CS 181u Applied Logic

Lecture 10

Today's class

Linear Temporal Logic

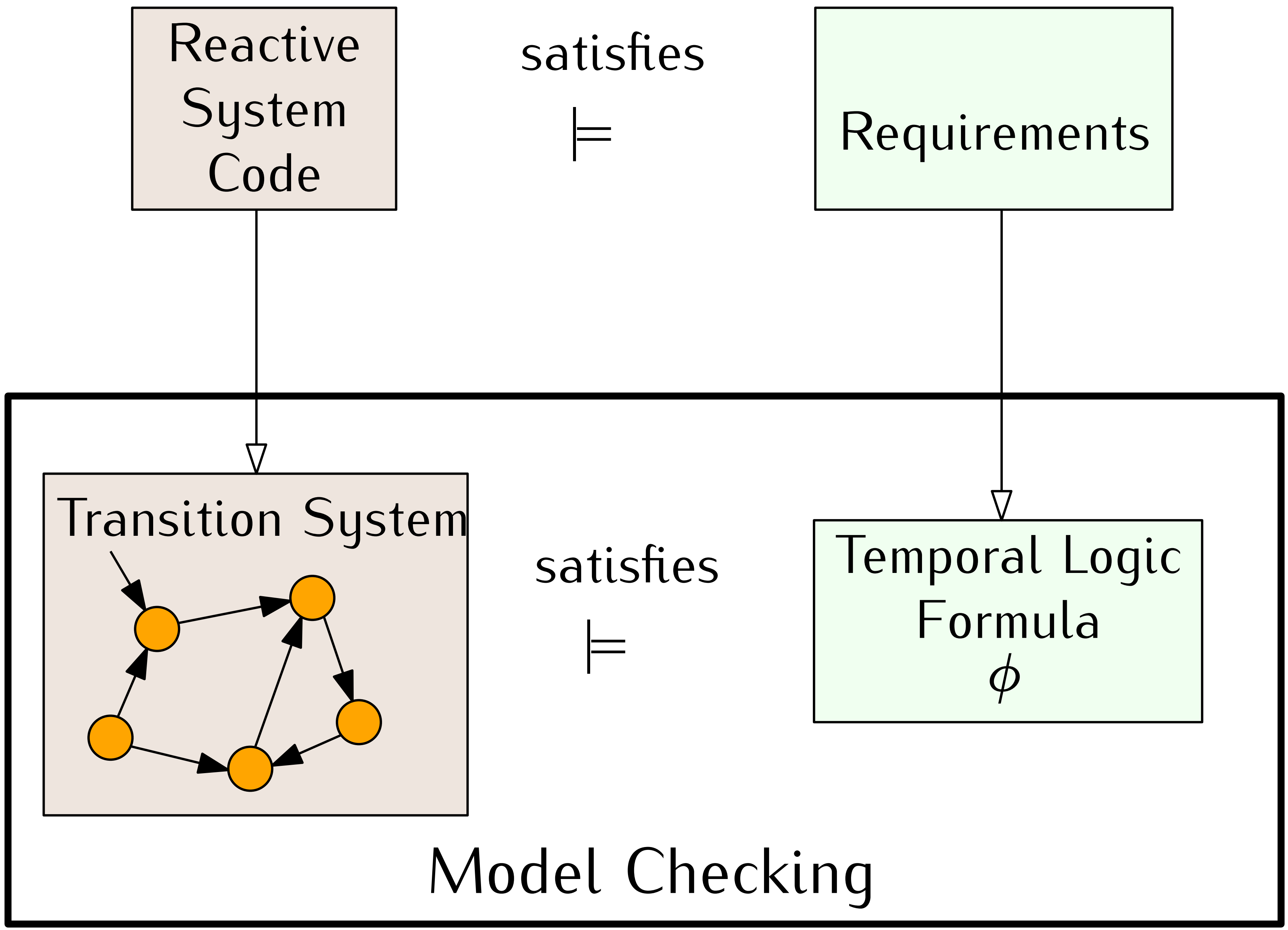
W and R operators. LTL operator basis.

Computation Tree Logic

Some practice, nested CTL formulas

LTL vs CTL

Equivalance of two temporal formulas


Reactive System Code
satisfies
⊨
Requirements
Transition System
satisfies
⊨
Temporal Logic Formula
ϕ
Model Checking

LTL W and R operators

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$$\vee \;\; \forall k \geq 1 \;\; \pi^k \models \phi$$

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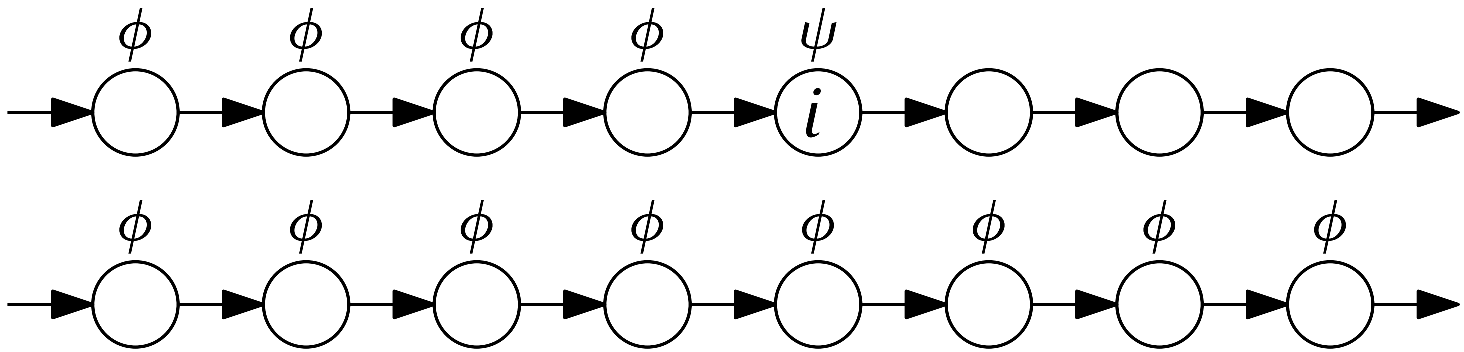

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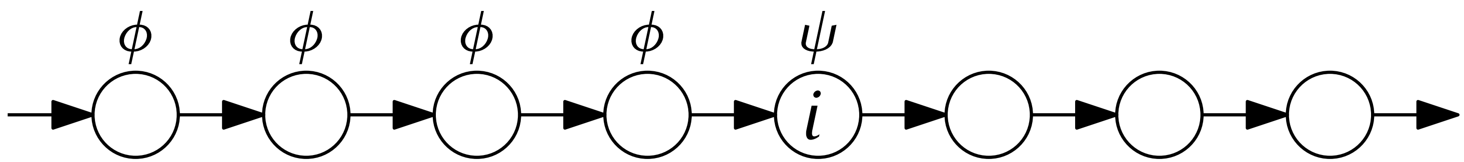

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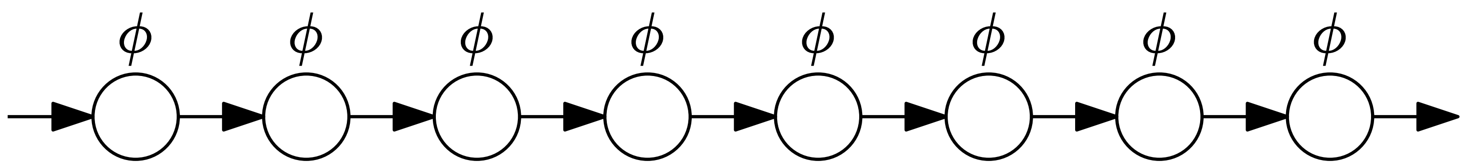

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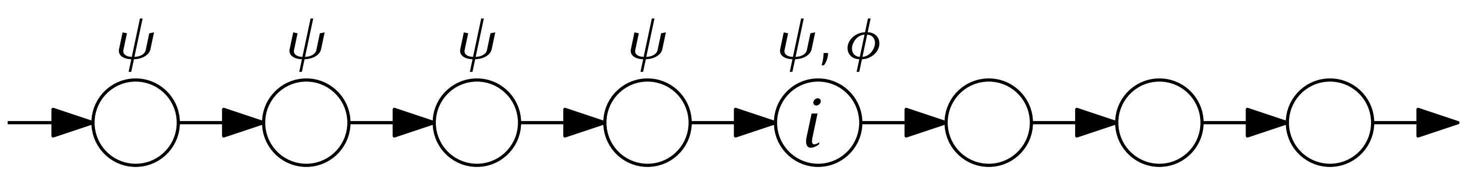
Release



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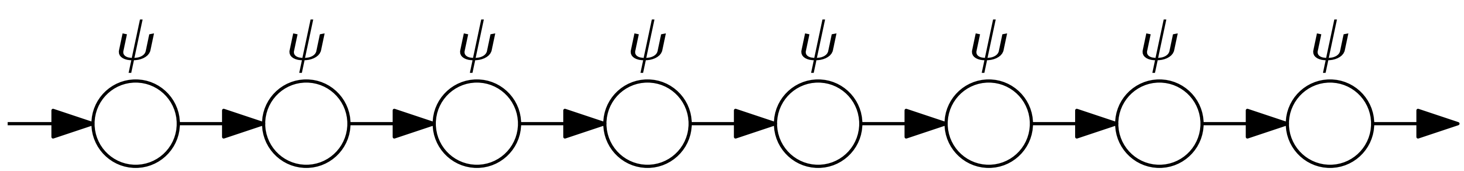

Why so many operators? G, X, U, F, W, R

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The acts of the mind, wherein it exerts its power over simple ideas, are chiefly these three: Combining several simple ideas into one compound one, and thus all complex ideas are made. The second is bringing two ideas, whether simple or complex, together, and setting them by one another so as to take a view of them at once, without uniting them into one, by which it gets all its ideas of relations. The third is separating them from all other ideas that accompany them in their real existence: this is called abstraction, and thus all its general ideas are made.

