CS 181U Applied Logic

Lecture 9

Computation Tree Logic

*v*SMV peculiarities

The `process` keyword will be deprecated.
Ignore the warning.

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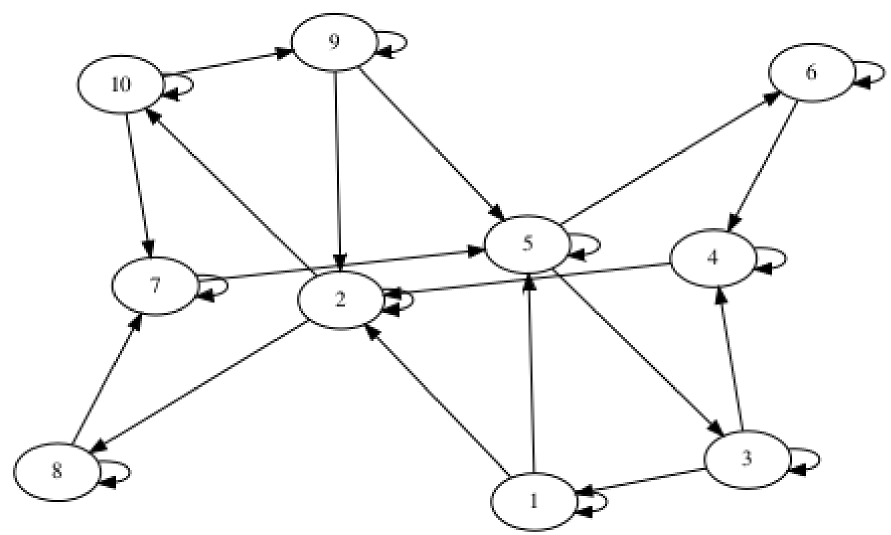
MODULE `main`, and processes `P0` and `P1` are three separate threads. `P0` and `P1` are subthreads of `main`.

```
MODULE proc(id, ...)
    ...
MODULE main
    ...
p0 = proc(0, ...)
p1 = proc(1, ...)
    ...
```



Use `FAIRNESS running` in `proc` specification.

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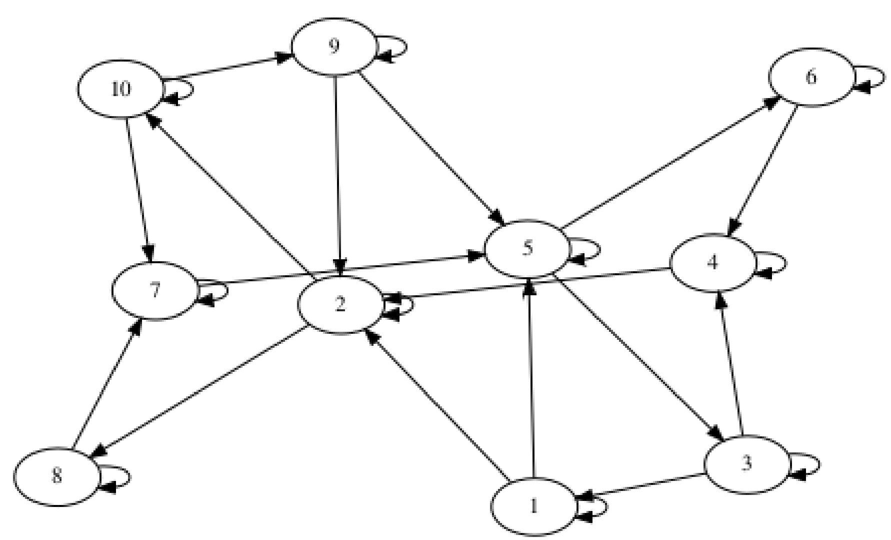

Use `FAIRNESS running` in `proc` specification.

Interesting Quote

If what is exactly stated can be done by a machine, the residue of the uniquely human becomes coextensive with the linguistic qualities that interfere with precise specification—ambiguity, metaphoric play, multiple encoding, and allusive exchanges between one symbol system and another. The uniqueness of human behavior thus becomes assimilated to the ineffability of language, and the common ground that humans and machines share is identified with the univocality of an instrumental language that has banished ambiguity from its lexicon.

-N. Katherine Hayles

How we Became Posthuman: Virtual Bodies in Cybernetics, Literature, and Informatics

Reminder

Linear Temporal Logic (LTL)

We will assign symbols for expressing temporal system requirements like *always* (G), *eventually* (F), *next* (X), *until* (U), and a few more. We will give a formal and unambiguous semantics to these symbols.

Transition Systems

We will learn a formal system of specifying transition systems (which we often depict as a transition diagram).

Concurrency Concepts

Safety, liveness, mutual exclusion, ...

Verification Software

Symbolic Model Verifier (NuSMV)

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We did all this.

Next

Computation Tree Logic (CTL)

We will learn a different way to write temporal properties of systems.

Verification Software + CTL

Symbolic Model Verifier (NuSMV) with CTL

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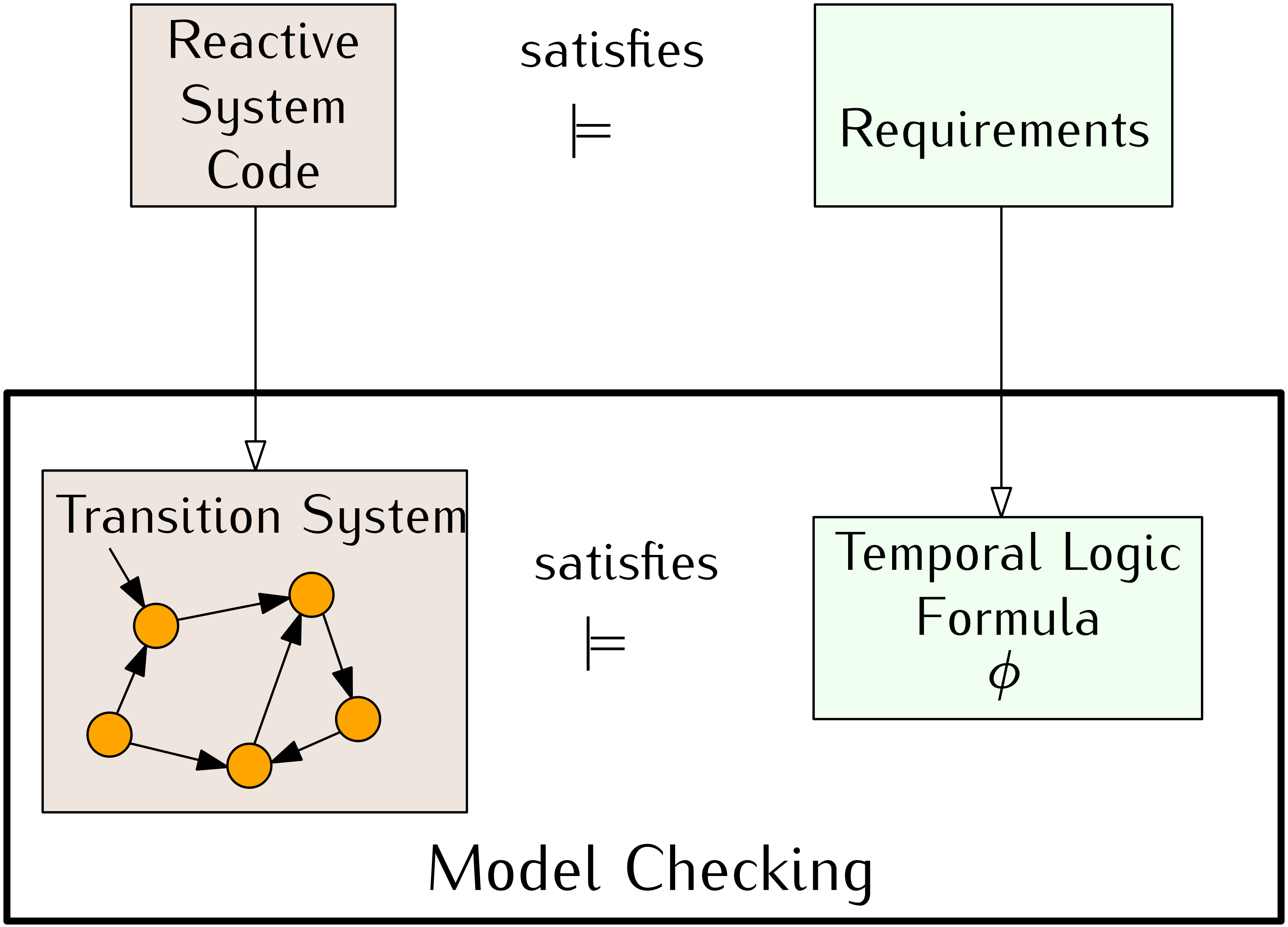
Today

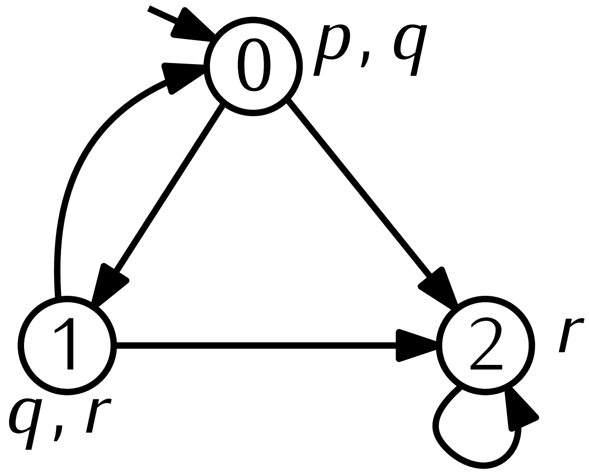
Verification Software + CTL

Symbolic Model Verifier (NuSMV) with CTL

Remember the big picture



Linear vs Branching Time Logic



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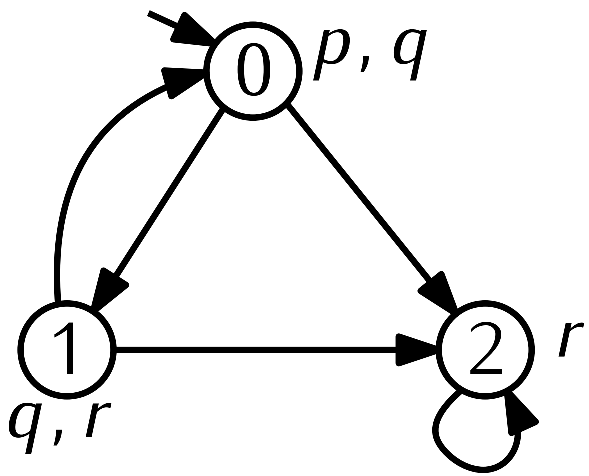
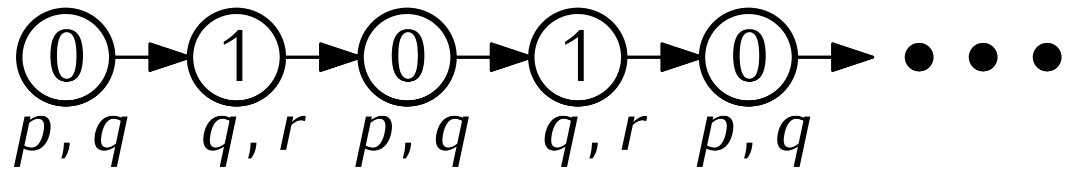


Some paths of $\mathcal{M}$



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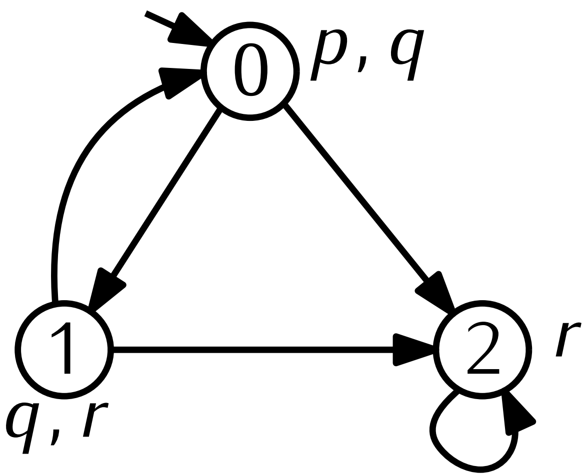

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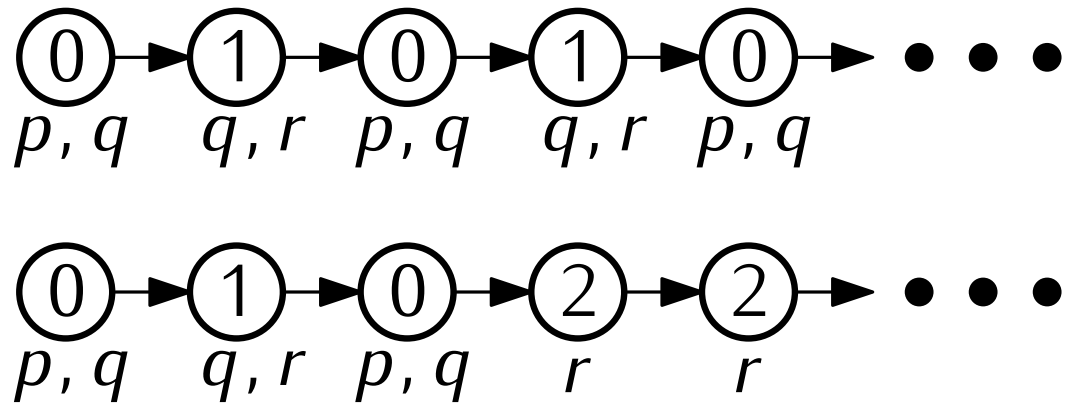

