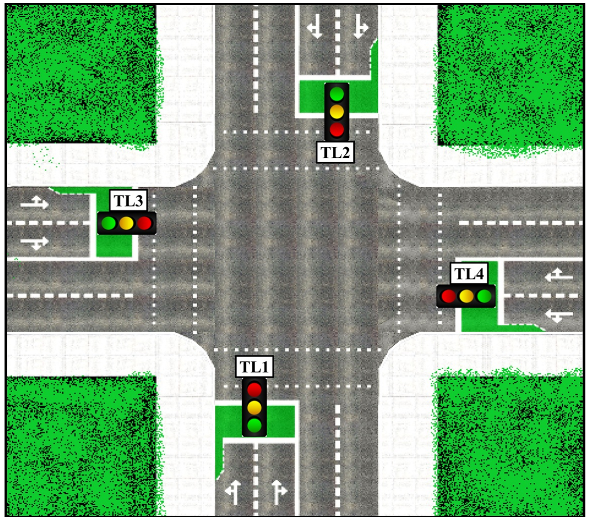
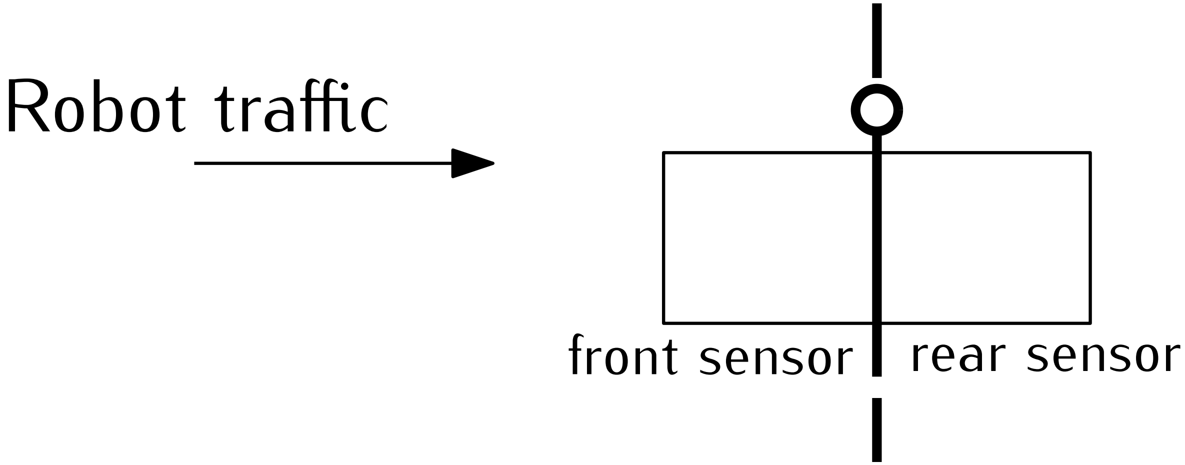


Safety: Any two perpendicular lanes never have corresponding lights that are green at the same time.

Short Break

Example: an automatic door controller.



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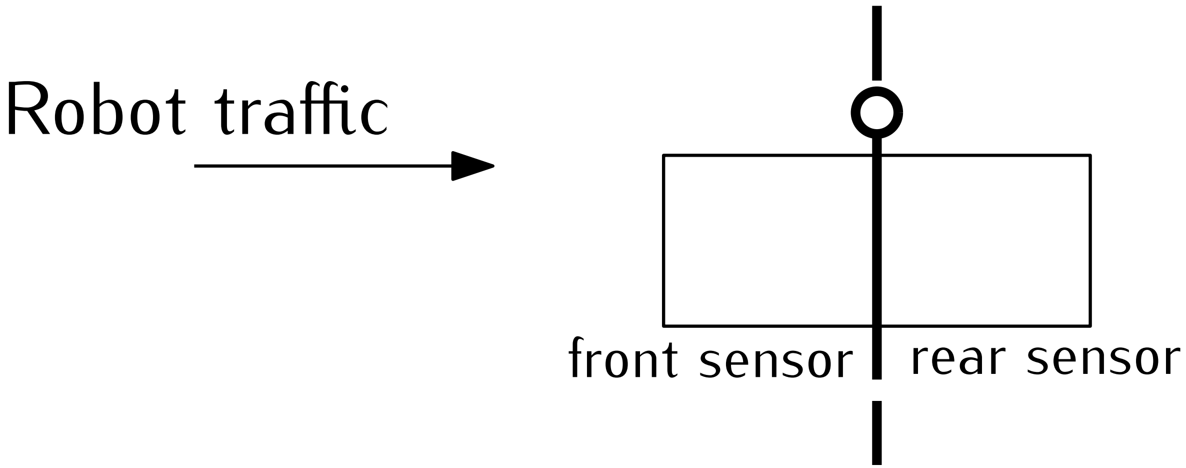

A Liveness Requirement: It is always the case that if the front pad is activated then eventually the door will be open.

Liveness: eventually something “good” happens.