SICP by Abelson, Sussman, and Sussman quoting John Locke from his *Essay Concerning Human Understanding*

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However, they are useful to have on hand to state things concisely, like when writing νSMV specifications.

$$GF(pc = w) \quad \text{vs} \quad \forall i \geq 1 \;\; \exists j \geq i \;\; \pi^j \models (pc = w)$$

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$\{\wedge, \neg\}$	$\{\lambda, (f \quad e)\}$	$\{S, K, I\}$
Propositional logic	Lambda calc	Combinatory Logic

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Hence, $\{U, X\}$ is a basis for LTL.

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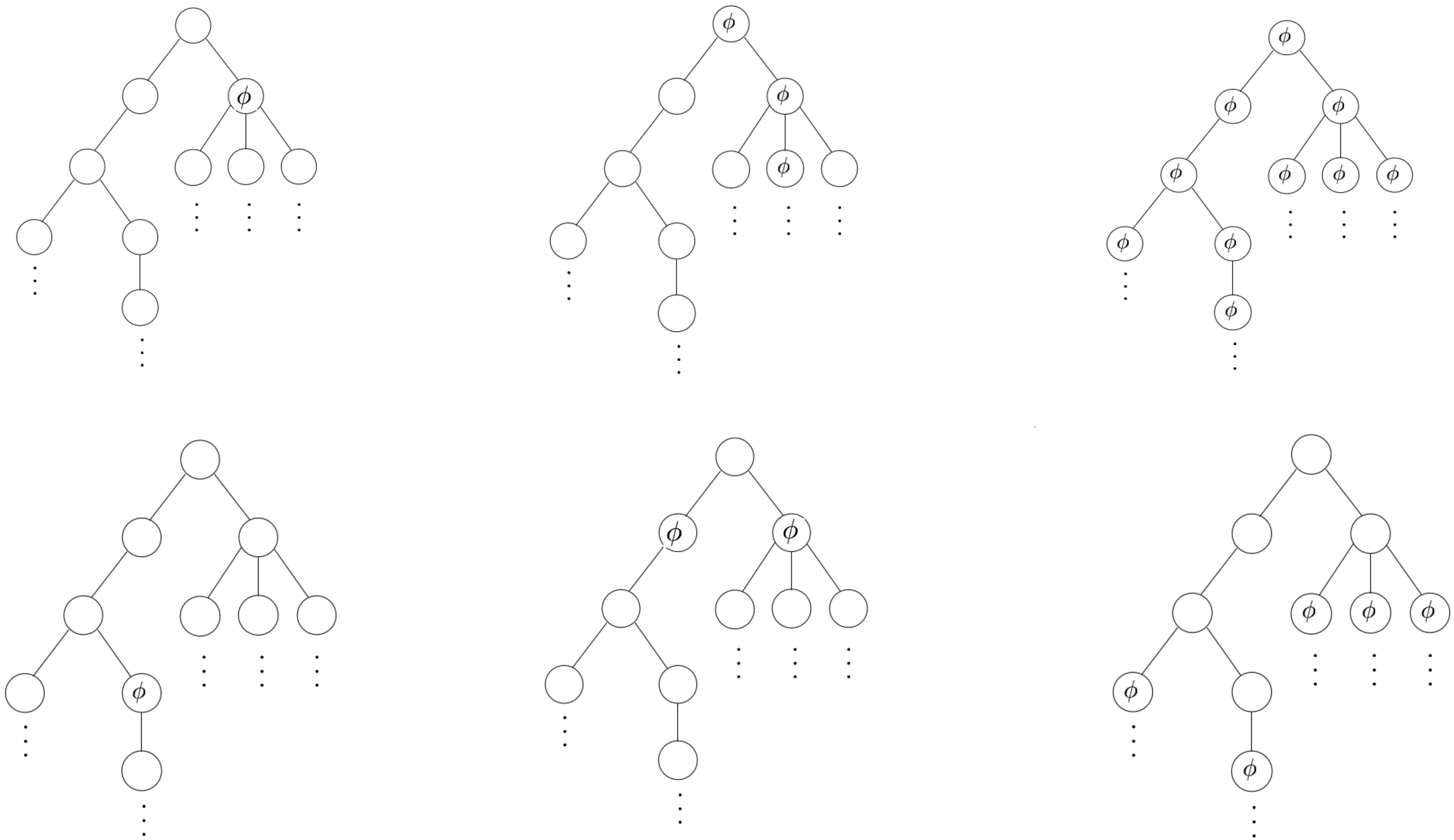
Hence, $\{U, X\}$ is a basis for LTL.

Similar reasoning shows that $\{R, X\}$
and $\{W, X\}$ are also bases.

CTL vs LTL

We would like to say something about the difference in expressiveness between CTL and LTL. Let's put that on hold for the moment and get a better grip on CTL first.

CTL review $AG\phi$ $EG\phi$ $AF\phi$ $EF\phi$ $AX\phi$ $EX\phi$



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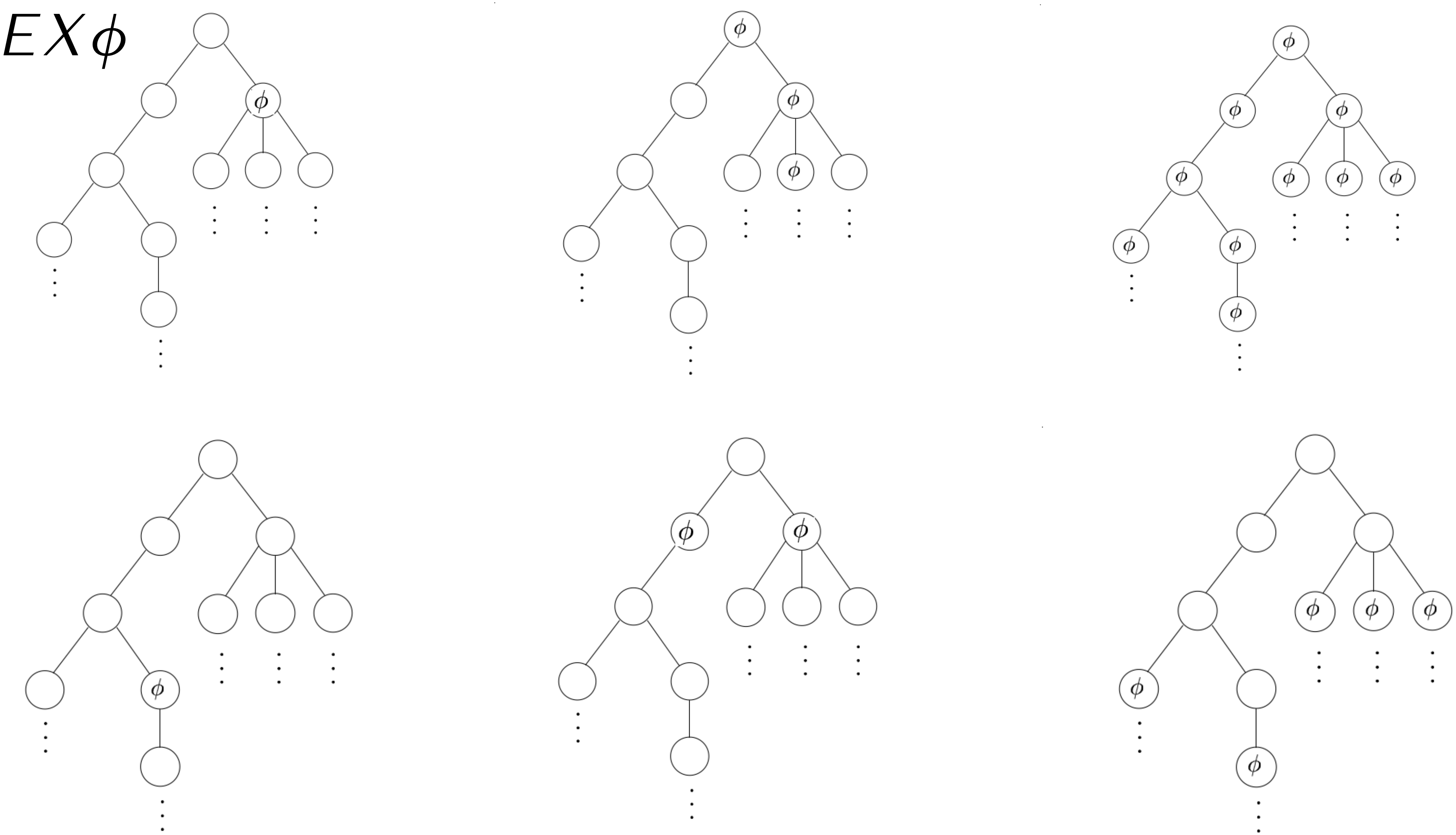