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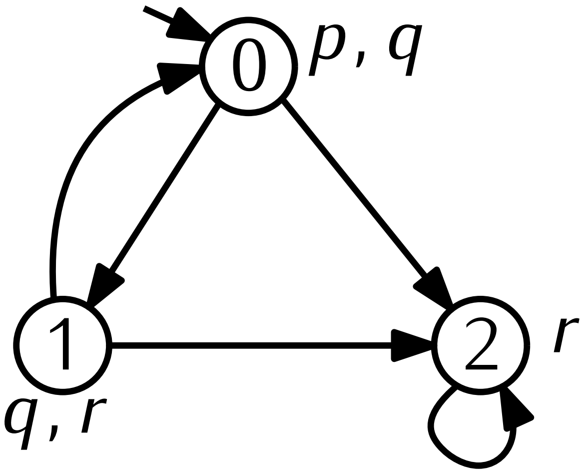

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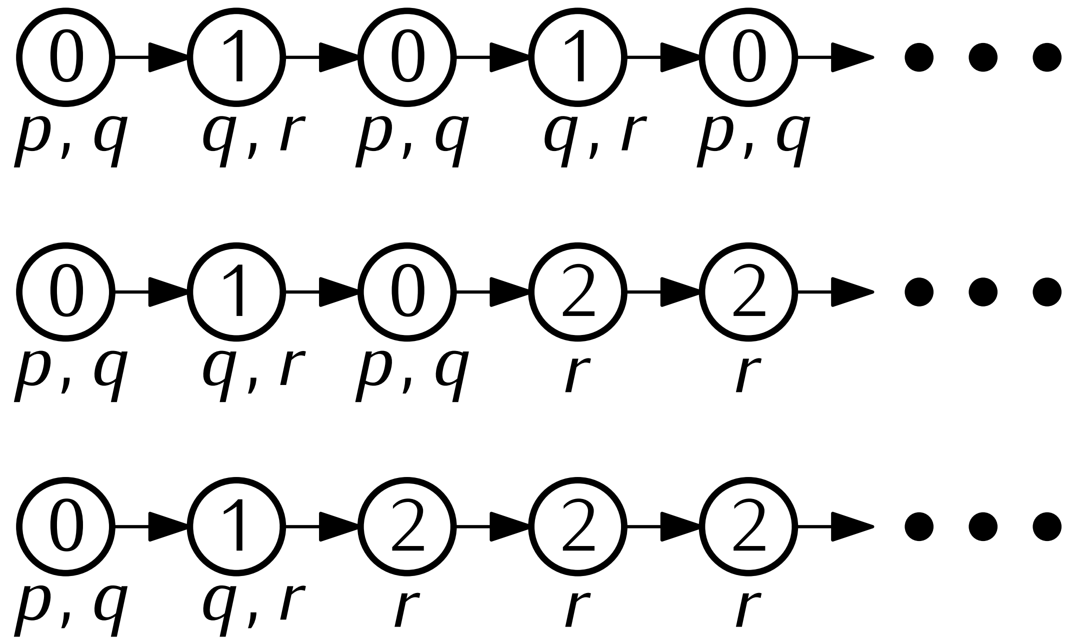

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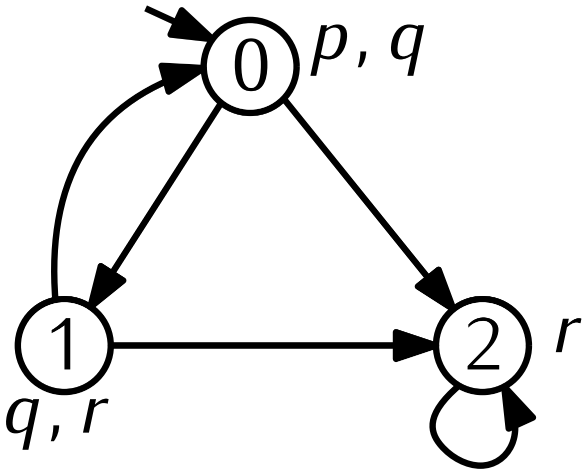

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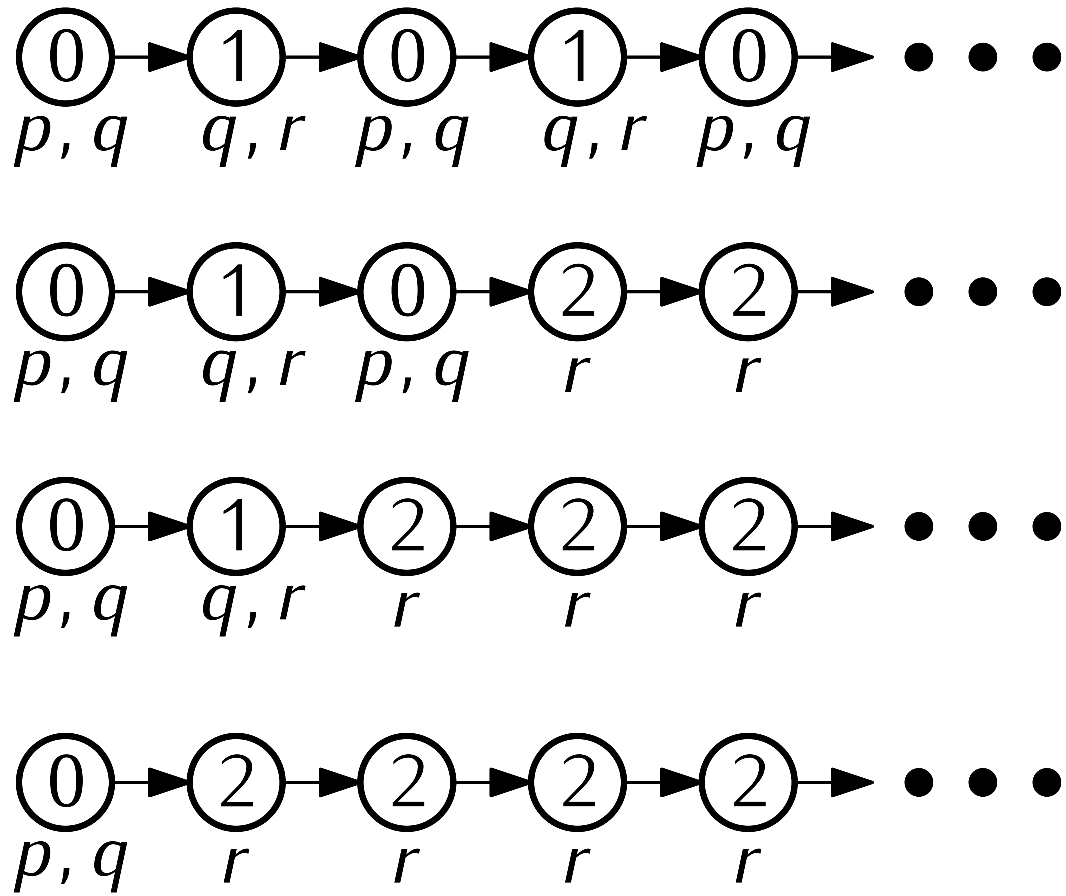

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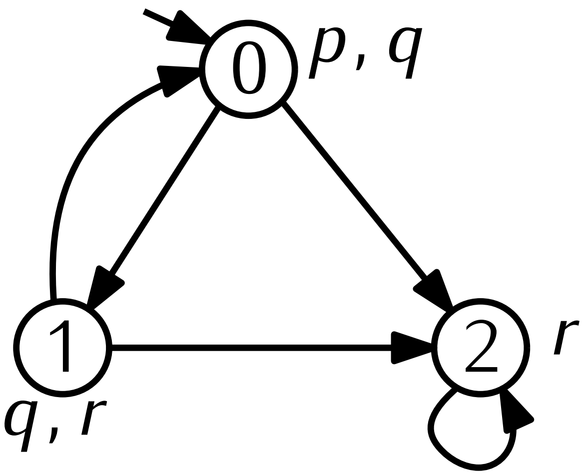

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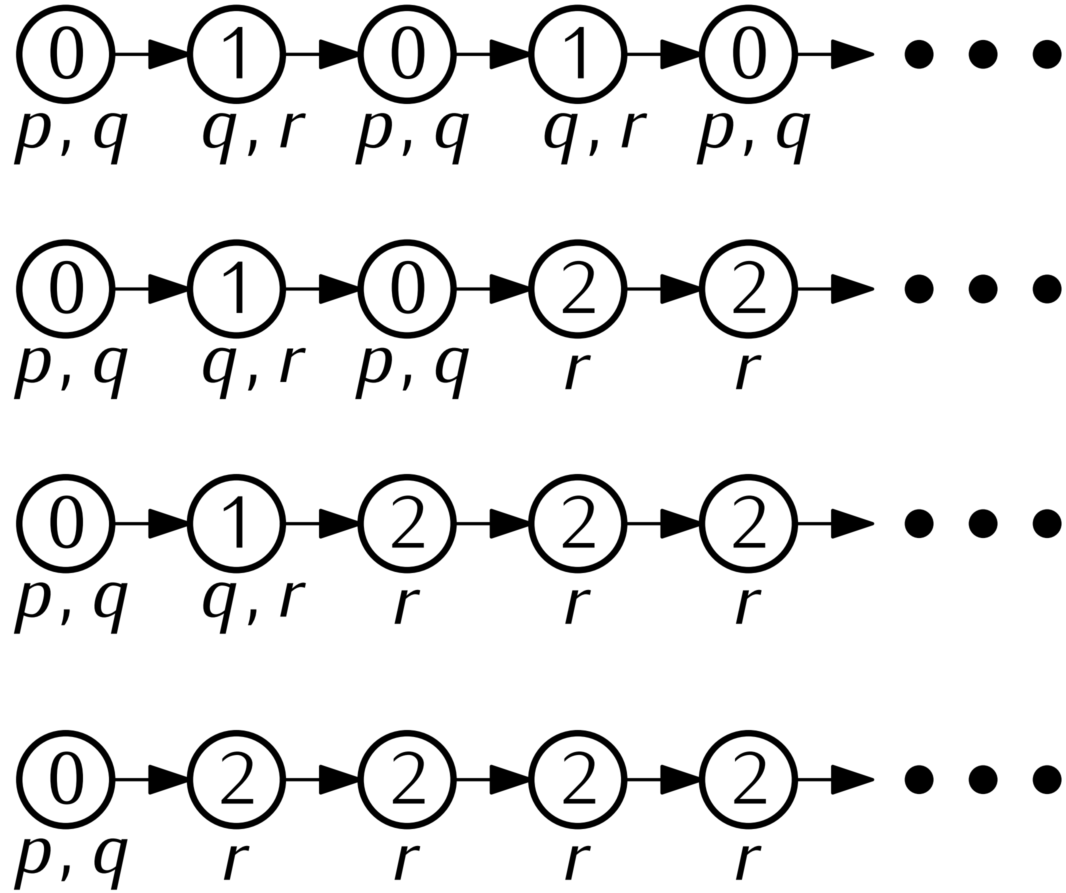
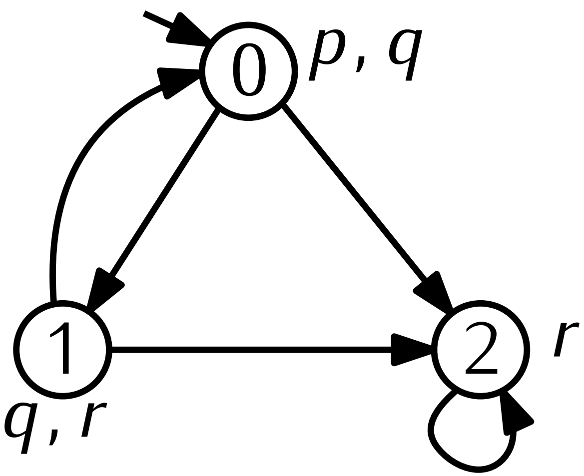


LTL Model
Checking

Linear vs Branching Time Logic



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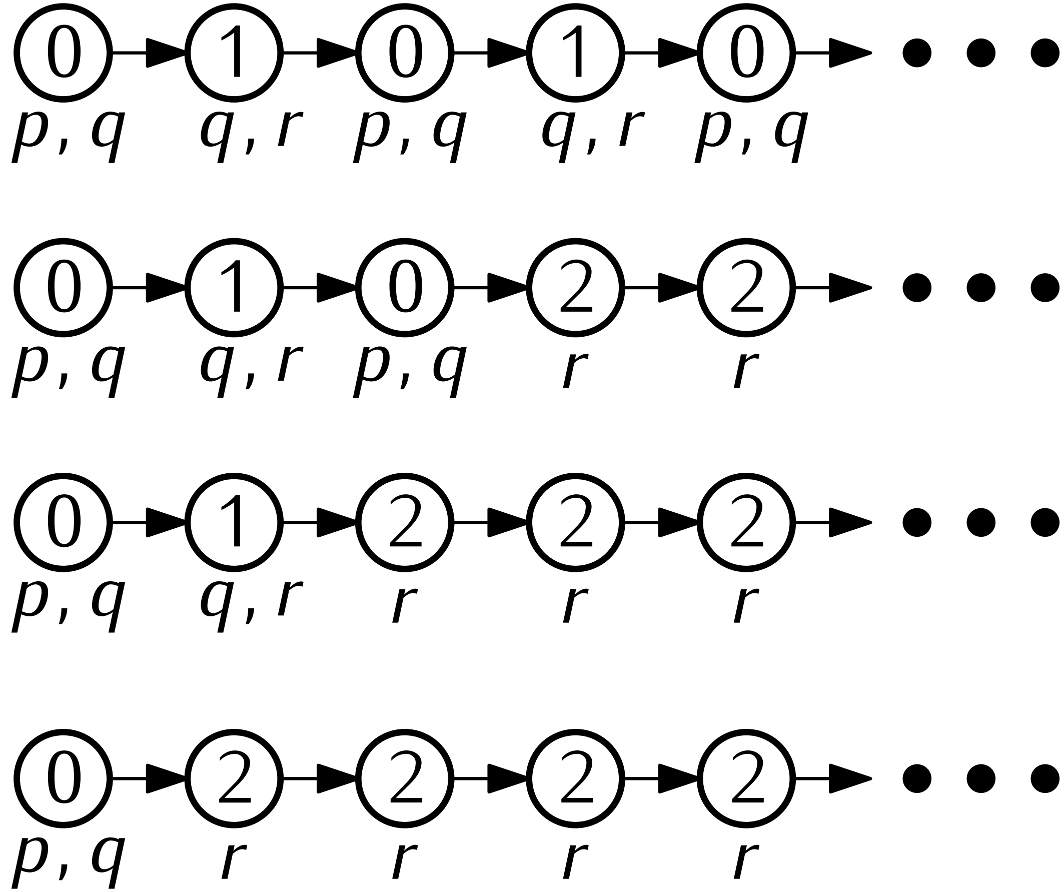
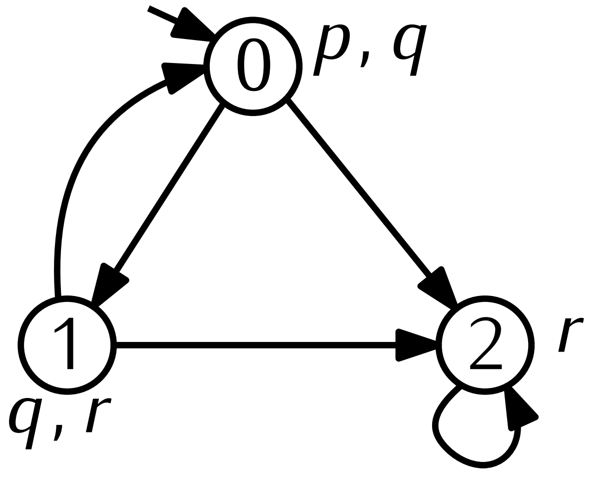