Safety: a bad thing never happens.

Example: an automatic door controller.

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$$G(front \rightarrow F(door = open))$$

Linear Temporal Logic

Example: an automatic door controller.

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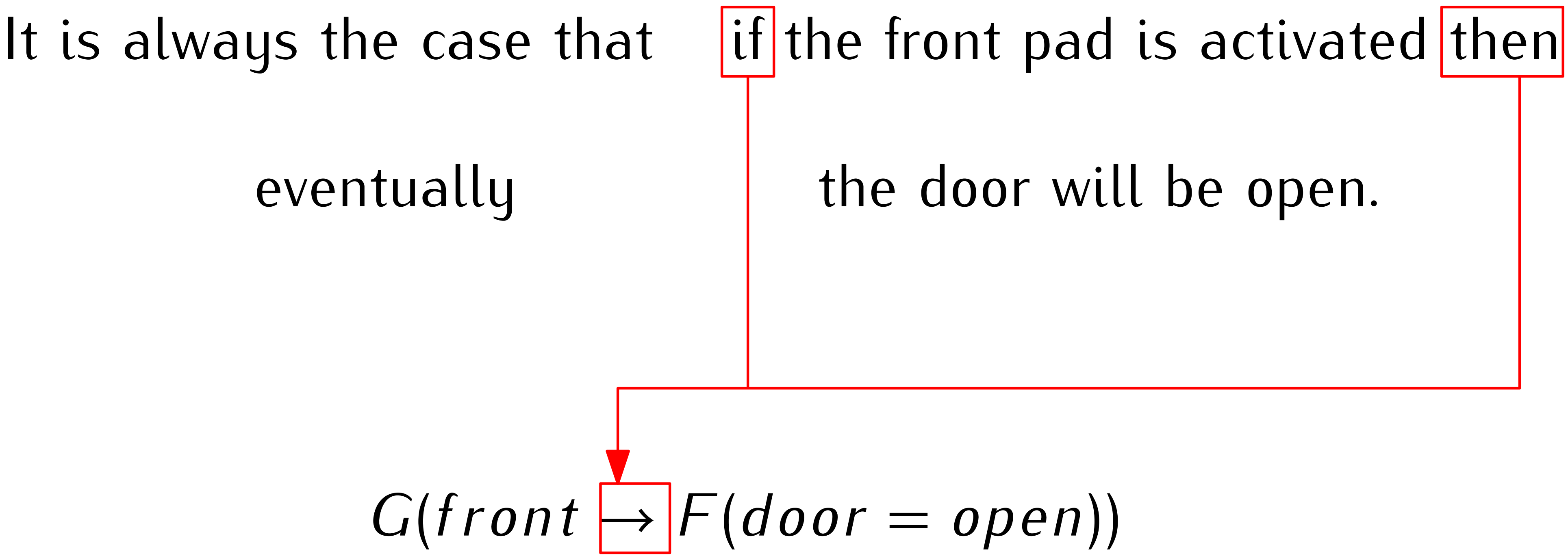

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We can check this property with NuSMV!

Example: an automatic door controller.

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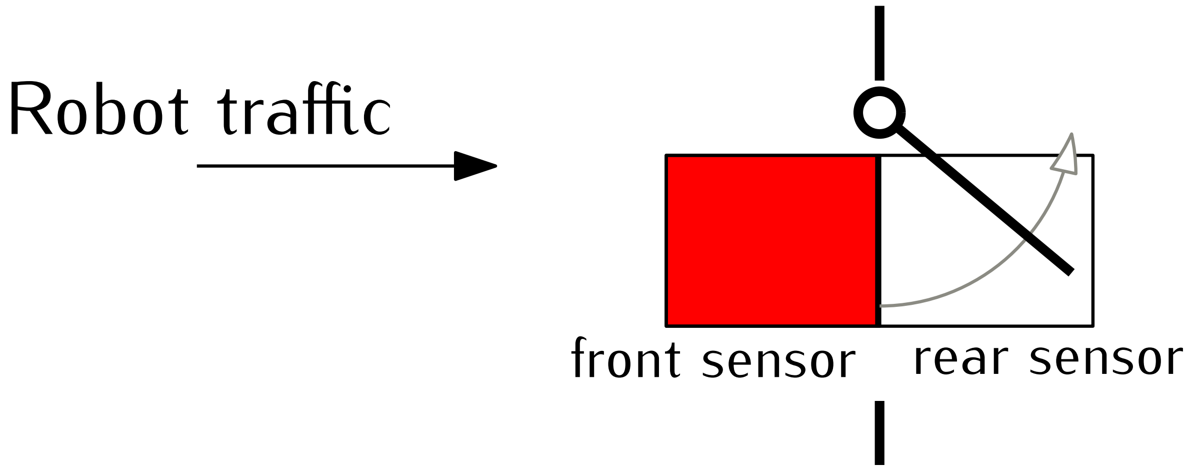
Let's also check this property with NuSMV …

Things to consider...



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What's missing?