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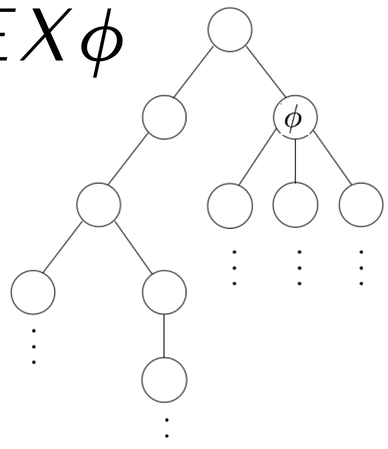
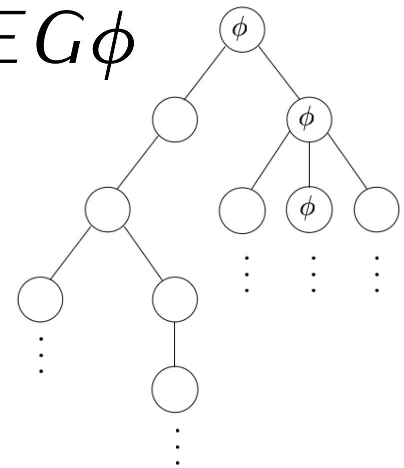
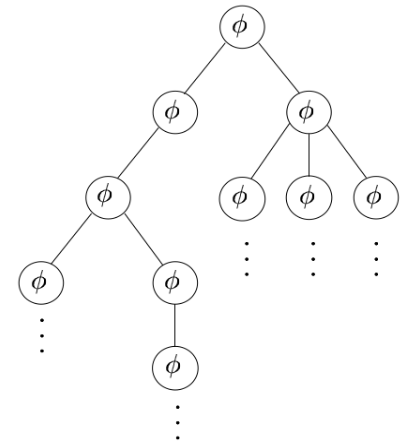
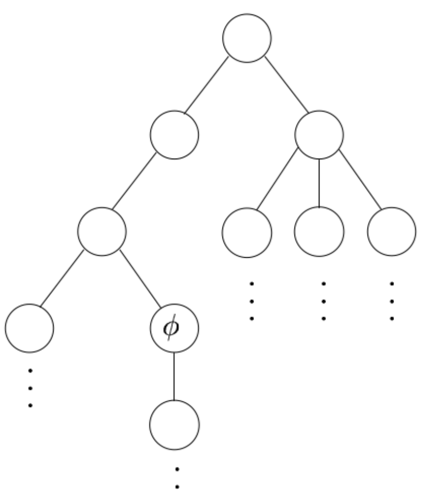
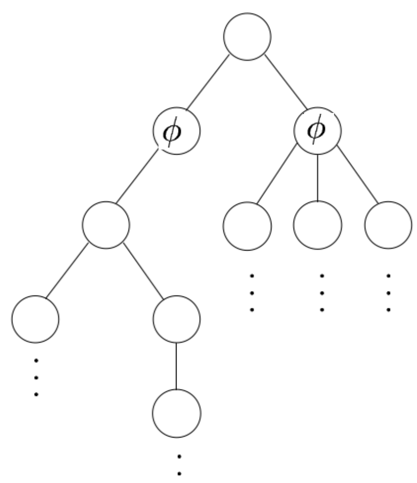


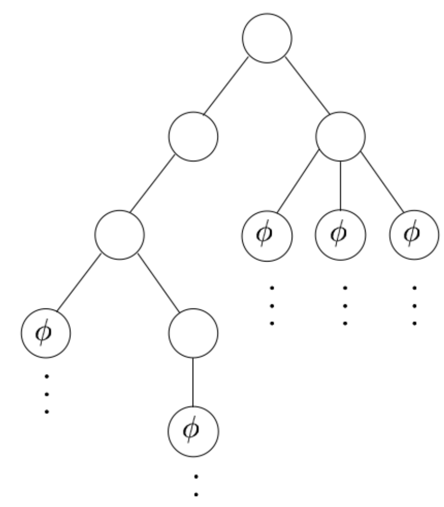
$EG\phi$











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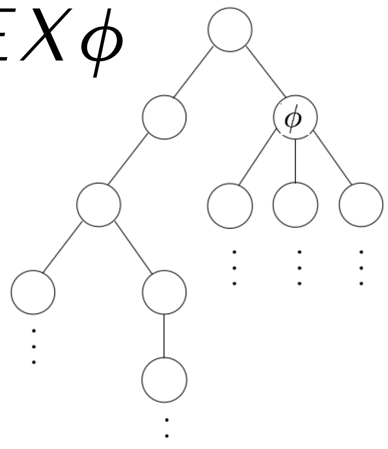

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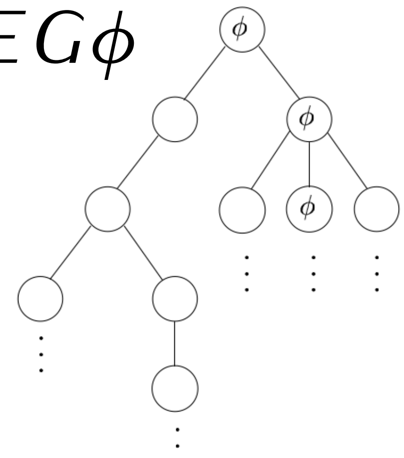
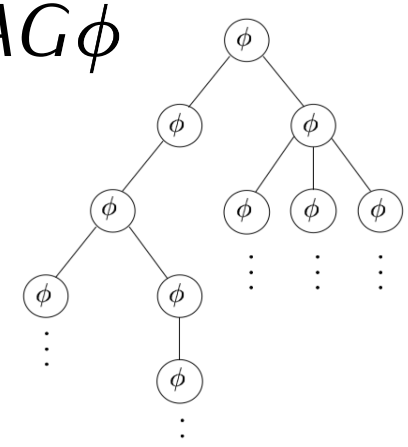
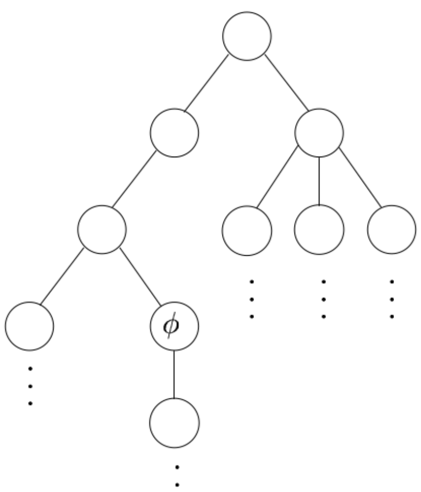
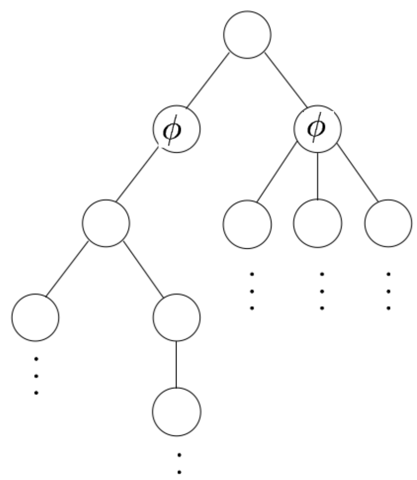
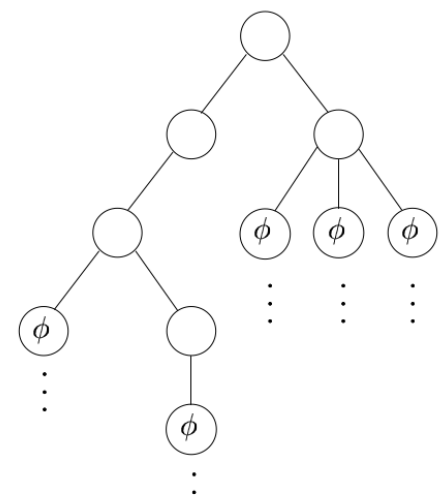


$AG\phi$









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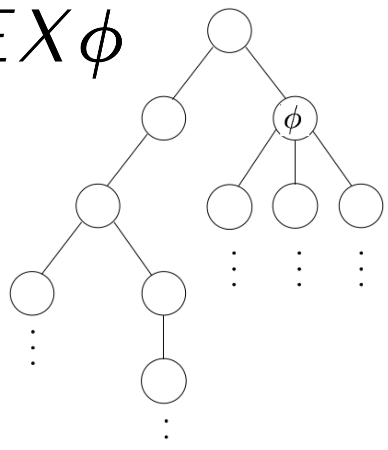

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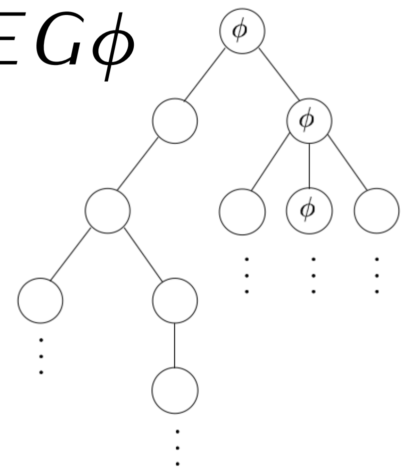
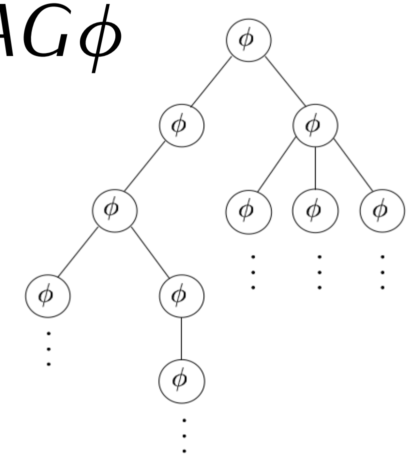
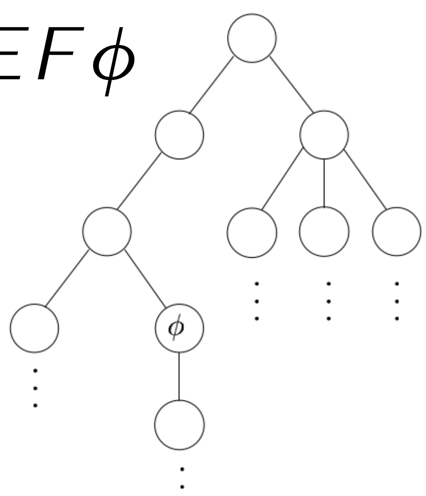
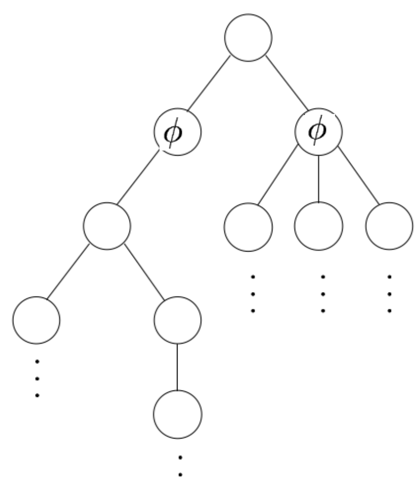