LTL Model Checking

$$\mathcal{M} \models \phi \Leftrightarrow \forall \pi \; [\pi \models \phi]$$

LTL formula

Linear vs Branching Time Logic



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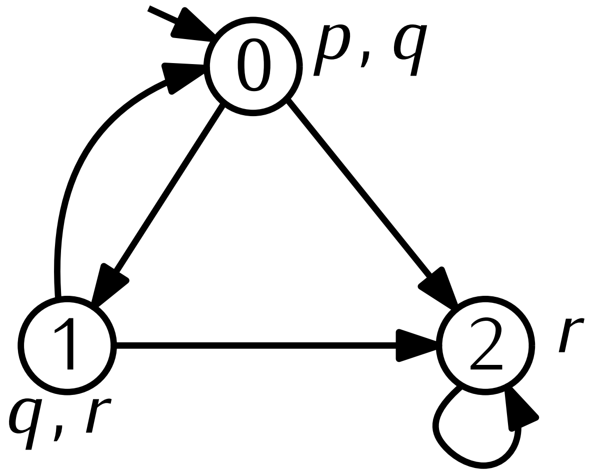

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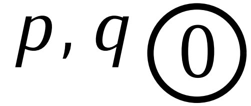

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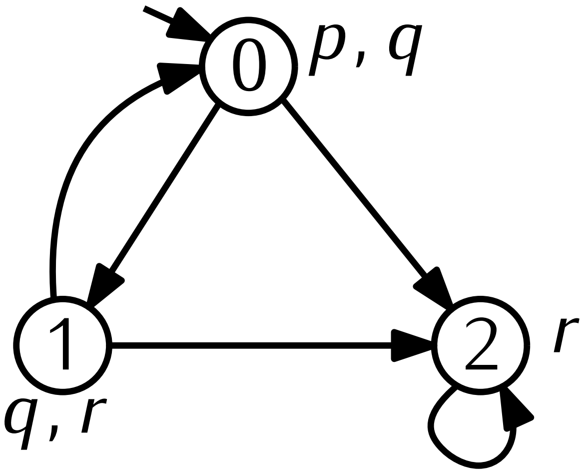

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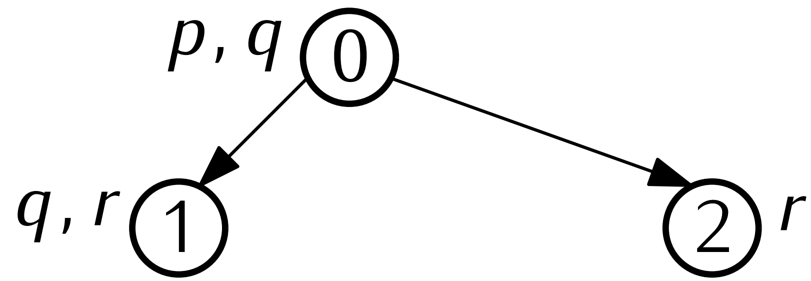

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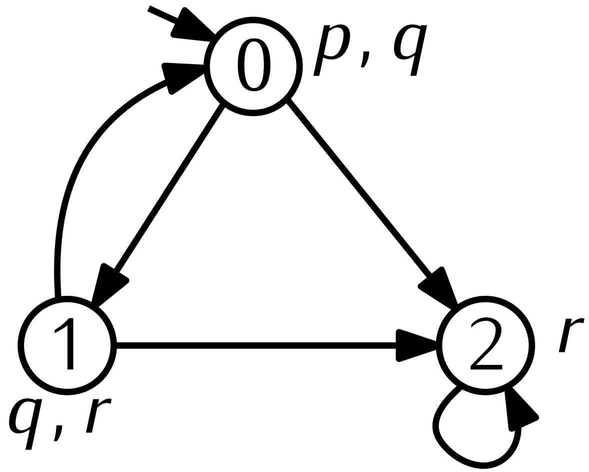

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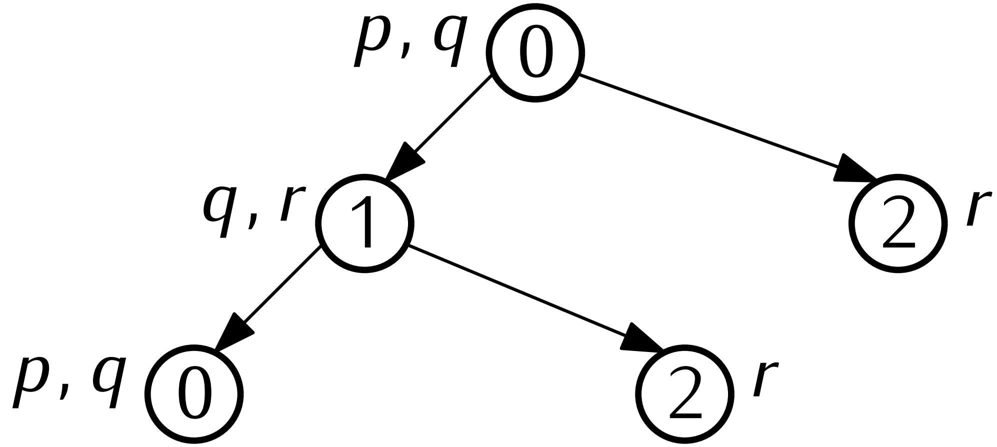

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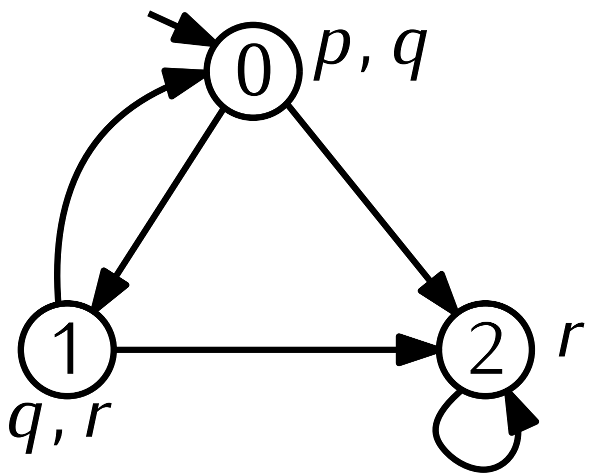

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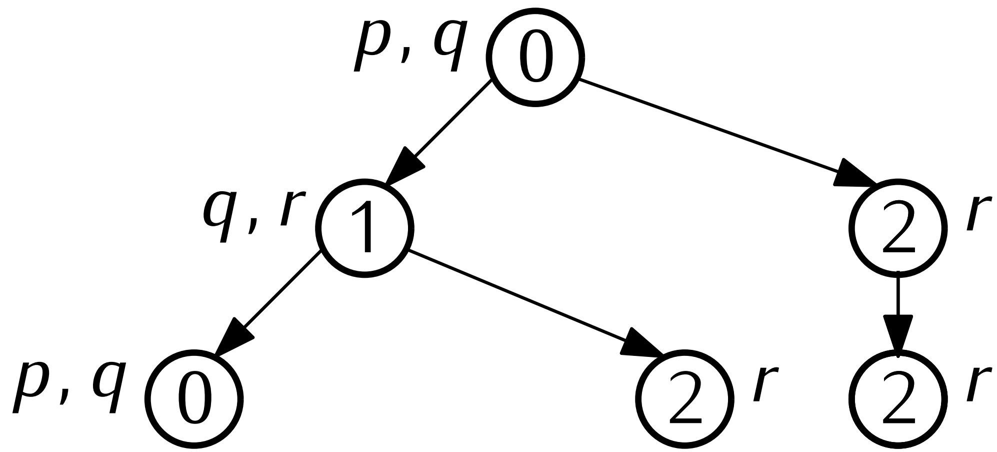

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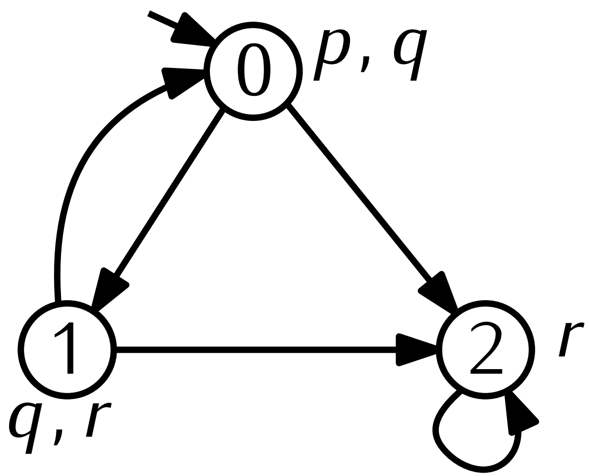

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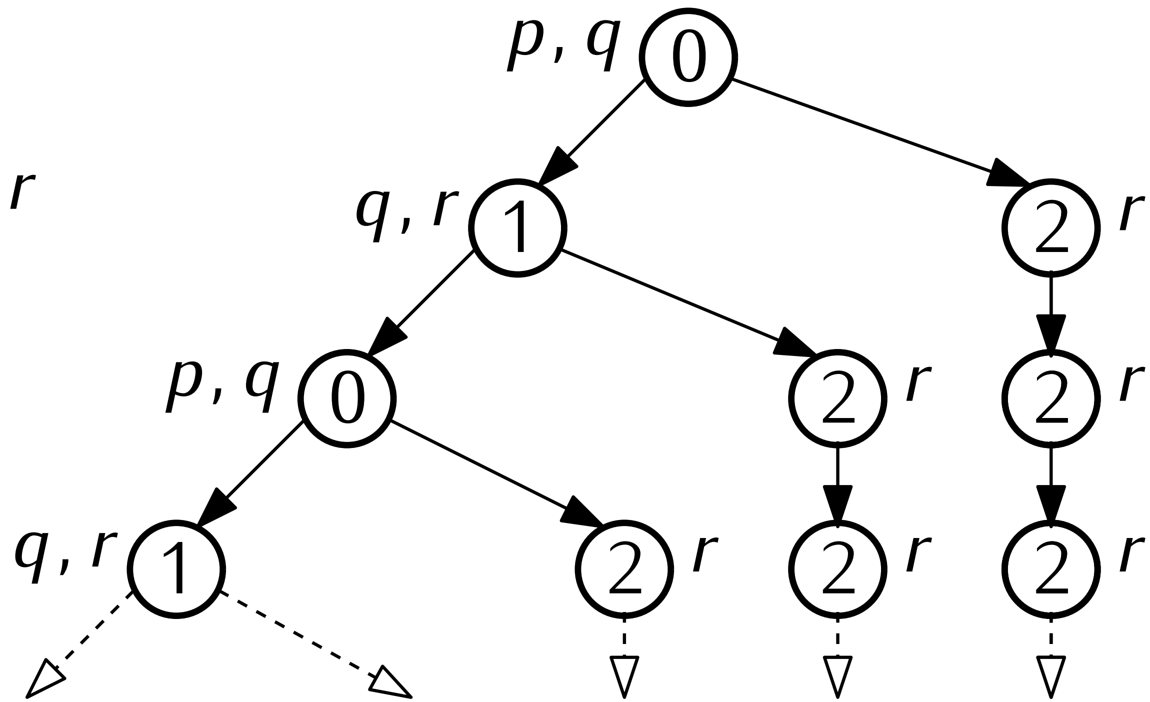

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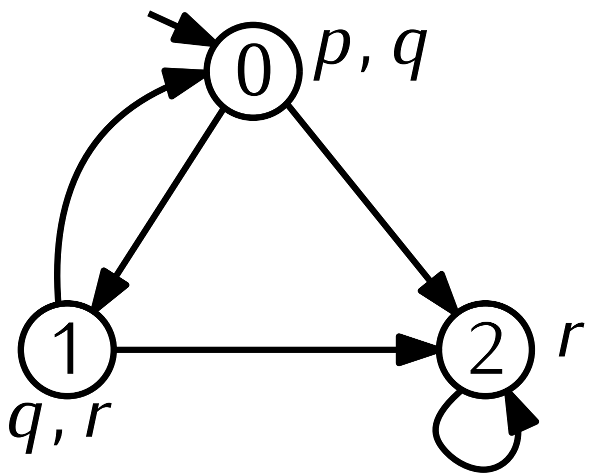