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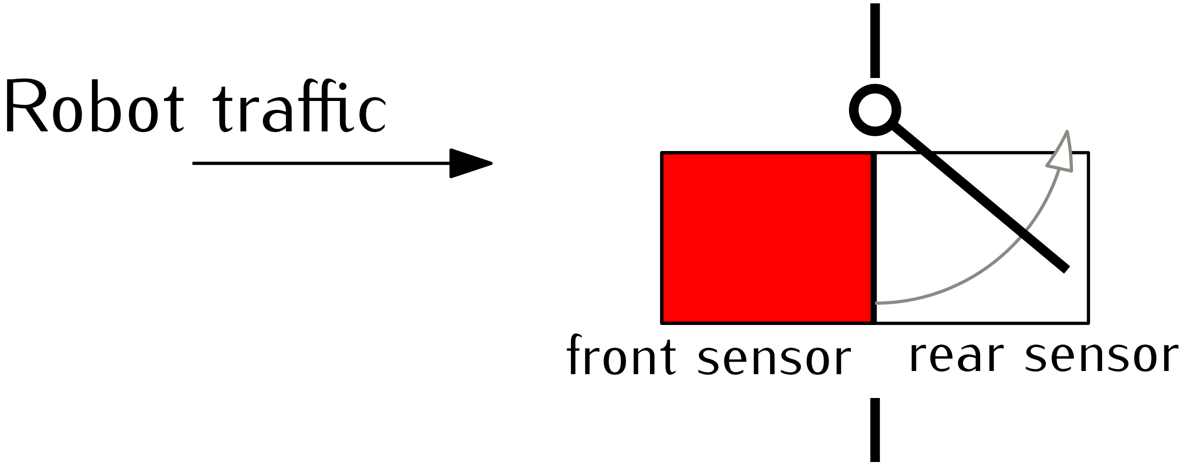

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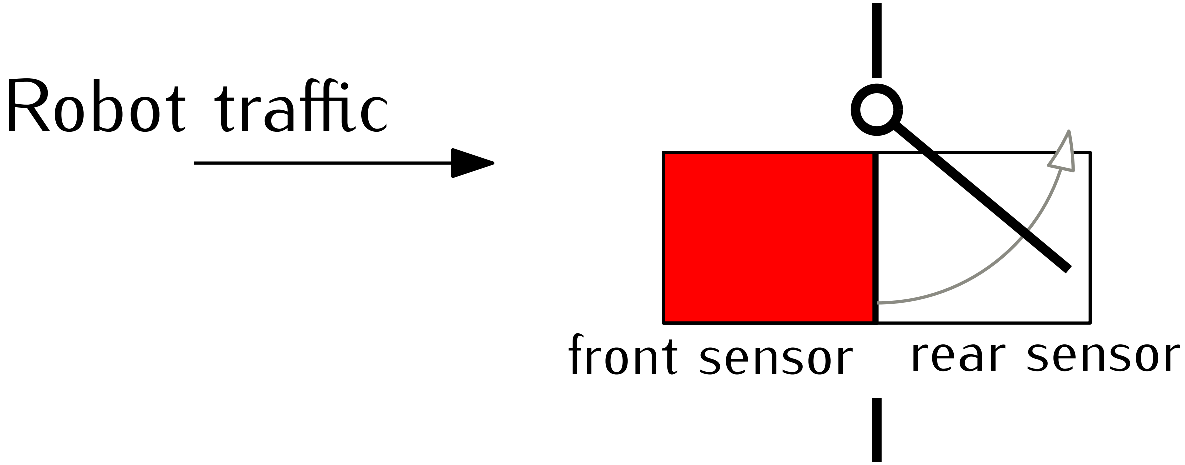

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Do we need to model the robot behavior and the sensor state??

$$next(robotPosition) := \ldots,\ next(frontSensor) := \ldots \text{ ??}$$

A common sentiment:

“I thought I knew how my [program , proof , simulation, model] worked until I ran [NuSMV, Z3, SPIN, JPF, Alloy, etc.] on it!”

Learning automated resoning techniques forces you to think *very carefully* about what you are doing, and often exposes subtle misunderstandings.

First Few Weeks:

Propositional Logic

A python-based domain specific language for propostional logic, satisfiability checking, model counting, and data structures for logic (BDDs).

Middle part of the class:

Transition Systems

We will learn a formal system of specifying transition systems (which we often depict as a transition diagram).

Temporal Logic (LTL)

We will assign symbols for expressing temporal system requirements like *always*, *eventually*, *next*, *until*.

Temporal Logic Software

Symbolic Model Verifier (NuSMV)

Later Weeks:

Automated Theorem Proving

We will use Z3 to help us automatically prove things,
e.g. a python program doesn't have assertion violations

or give us counterexamples
e.g. inputs that cause an assertion violation

Finally:

Presentation about a logic or automated reasoning:

tool or software
theory or foundation
cultural, social, historical, cognitive, linguistic context

Next Class

Human reasoning and logic

Propositional logic

First HW

Assignment how-to: writing, coding, submitting