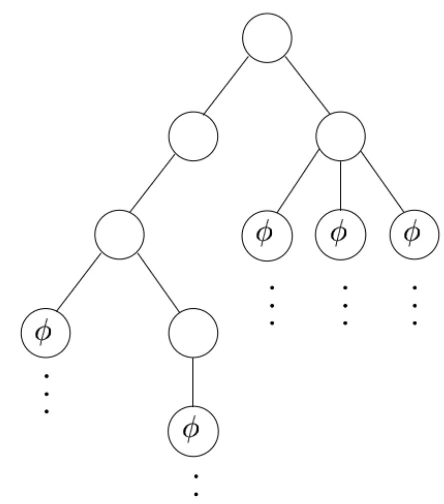
$AG\phi$



$EF\phi$







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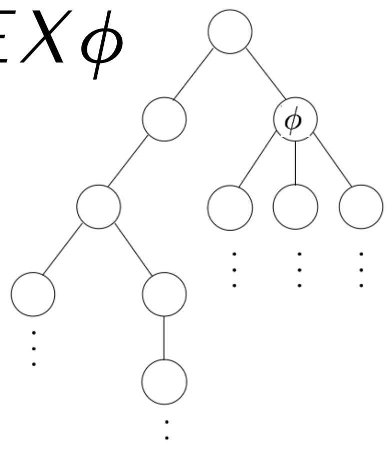
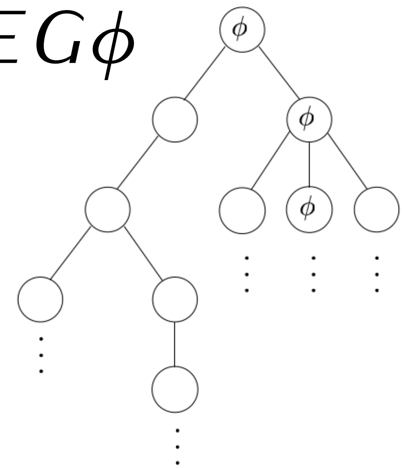
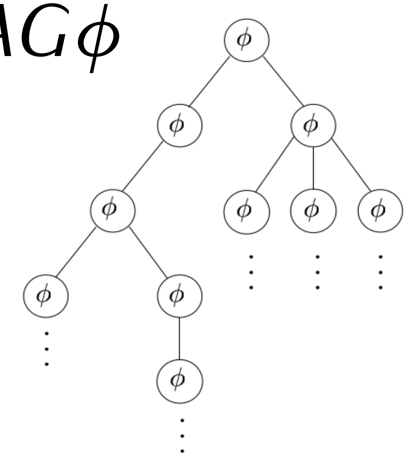


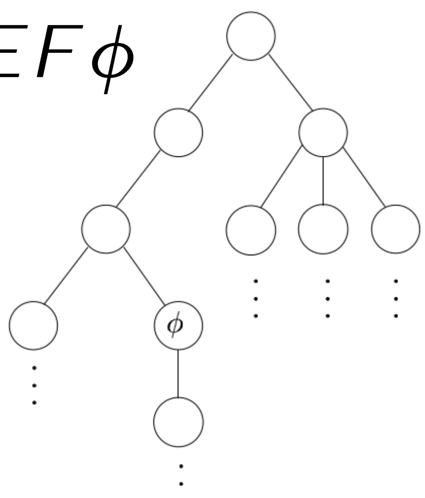
$EG\phi$



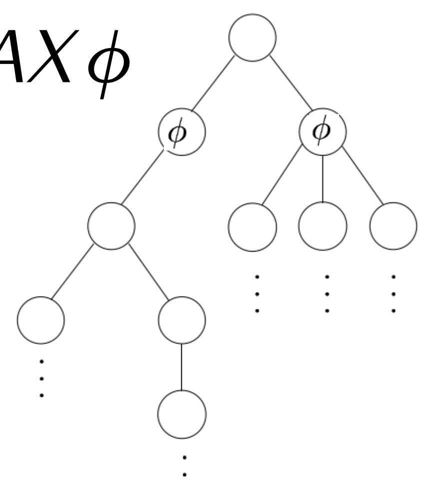
$AG\phi$

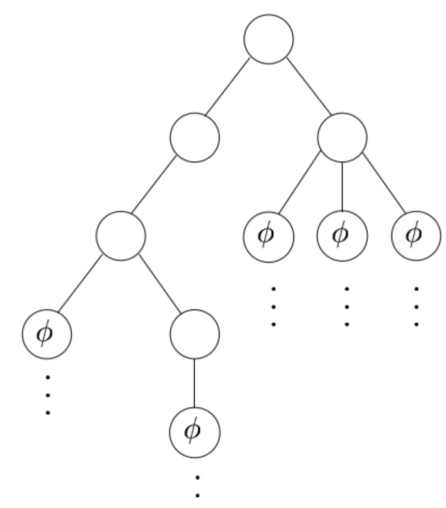


$EF\phi$



$AX\phi$





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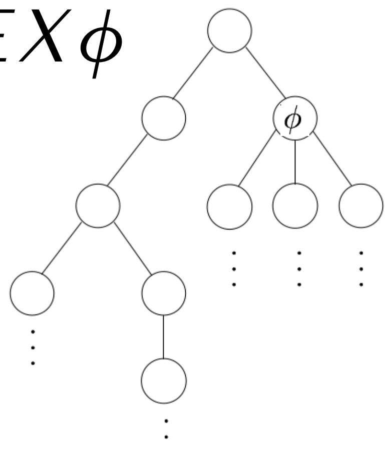
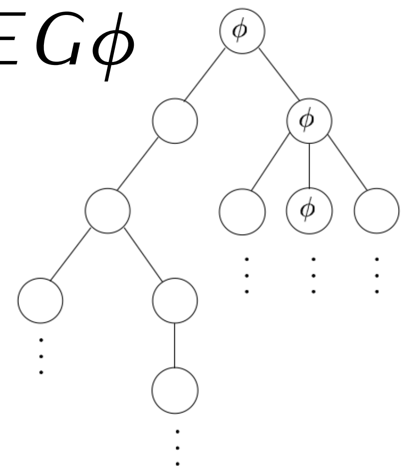
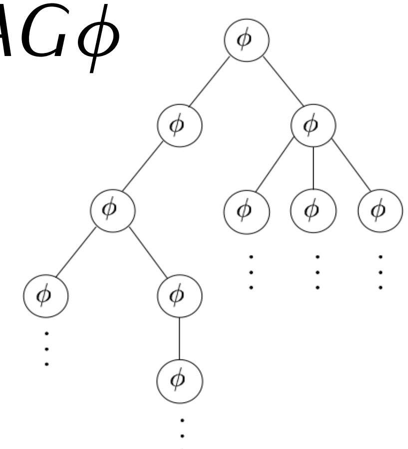