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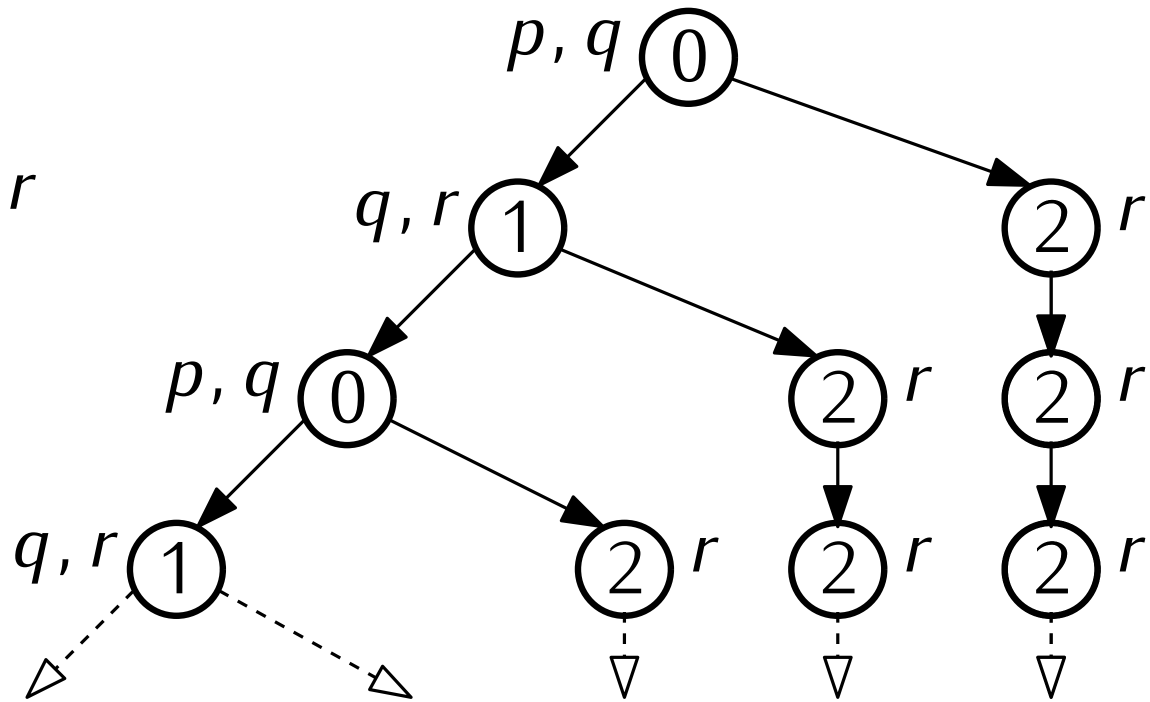
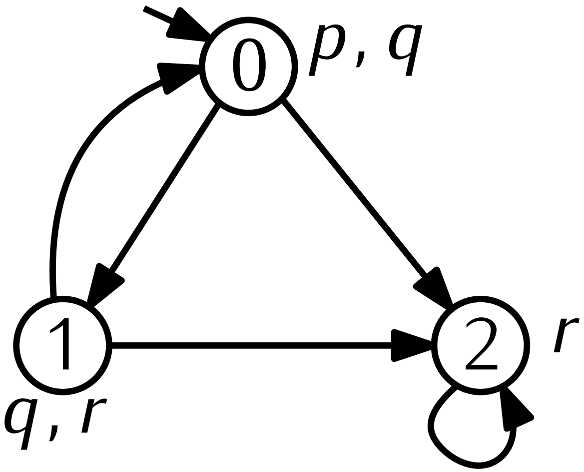


Computation Tree Logic (CTL) expresses properties of “alternative timelines”.

Linear vs Branching Time Logic



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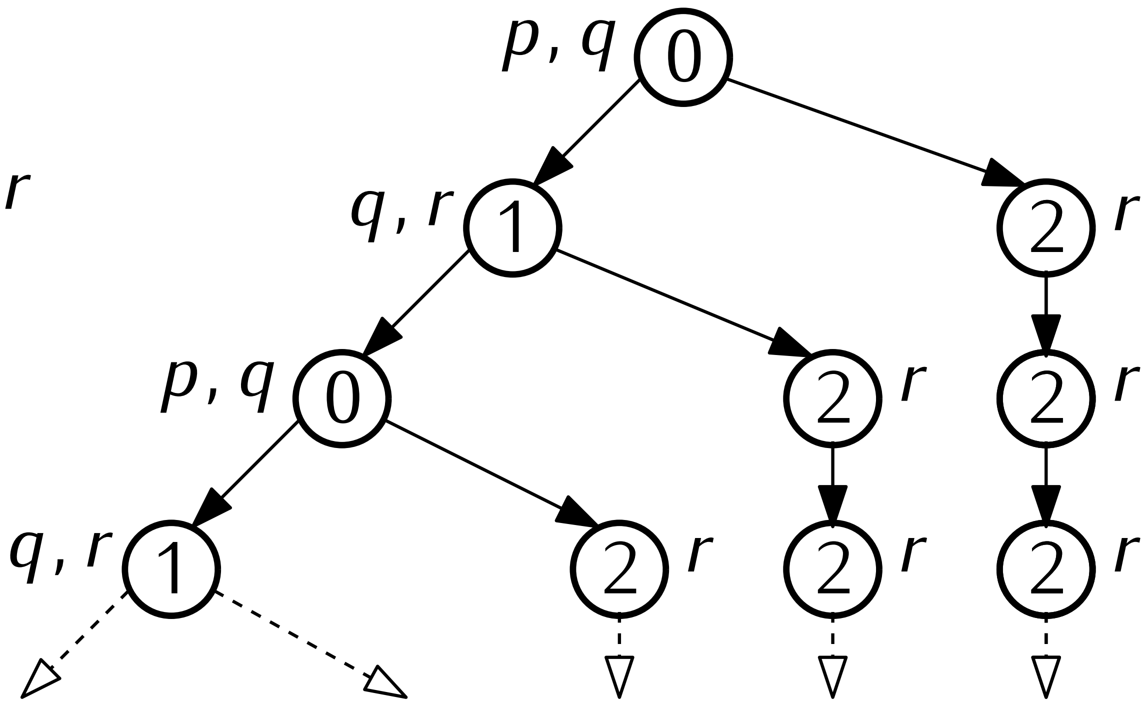

Computation Tree Logic (CTL) expresses properties of "alternative timelines".

$$\mathcal{M} \models \phi \Leftrightarrow ??????????$$

CTL formula

Computation Tree Logic Syntax

Suppose α and β are LTL formulas.
Suppose p_i is a propositional atom.
Then the following are all LTL formulas.

$$\top \quad \bot \quad p_i$$

$$\neg\alpha \quad \alpha \vee \beta \qquad \alpha \wedge \beta \quad \alpha \to \beta$$

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We use the same temporal operators: G, F, X, U

We attach path quantifiers, A (inevitably) or E (possibly), to each temporal operator.

$$AG \quad AF \quad AX \quad AU$$

$$EG \quad EF \quad EX \quad EU$$

Computation Tree Logic Semantics

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- If ϕ is an A-operator, then $s \models \phi$ iff **all paths** starting at s satisfy the 'LTL formula' made by removing the A symbol.
- If ϕ is an E-operator, then $s \models \phi$ iff **there exists a path** starting at s satisfy the 'LTL formula' made by removing the E symbol.

Computation Tree Logic Semantics

We just defined what it means for a state to satisfy a CTL property, $s \models \phi$.

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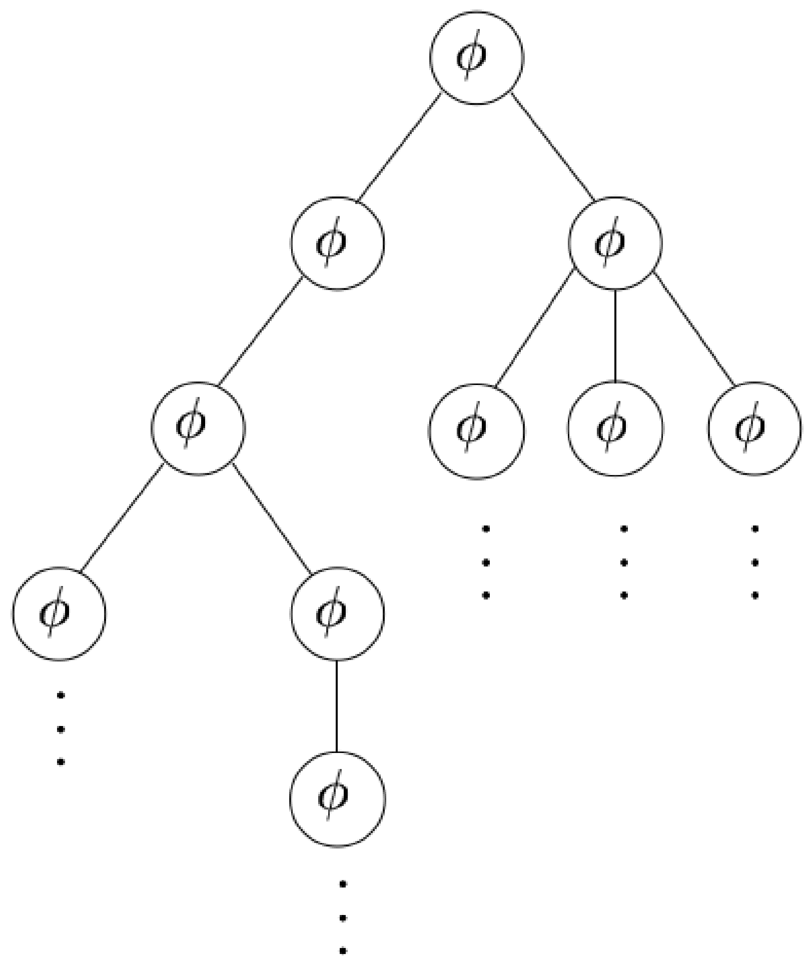
$$\mathcal{M} \models \phi \Leftrightarrow \forall s \in I \;\; s \models \phi$$