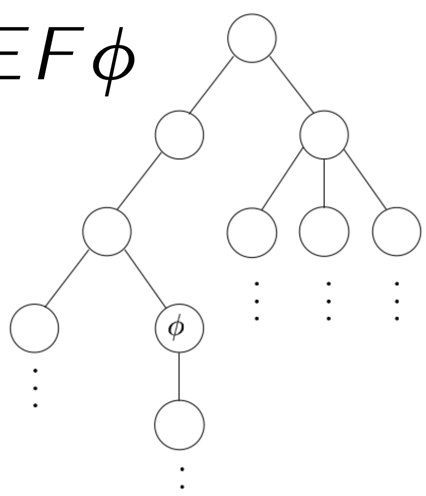
$EG\phi$



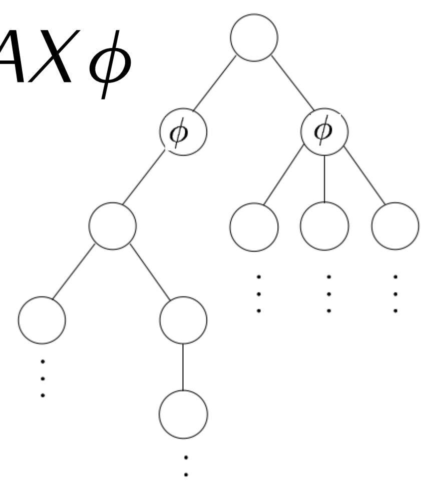
$AG\phi$



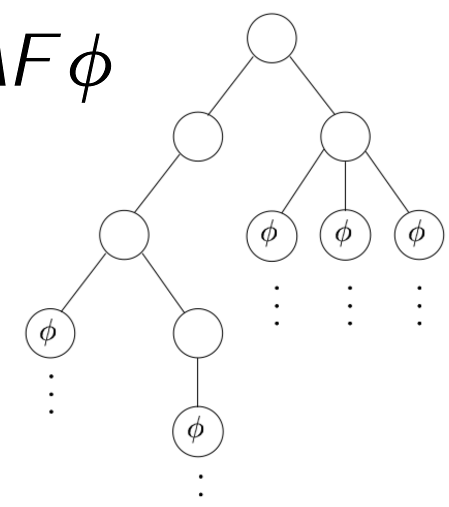
$EF\phi$



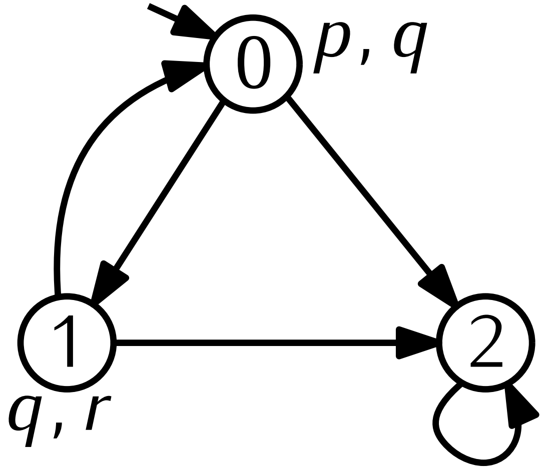
$AX\phi$



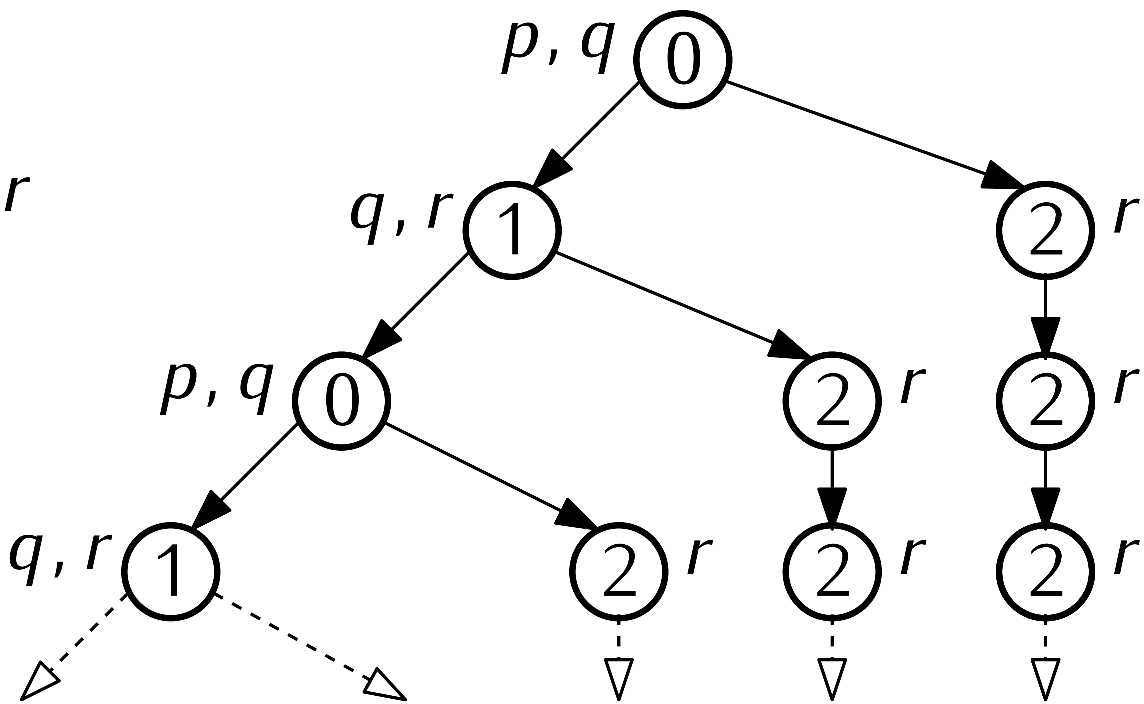
$AF\phi$



Computation Tree Logic Example Properties



Computation tree for $\mathcal{M}$



$\mathcal{M} \models p \land q$?

$\mathcal{M} \models \neg r$?

$\mathcal{M} \models EX(q \land r)$?

$\mathcal{M} \models AX(q \land r)$?

$\mathcal{M} \models \neg AX(q \land r)$?

$\mathcal{M} \models \neg EF(p \land r)$?

$\mathcal{M} \models EG\neg r$?

$\mathcal{M} \models AFq$?

$\mathcal{M} \models p\ AU\ r$?

$\mathcal{M} \models \neg(p \land q)\ EU\ r$?

Nested CTL operators

Consider the following sentence:

> No matter what, at some point the transition system reaches a state where there exists a path such that from that point on, p holds forever.

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Inevitably

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Inevitably Eventually

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Inevitably Eventually Possibly

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Inevitably Eventually Possibly Always p

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Inevitably Eventually Possibly Always p

$$AF\ EG\ p$$

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Inevitably Eventually Possibly Always p

$$AF\ EG\ p$$

A typical HW problem: translate a few sentences into CTL.

A basis for CTL

The CTL operators AG, AF, AX, EF, and EG can all be written using only negations ($\neg$), and EX, EU, and AU.

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$$AX\phi \equiv \ ????$$

$$AX\phi \equiv \neg EX\ \neg\phi$$

A typical HW problem: show that CTL operators can be written using only $\{\neg, EX, EU, AU\}$.

Another interesting property of CTL formulas

Any CTL operator $\oplus$ can be written using only $\oplus$, AX, EX, and Boolean connectives.

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Example: Operator AG can be written using only AG, AX, EX, and Boolean connectives.

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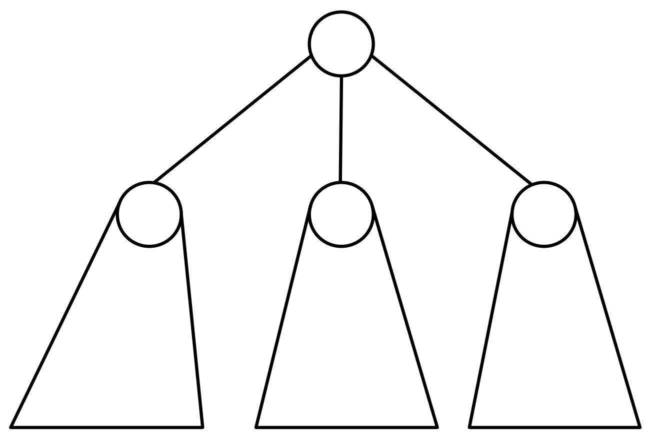

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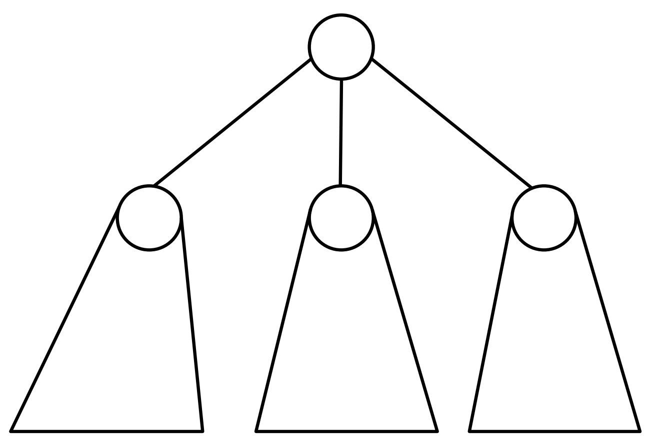

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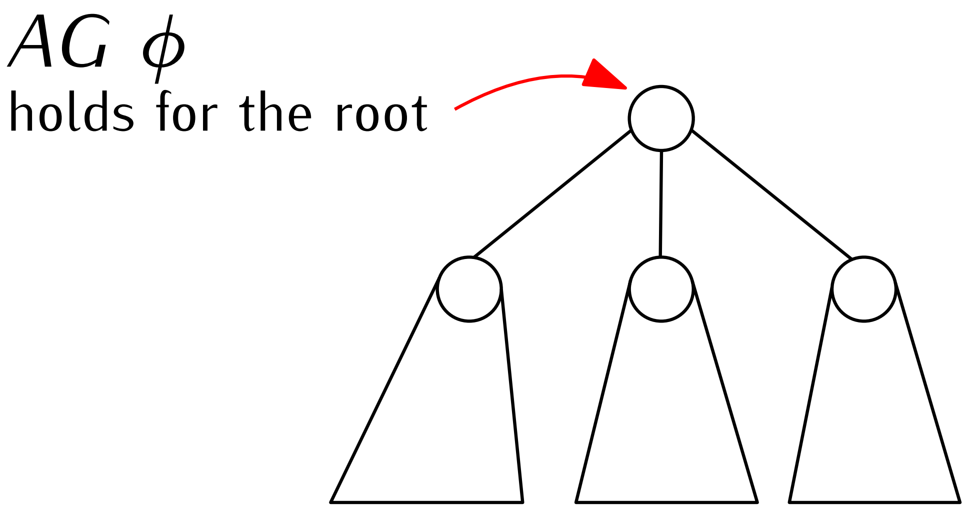

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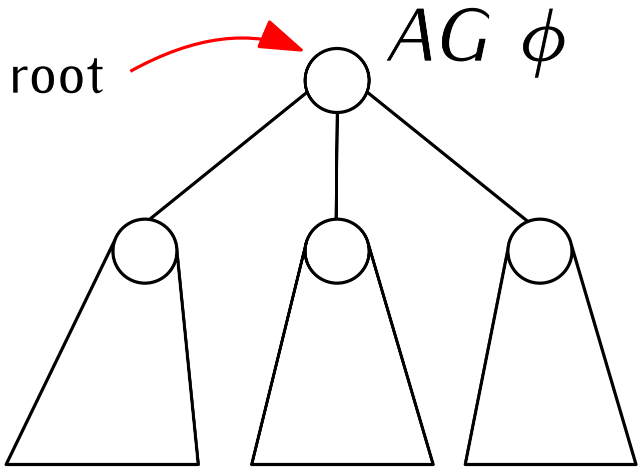