CTL Model Checking

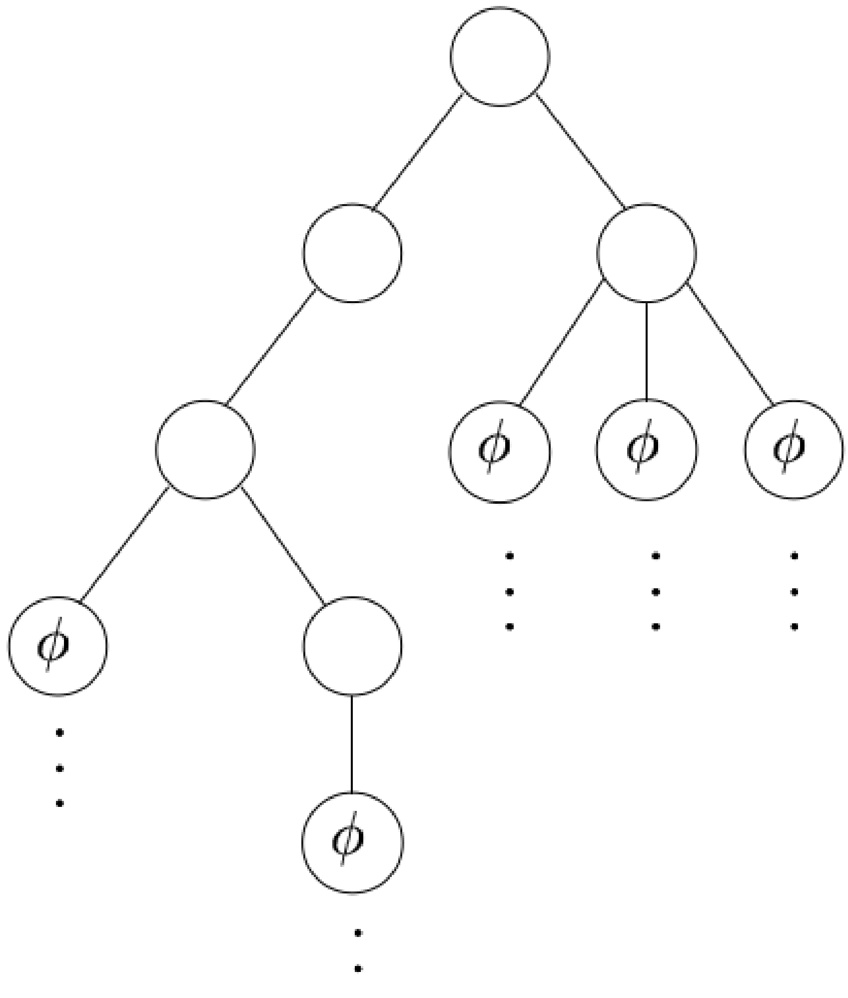
Computation Tree Logic Semantics

The idea of $AG\phi$: inevitably always ϕ



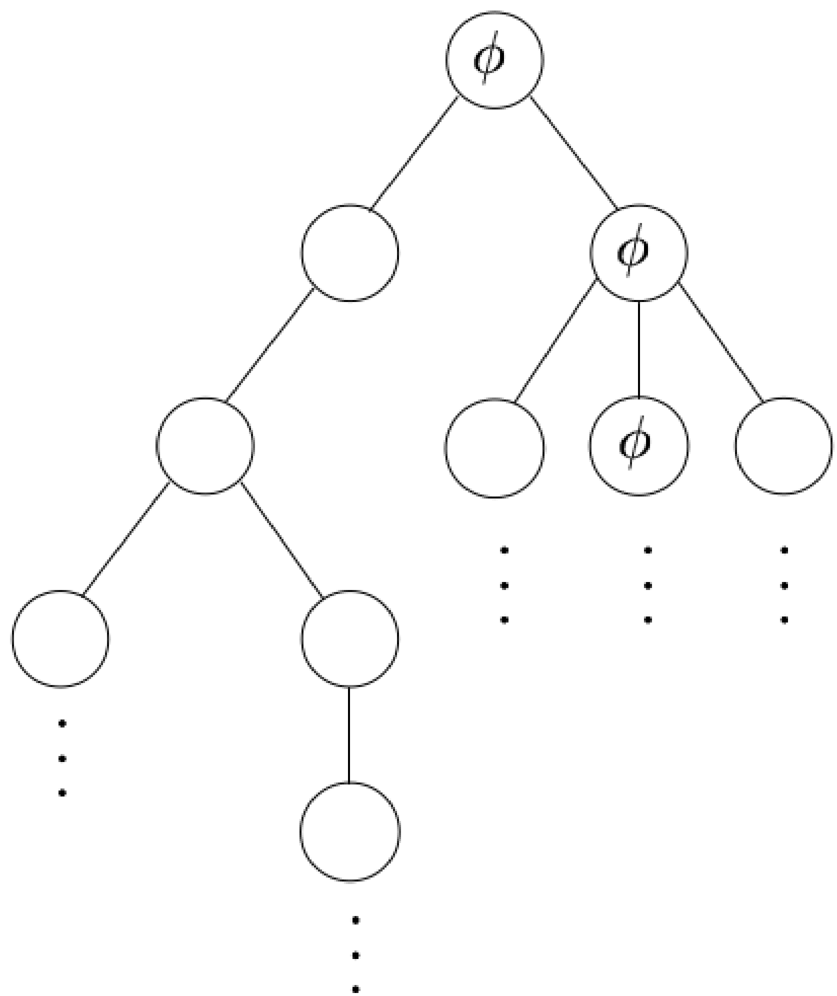
Computation Tree Logic Semantics

The idea of $AF\phi$: inevitably eventually ϕ



Computation Tree Logic Semantics

The idea of $EG\phi$: possibly always ϕ



Computation Tree Logic Semantics

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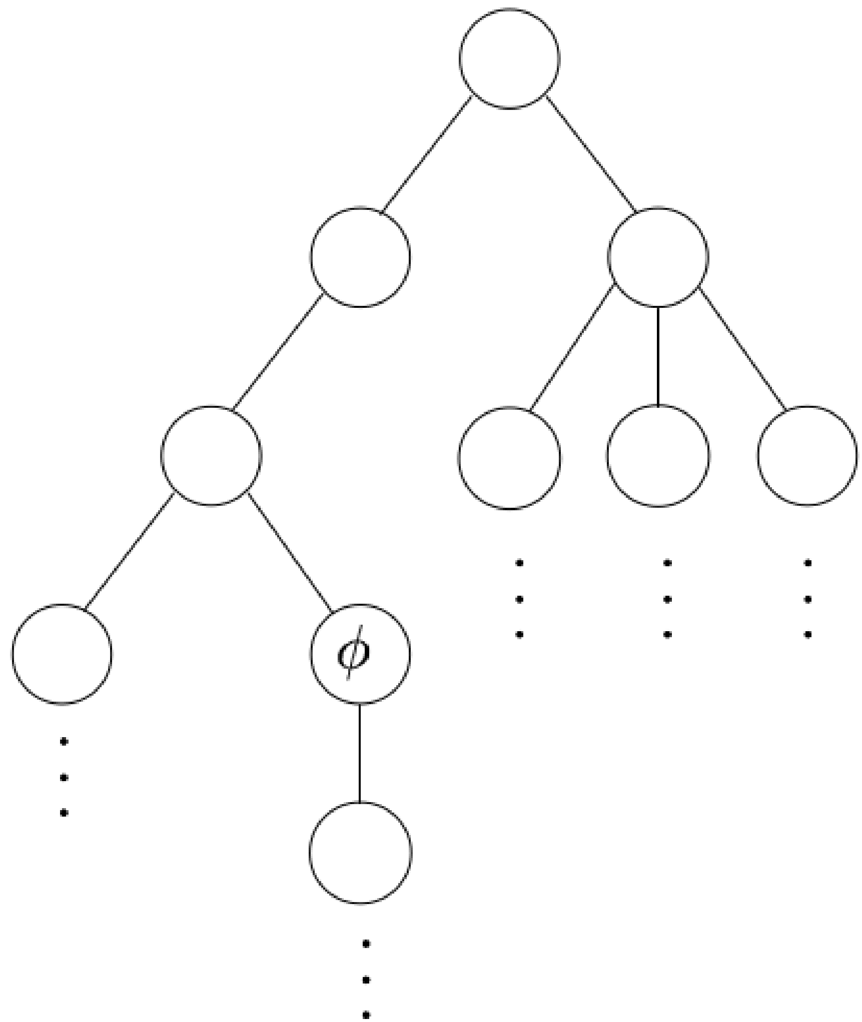
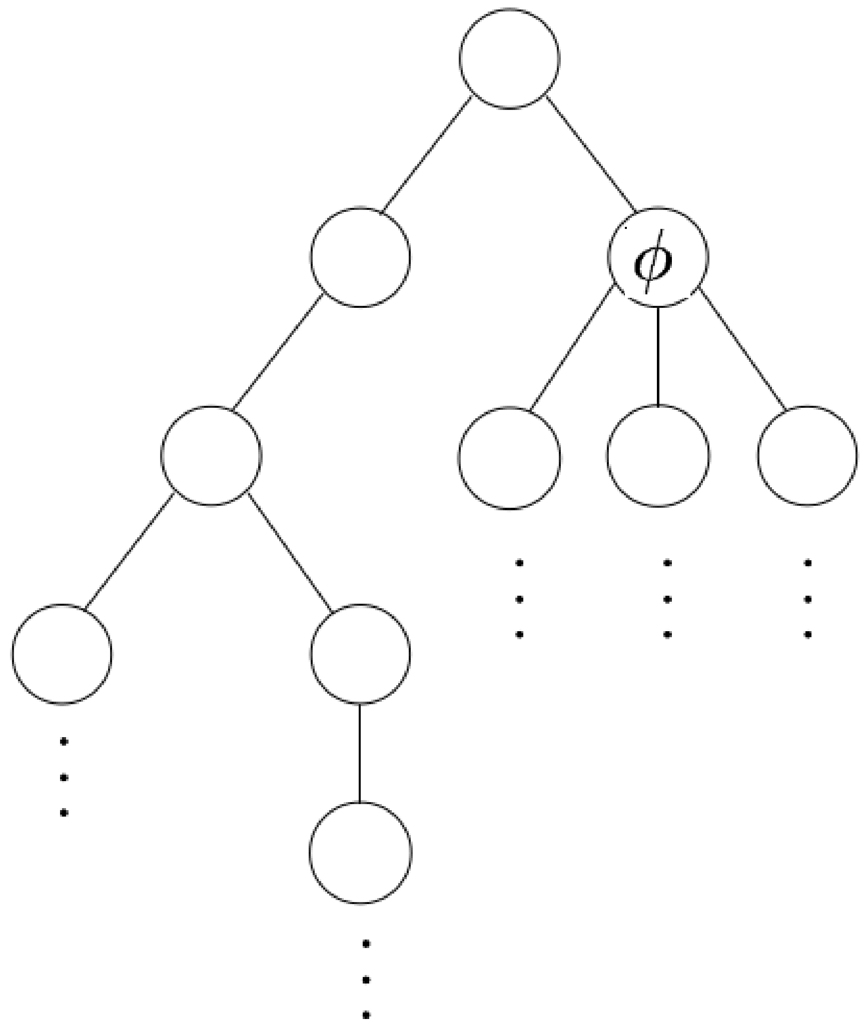


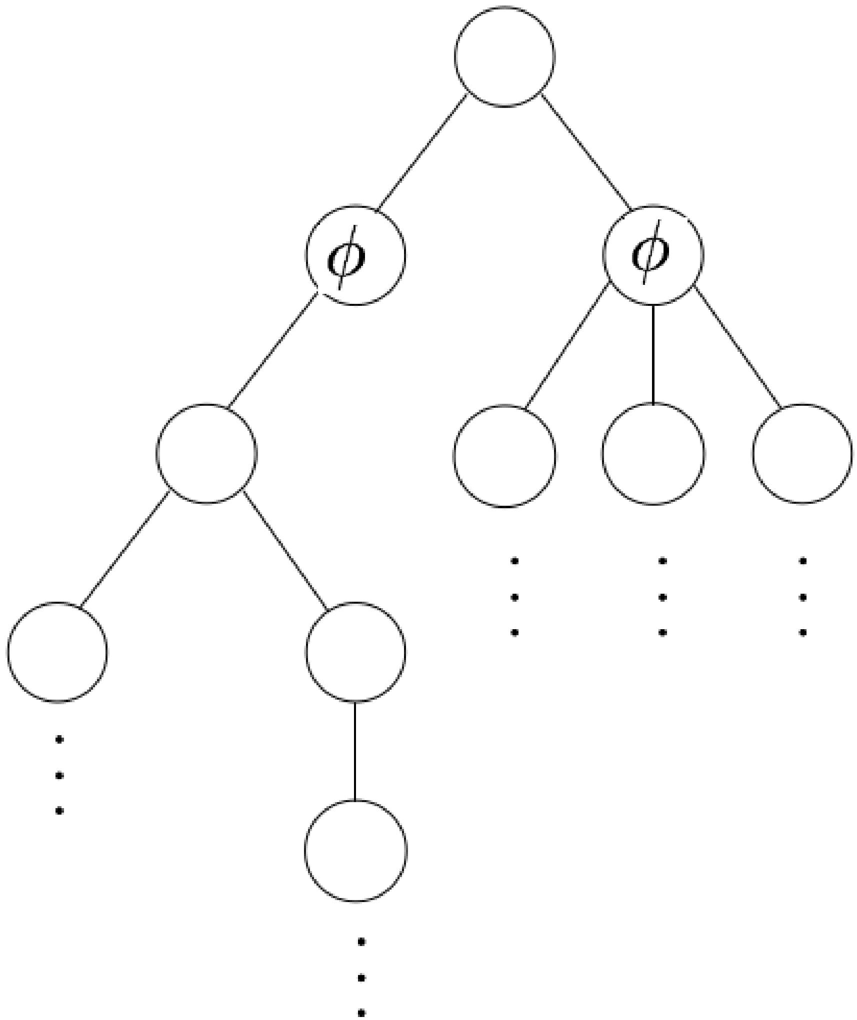
Computation Tree Logic Semantics

The idea of $EX\phi$: possibly next ϕ



Computation Tree Logic Semantics

The idea of $AX\phi$: inevitably next ϕ



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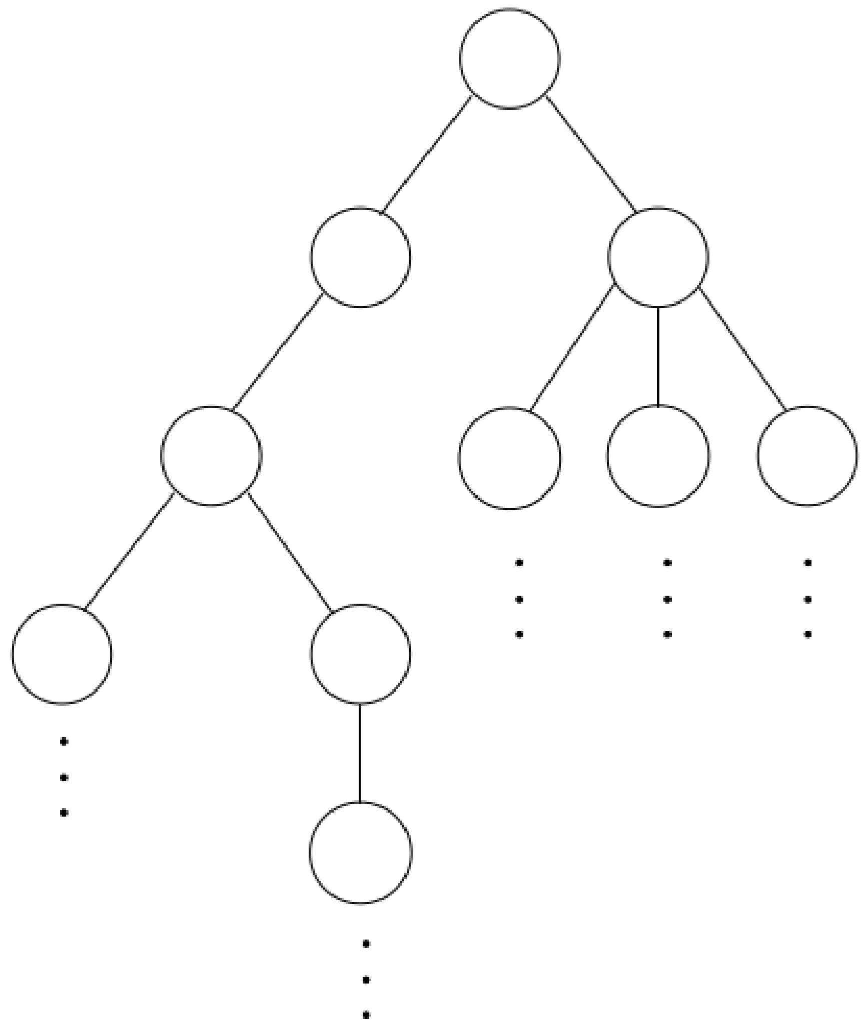
The idea of $\phi AU \psi$: inevitably ϕ until ψ