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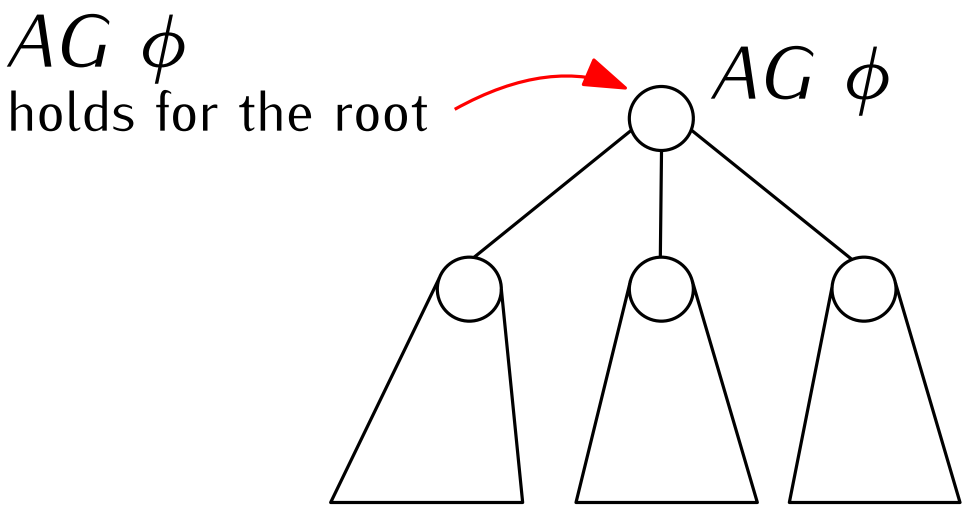

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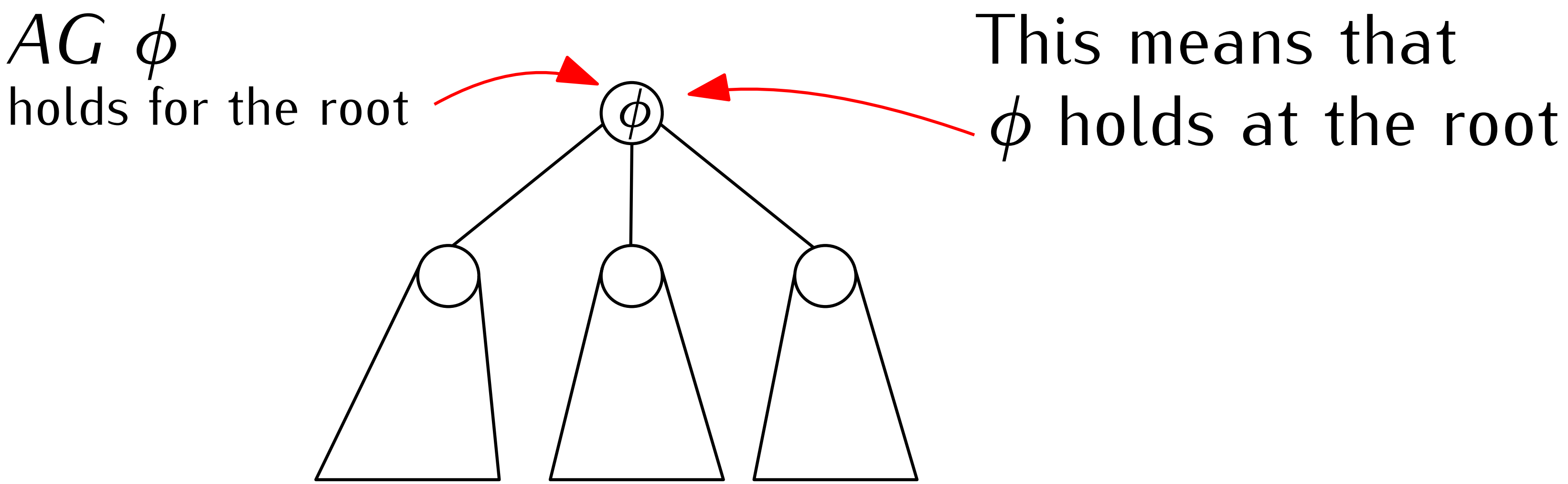

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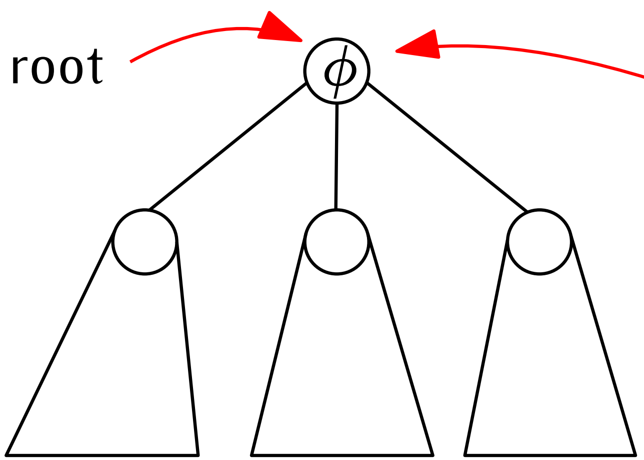

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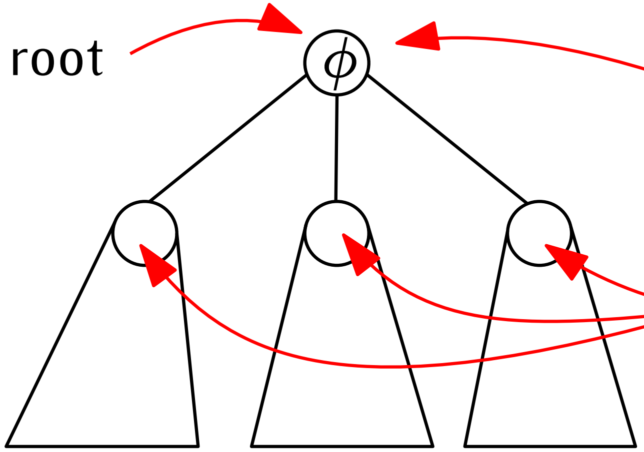

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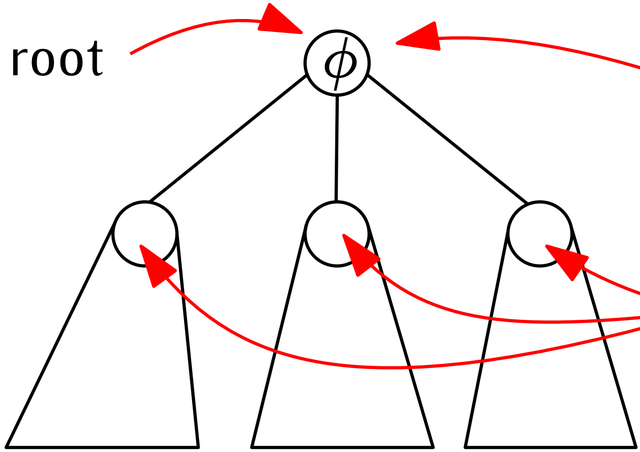

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$$AG\ \phi \equiv \phi \wedge (AX\ AG\ \phi)$$

A typical HW problem: Do the same for EG, EF, AF, EU, AU.

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In words, two formulas are not equivalent if we can find a transition system that satisifes one formula but not the other.

Showing that $\alpha \not\equiv \beta$

Consider these two temporal formulas

$F\ G\ p$ $\qquad\qquad$ $AF\ AG\ p$

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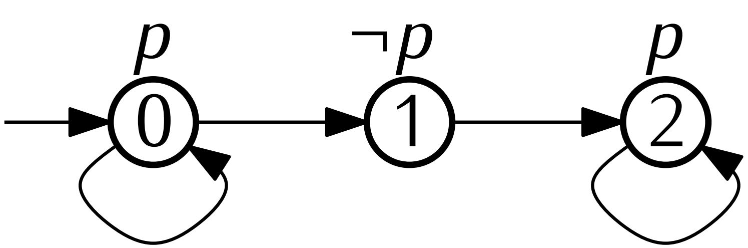

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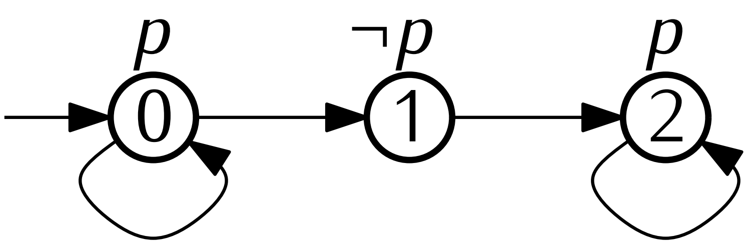

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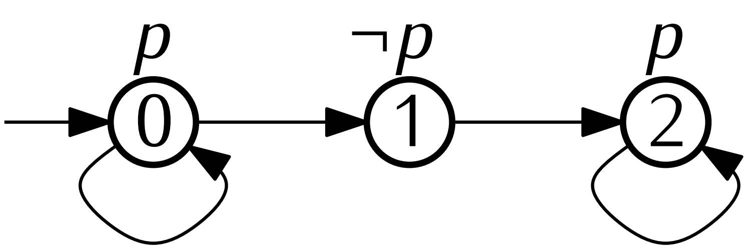

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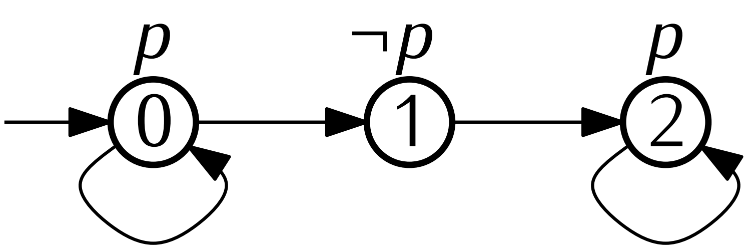

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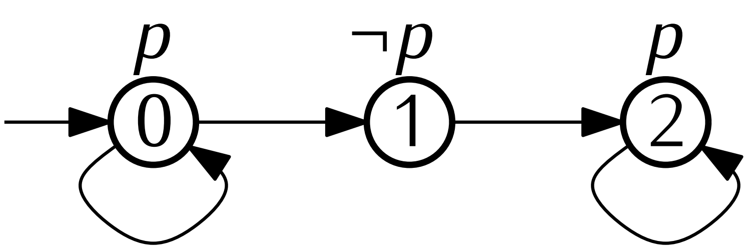

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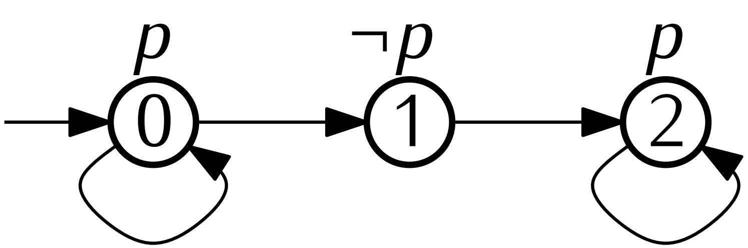

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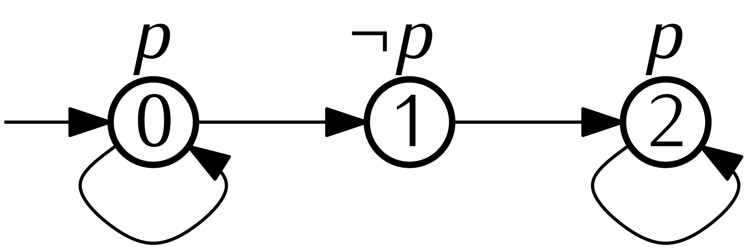

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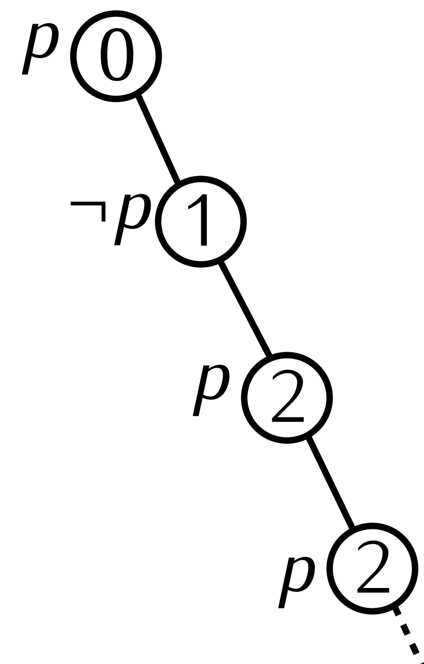

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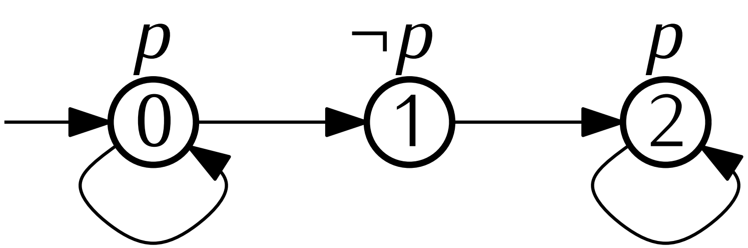

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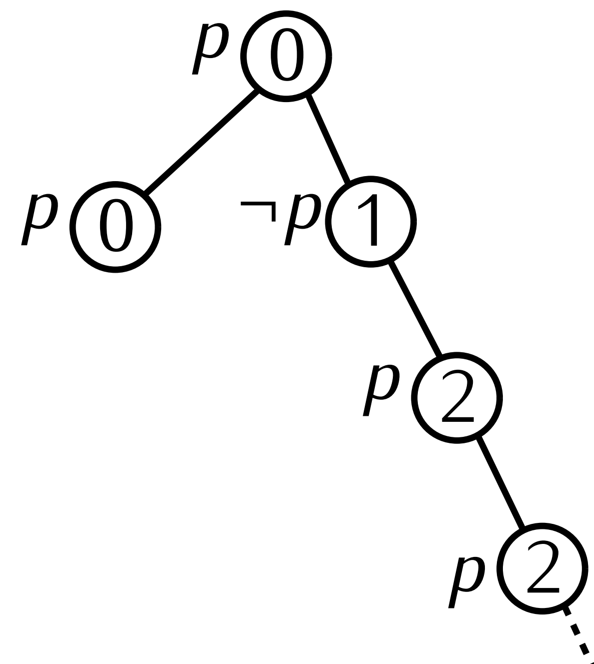

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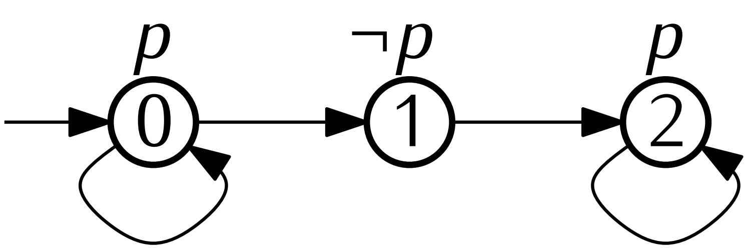

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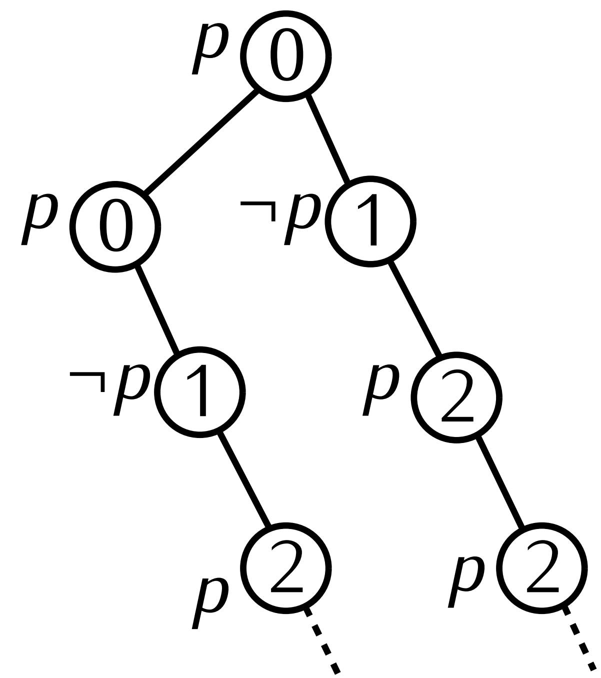

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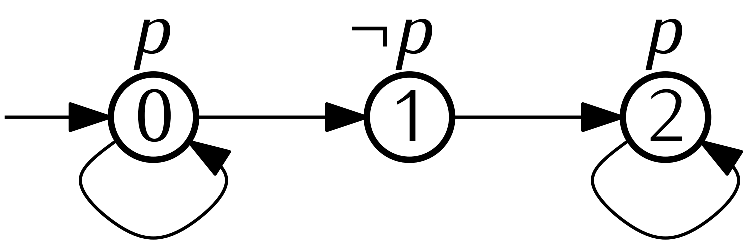

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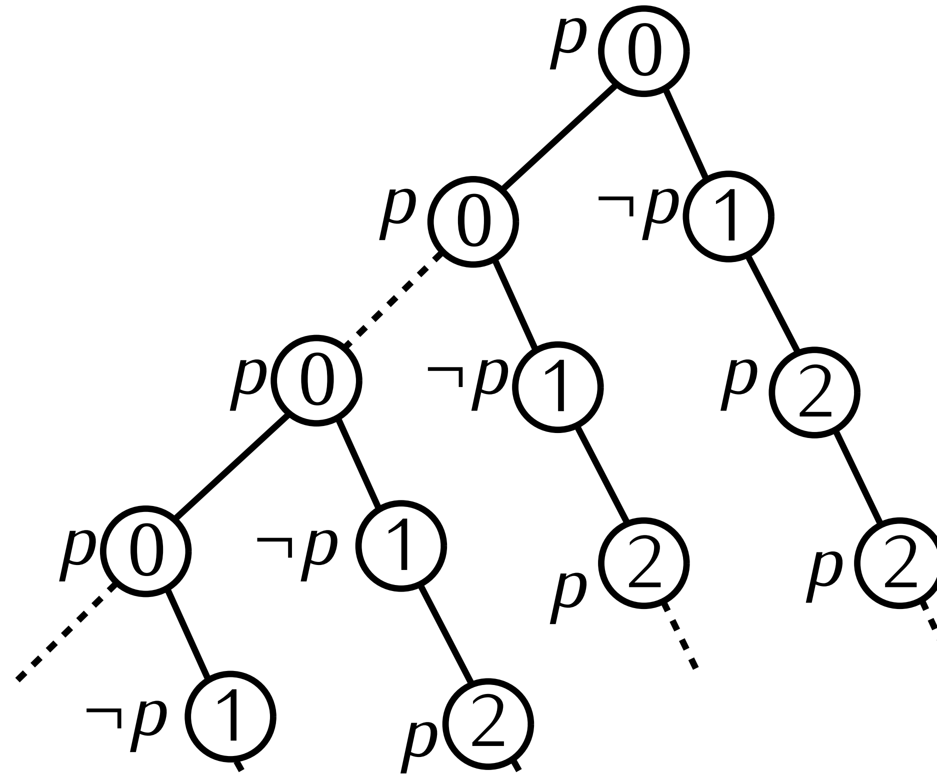