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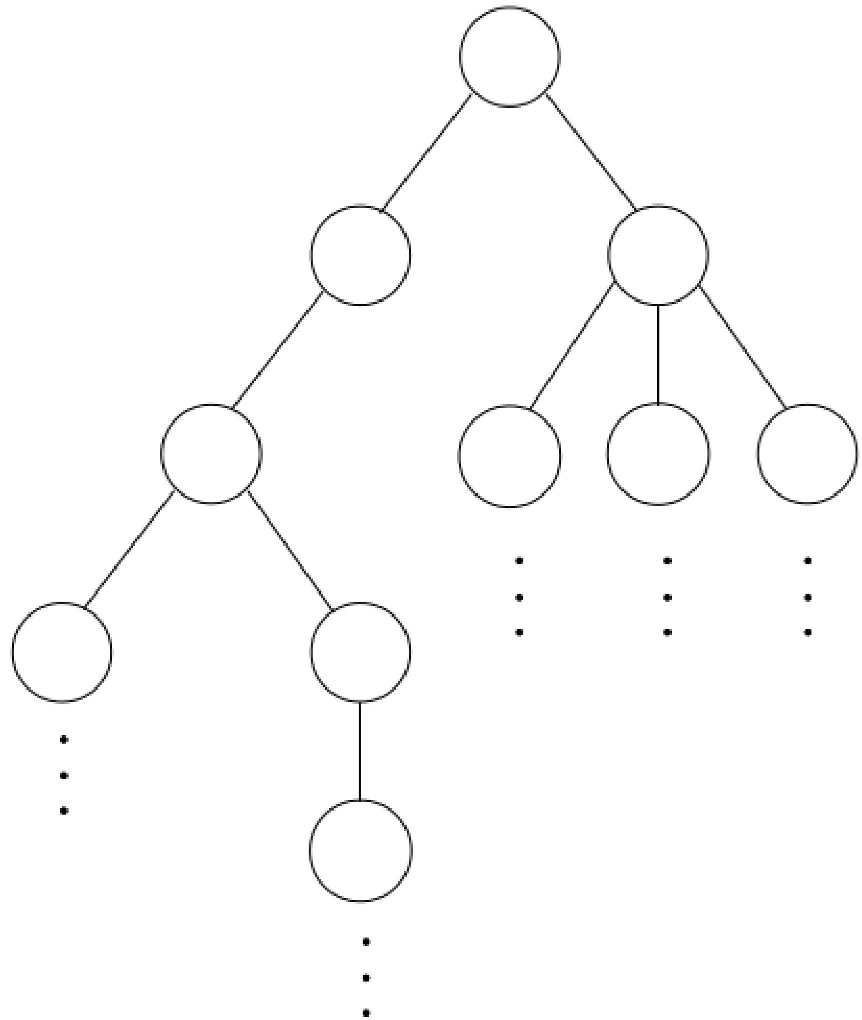

Fill in your answer in class.

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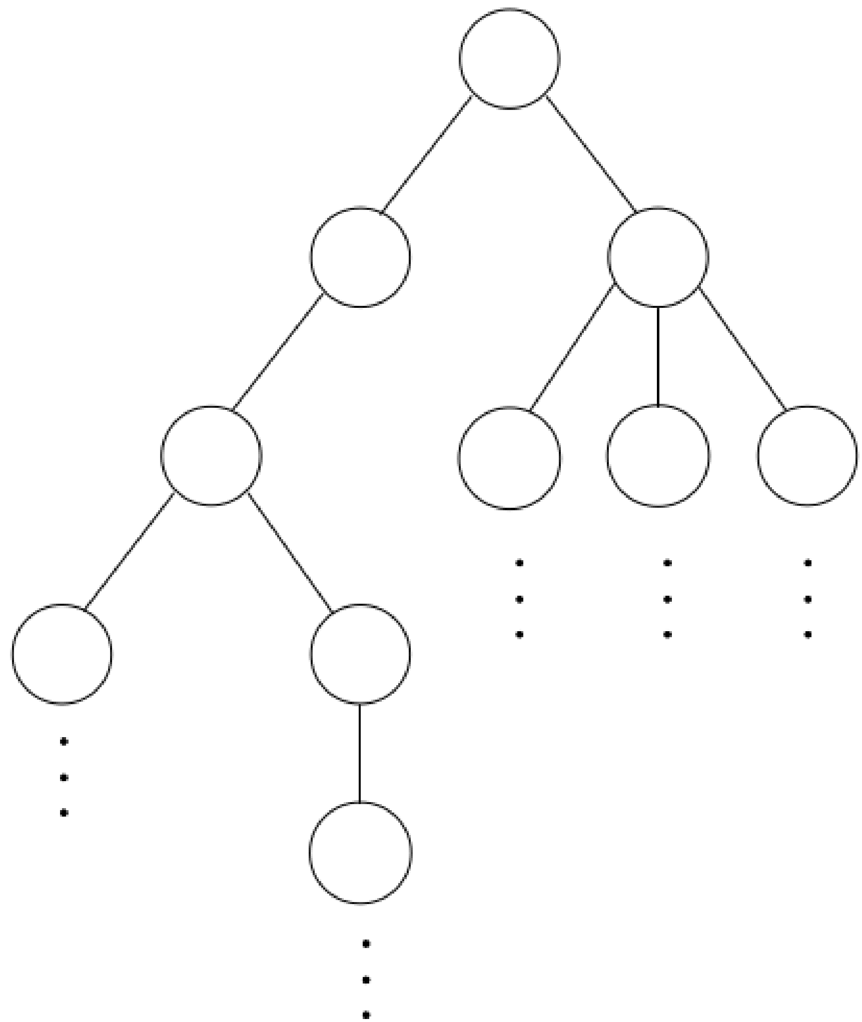

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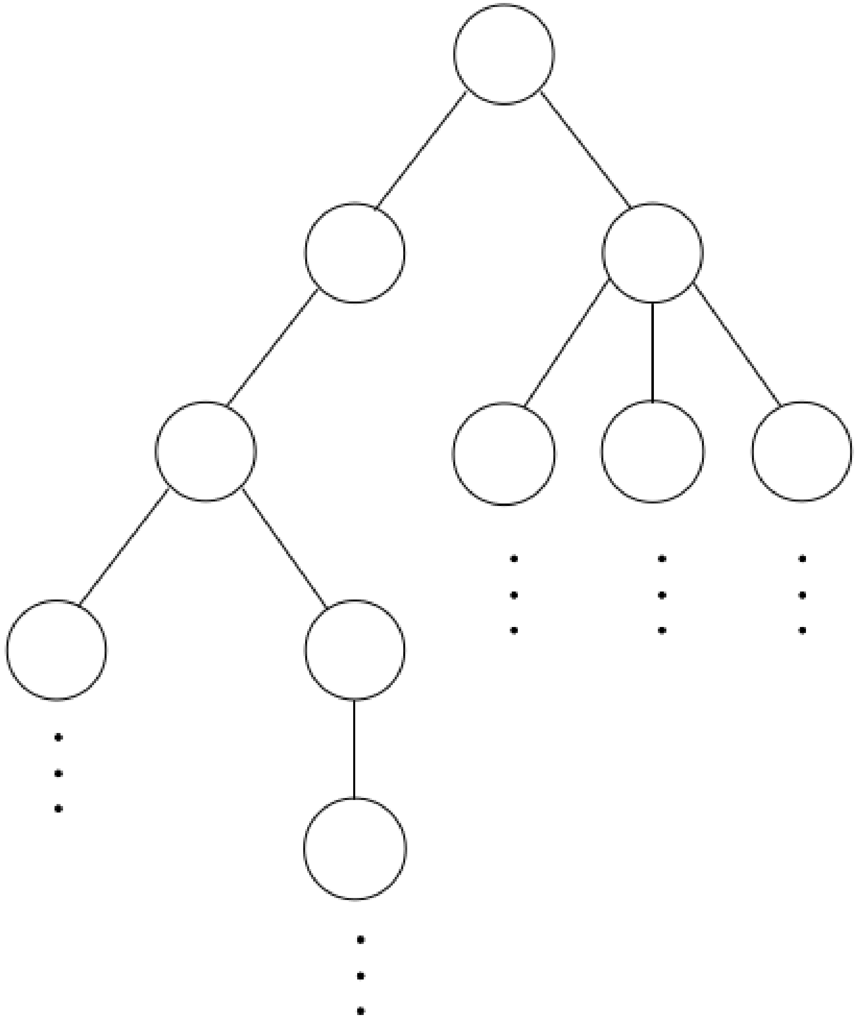

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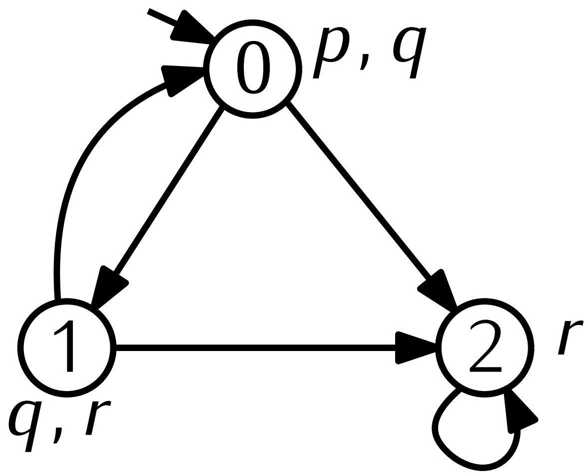
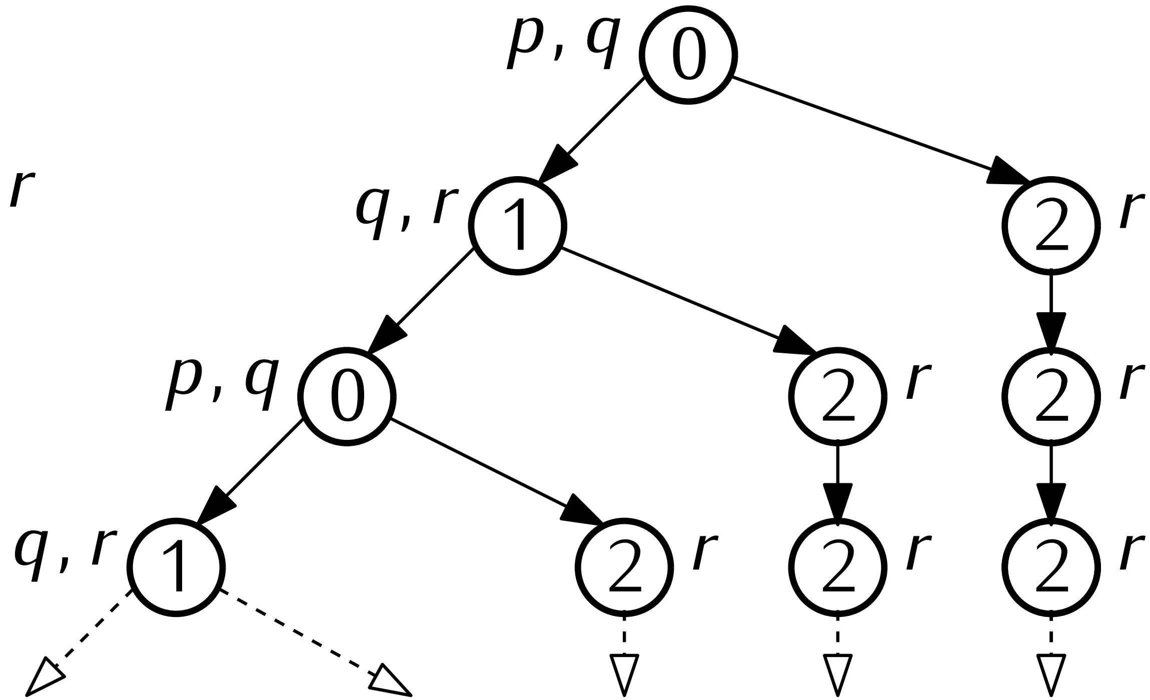
Computation Tree Logic Example Properties



Computation tree for $\mathcal{M}$



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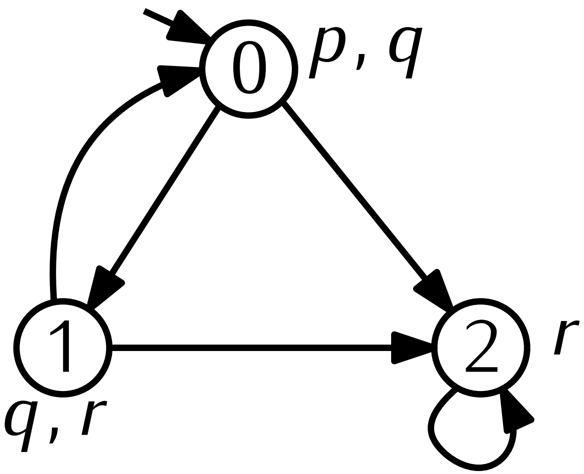

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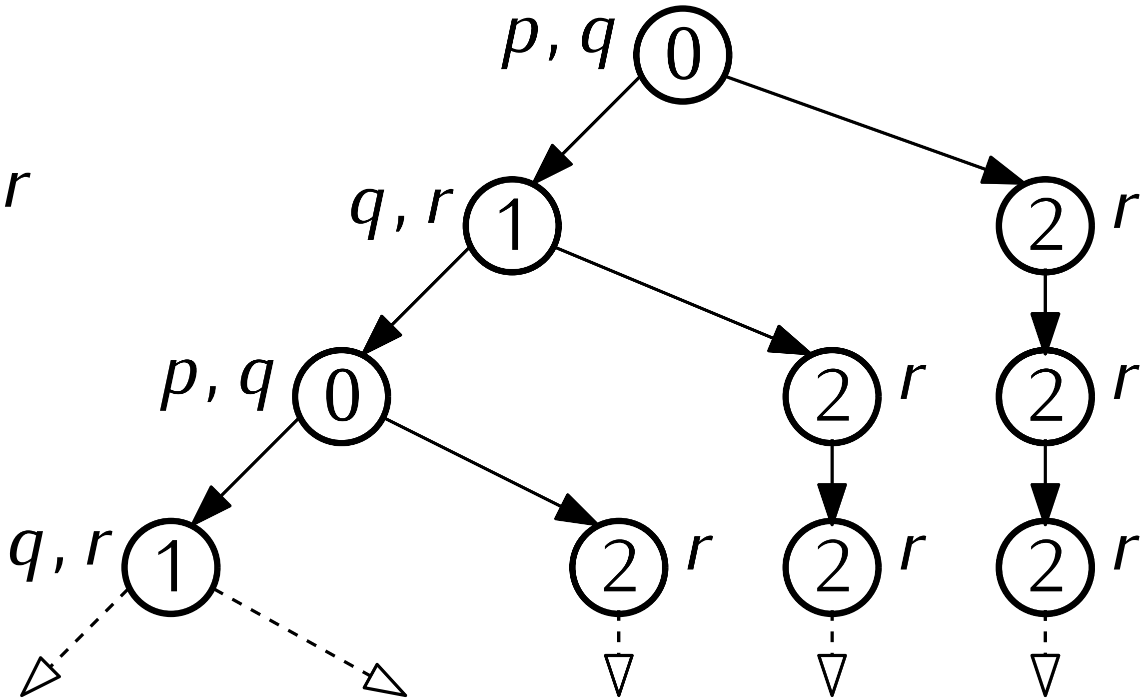

$\mathcal{M} \models p \wedge q$?