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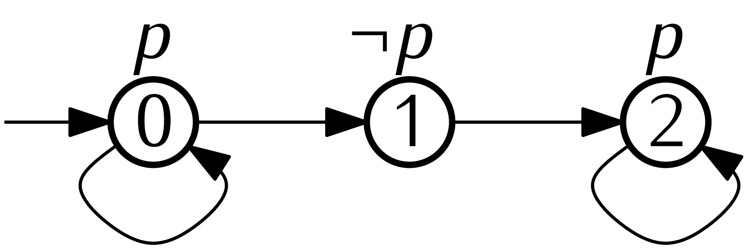

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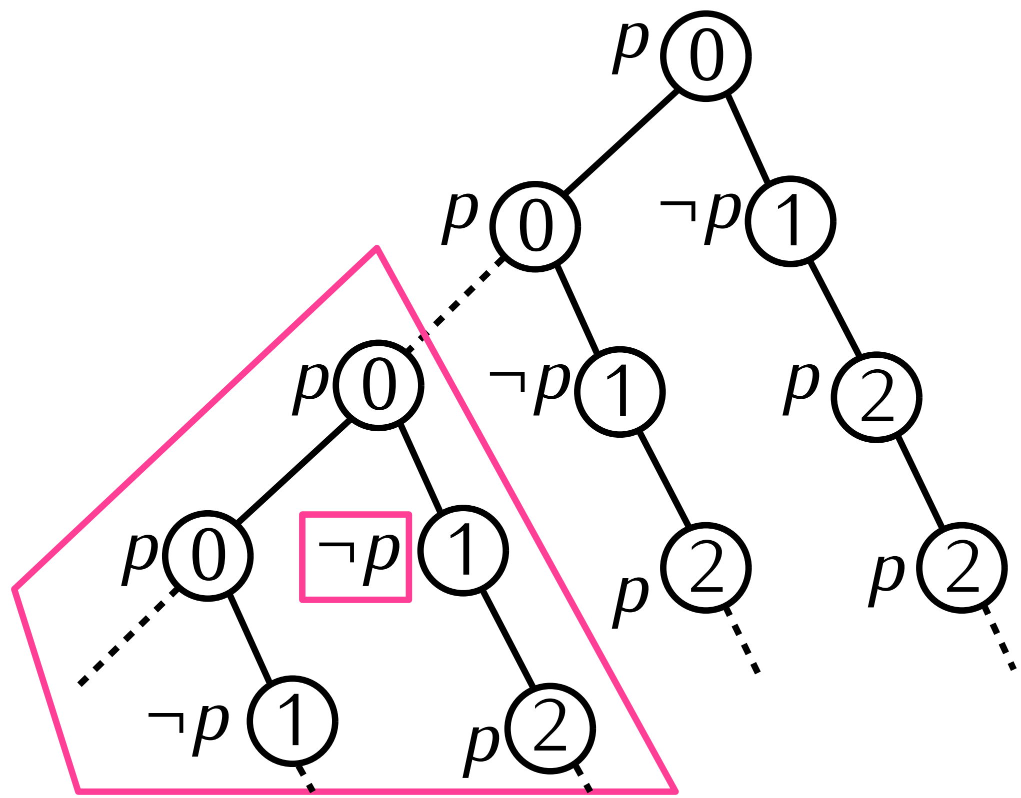

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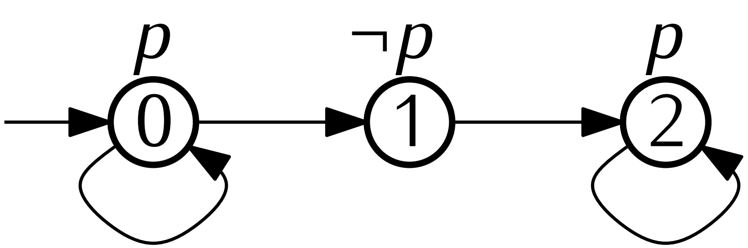

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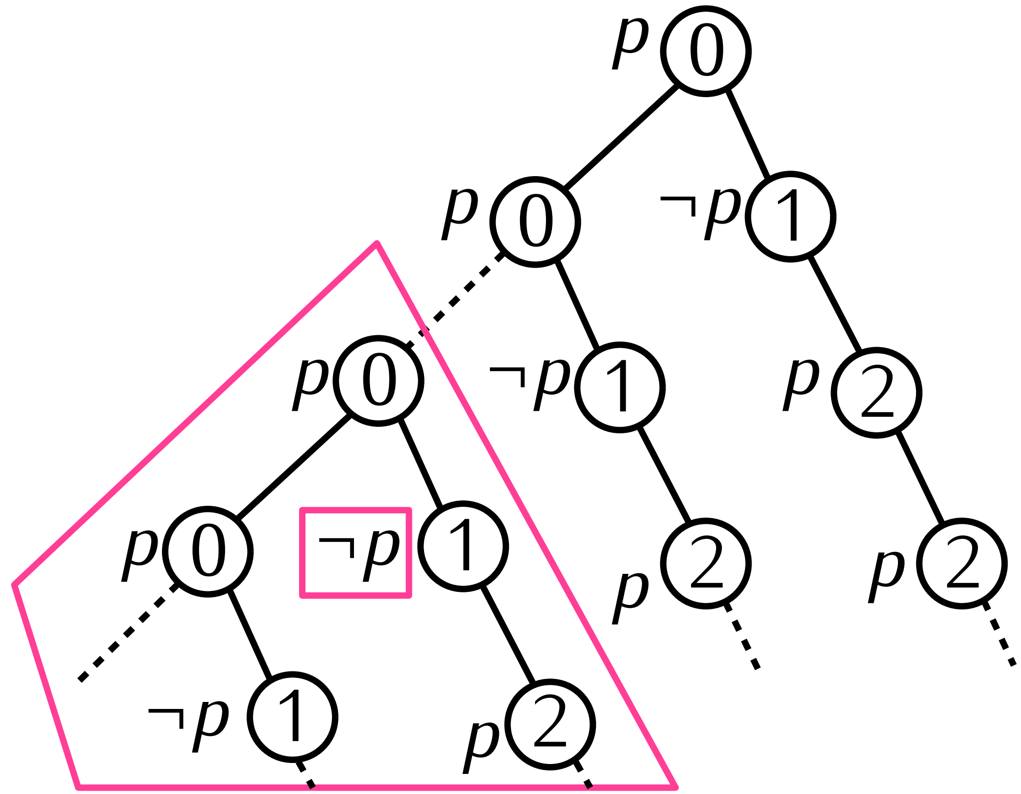

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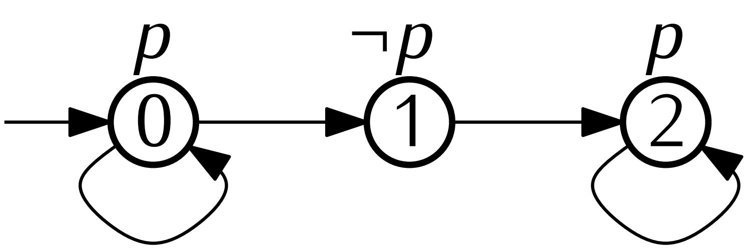

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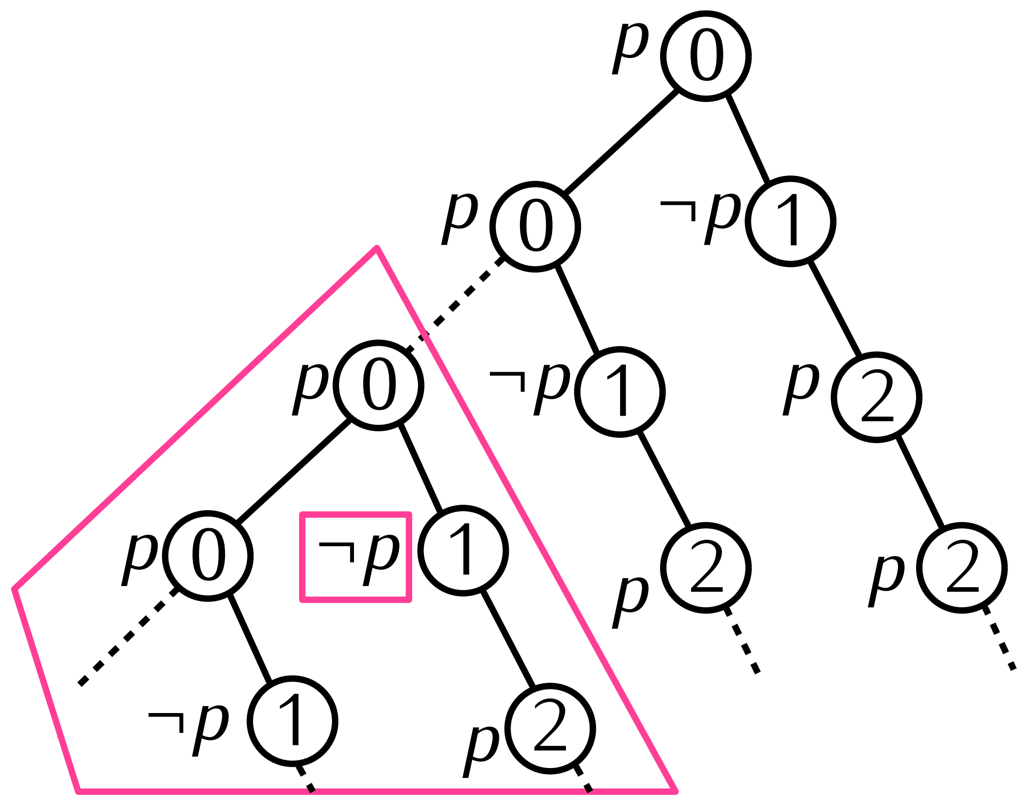

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