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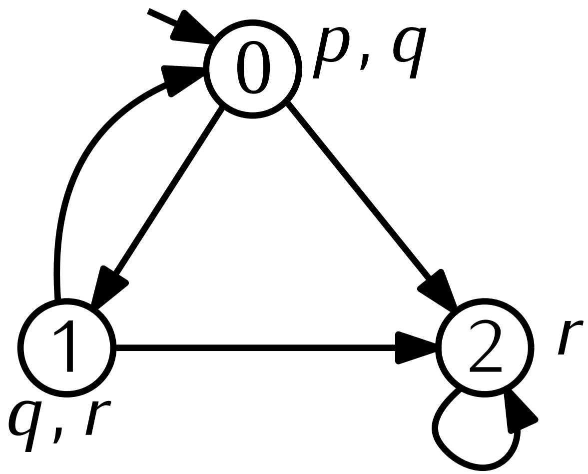

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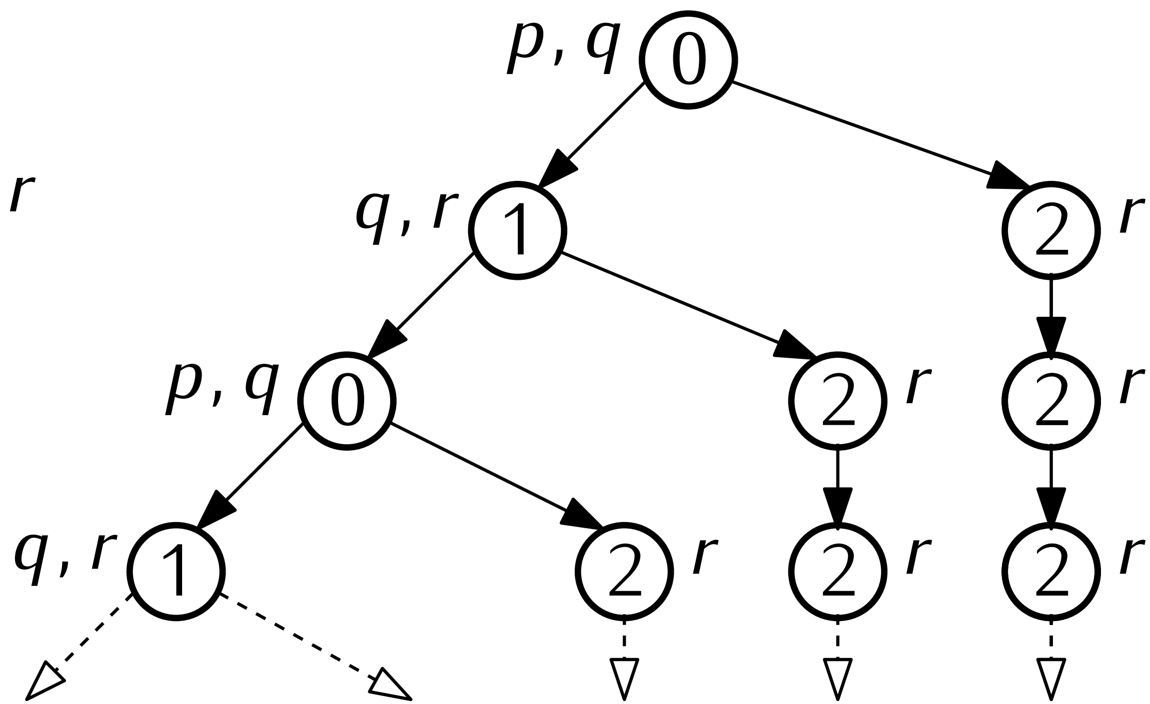

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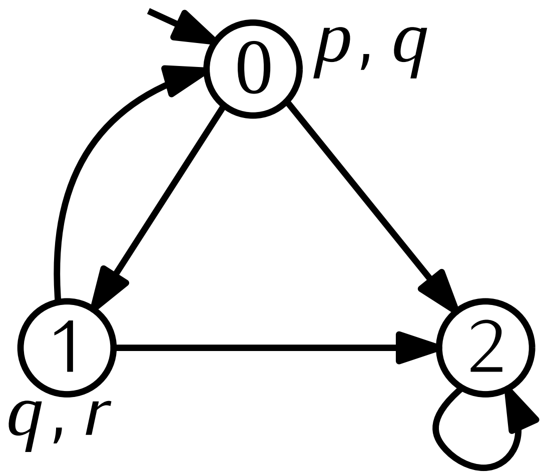

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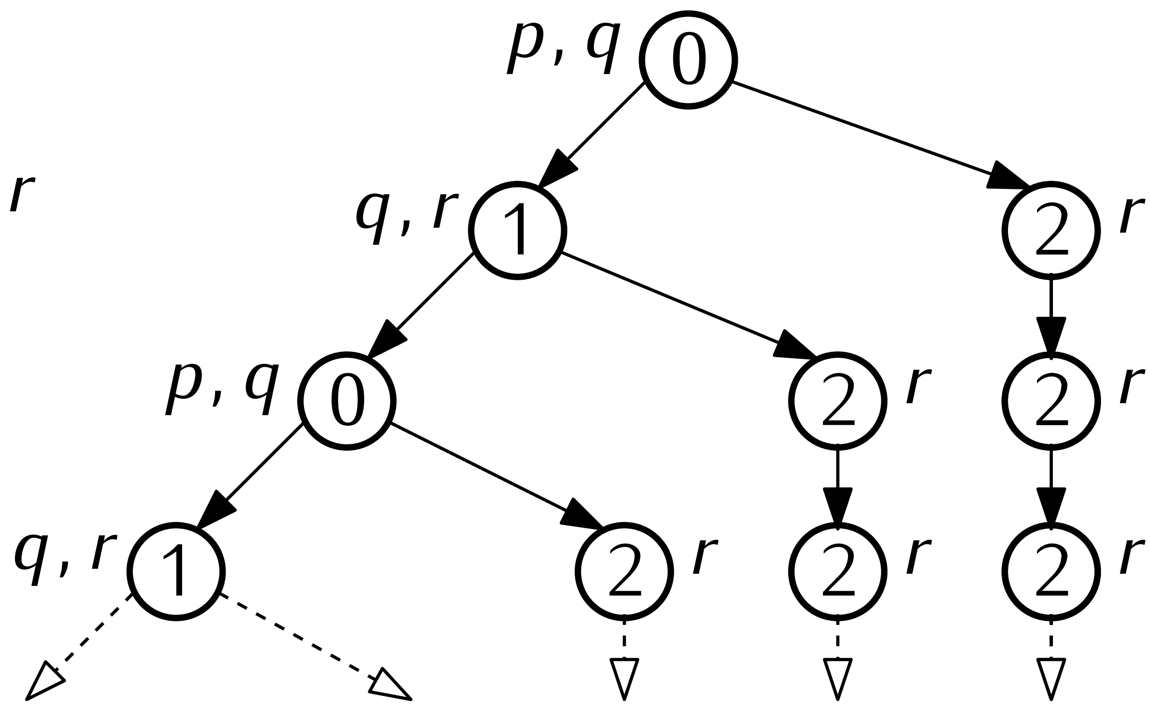

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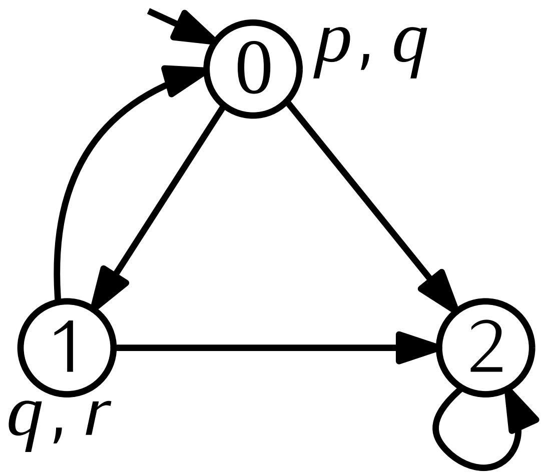

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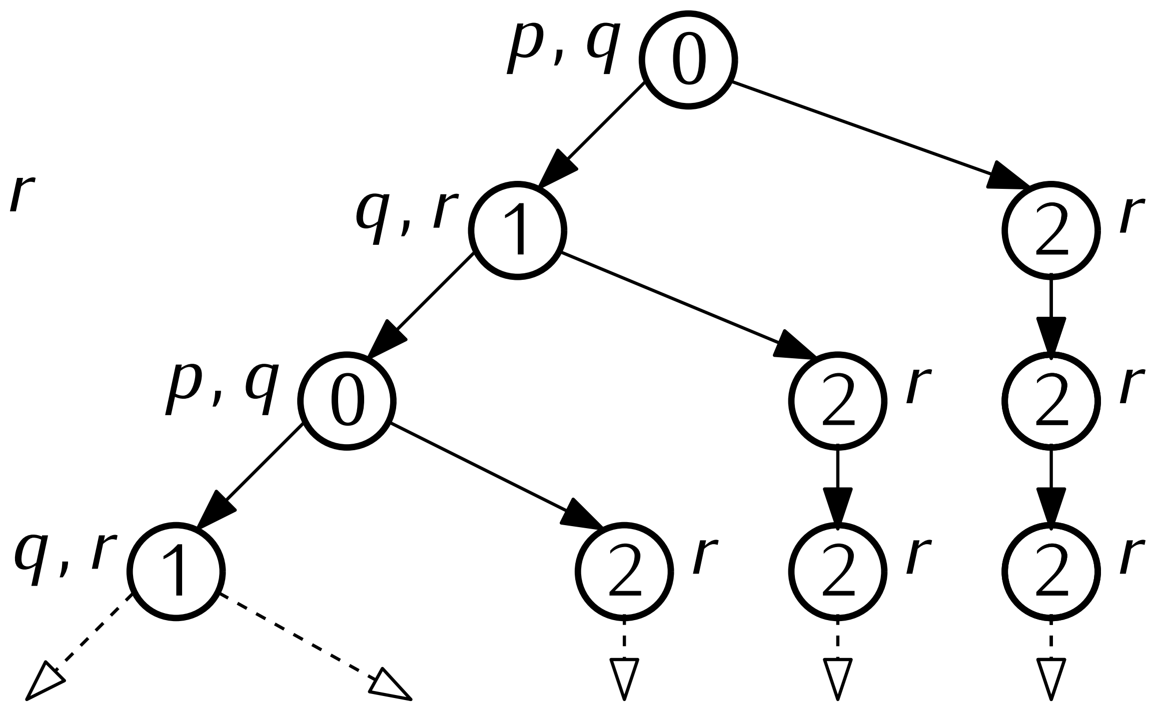

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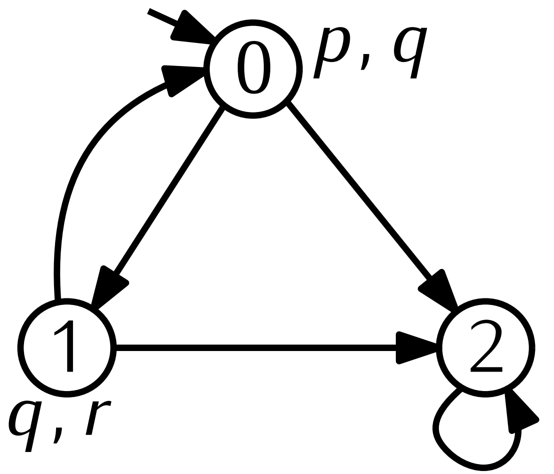
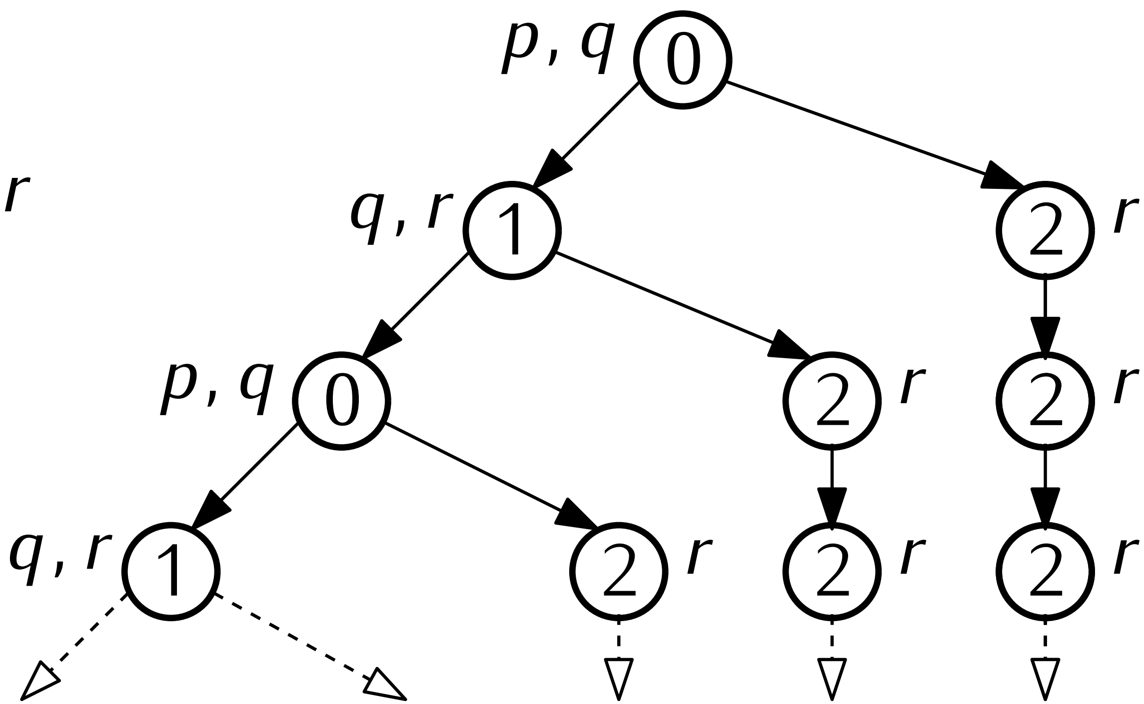
$\mathcal{M} \models AX(q \wedge r)$?

Computation Tree Logic Example Properties



Computation tree for $\mathcal{M}$



$\mathcal{M} \models p \wedge q$?

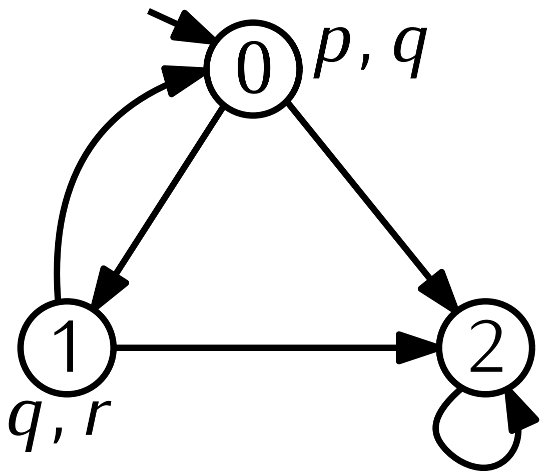
$\mathcal{M} \models \neg r$?

$\mathcal{M} \models EX(q \wedge r)$?

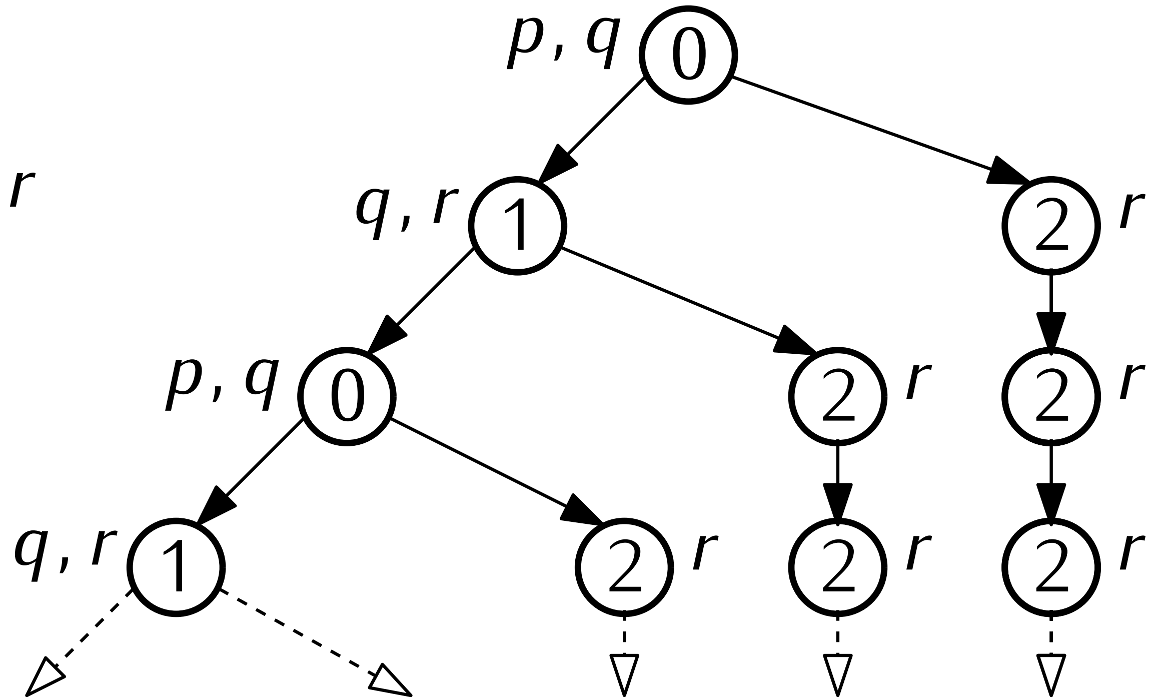
$\mathcal{M} \models AX(q \wedge r)$?

$\mathcal{M} \models \neg AX(q \wedge r)$?

Computation Tree Logic Example Properties



Computation tree for $\mathcal{M}$



$\mathcal{M} \models p \wedge q$?

$\mathcal{M} \models \neg r$?

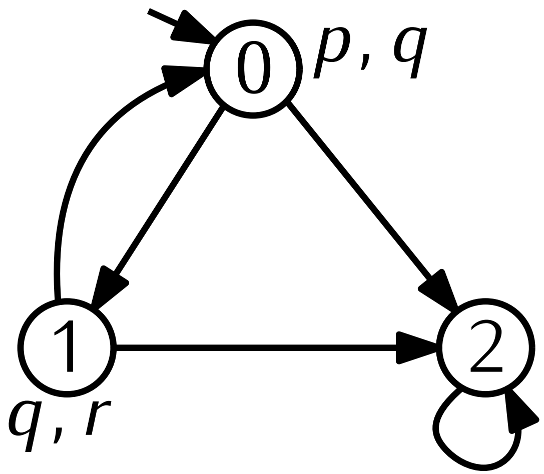
$\mathcal{M} \models EX(q \wedge r)$?

$\mathcal{M} \models AX(q \wedge r)$?

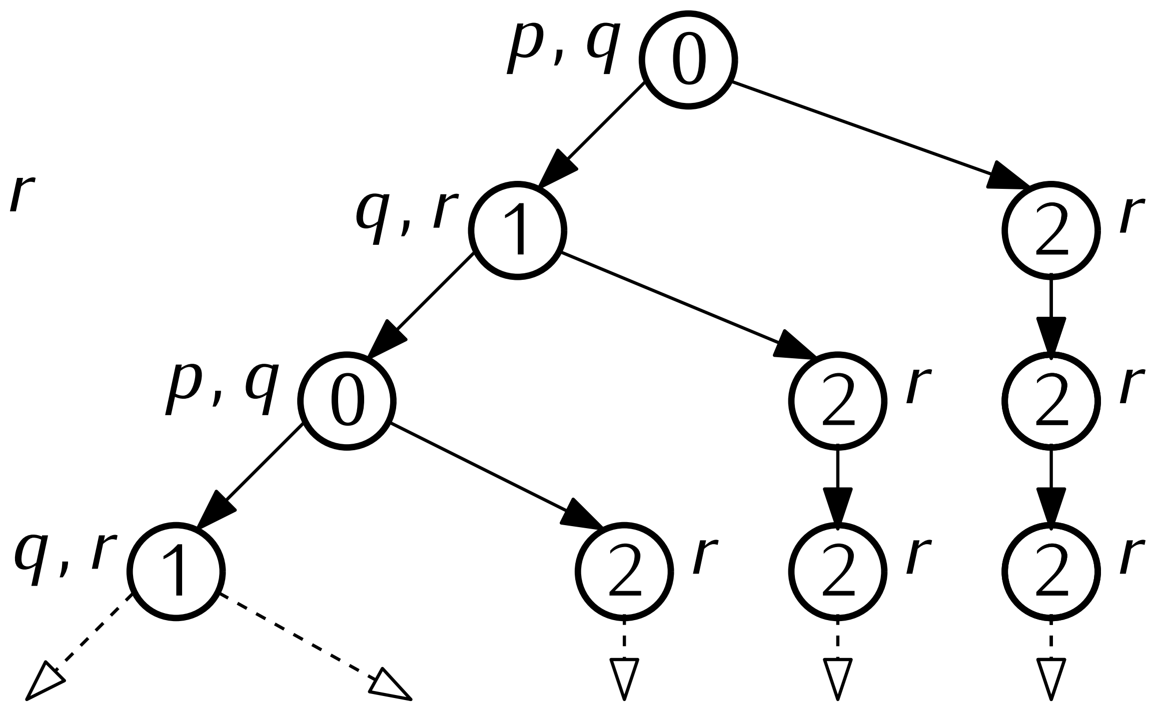
$\mathcal{M} \models \neg AX(q \wedge r)$?

$\mathcal{M} \models \neg EF(p \wedge r)$?

Computation Tree Logic Example Properties



Computation tree for $\mathcal{M}$



$\mathcal{M} \models p \wedge q$?

$\mathcal{M} \models \neg r$?

$\mathcal{M} \models EX(q \wedge r)$?

$\mathcal{M} \models AX(q \wedge r)$?

$\mathcal{M} \models \neg AX(q \wedge r)$?

$\mathcal{M} \models \neg EF(p \wedge r)$?

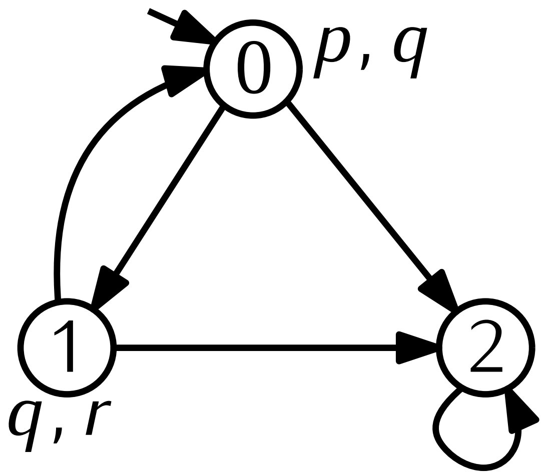
$\mathcal{M} \models EG\neg r$?

$\mathcal{M} \models AFq$?

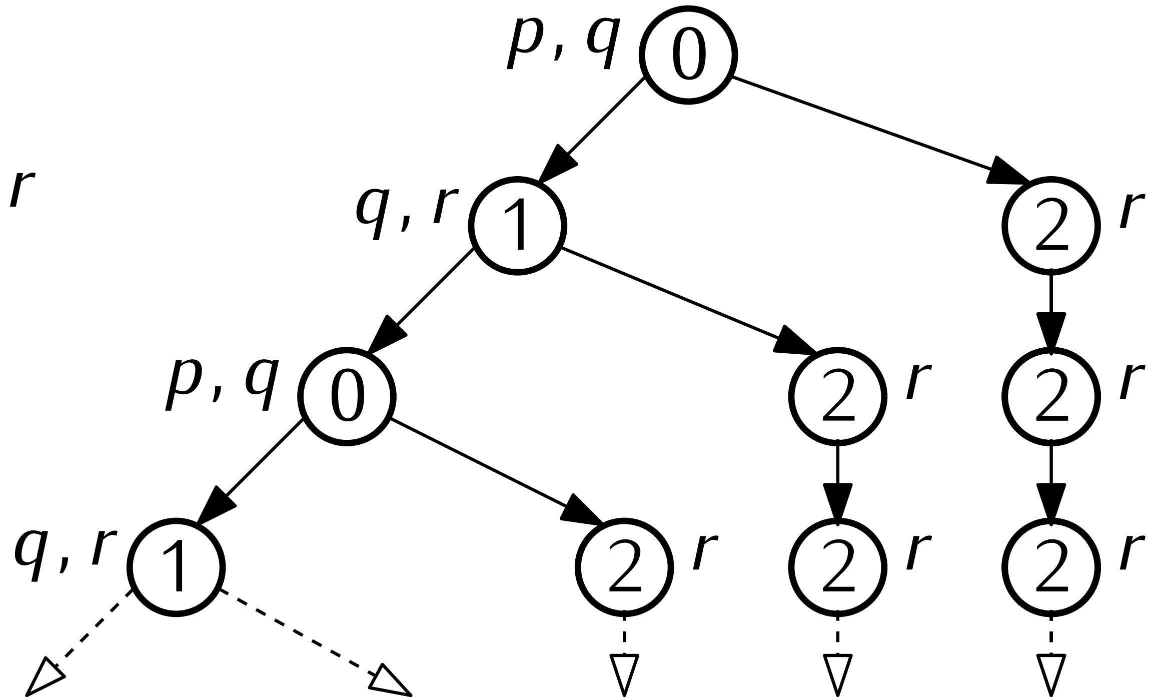
$\mathcal{M} \models p\ AU\ r$?

$\mathcal{M} \models \neg(p \wedge q)\ EU\ r$?

Computation Tree Logic Example Properties



Computation tree for $\mathcal{M}$



$\mathcal{M} \models p \wedge q$?

$\mathcal{M} \models \neg r$?

$\mathcal{M} \models EX(q \wedge r)$?

$\mathcal{M} \models AX(q \wedge r)$?

$\mathcal{M} \models \neg AX(q \wedge r)$?

$\mathcal{M} \models \neg EF(p \wedge r)$?

$\mathcal{M} \models EG \neg r$?

$\mathcal{M} \models AFq$?

$\mathcal{M} \models p \; AU \; r$?

$\mathcal{M} \models \neg(p \wedge q) \; EU \; r$?