CS 181u Applied Logic

Lecture 12
The Big Picture

Reactive System Code satisfies Requirements

Transition System satisfies Temporal Logic Formula $\phi$

Model Checking
Computation Tree Logic (CTL) expresses properties of "alternative timelines".

\[ \mathcal{M} \models \phi \iff \forall s \in I \ s \models \phi \]

CTL Model Checking
CTL review  $AG\phi$  $EG\phi$  $AF\phi$  $EF\phi$  $AX\phi$  $EX\phi$

$EX\phi$  $EG\phi$  $AG\phi$

$EF\phi$  $AX\phi$  $AF\phi$
CTL Model Checking

Given $\mathcal{M}$ and CTL formula $\phi$
we want to check if $\mathcal{M} \models \phi$. 

$$\mathcal{M} \models \phi \iff \forall s \in I \ s \models \phi$$
Given $\mathcal{M}$ and CTL formula $\phi$
we want to check if $\mathcal{M} \models \phi$.

One idea: come up with some algorithm that looks at
the set of initial states $I$ and outputs *true* or *false*
depending on if $s \models \phi$ for all $s \in I$. 
CTL Model Checking

Given $M$ and CTL formula $\phi$, we want to check if $M \models \phi$.

One idea: come up with some algorithm that looks at the set of initial states $I$ and outputs true or false depending on if $s \models \phi$ for all $s \in I$.

A slightly different idea: figure out the set of states $S' \subseteq S$ such that for all $s \in S'$, $s \models \phi$. Then check if $I \subseteq S'$.
CTL Model Checking

\[ M \models \phi \iff \forall s \in I \ s \models \phi \]

Given \( M \) and CTL formula \( \phi \), we want to check if \( M \models \phi \).

One idea: come up with some algorithm that looks at the set of initial states \( I \) and outputs true or false depending on if \( s \models \phi \) for all \( s \in I \).

A slightly different idea: figure out the set of states \( S' \subseteq S \) such that for all \( s \in S' \), \( s \models \phi \). Then check if \( I \subseteq S' \).

This is easier.
CTL Model Checking

\[ M \models \phi \iff \forall s \in I \; s \models \phi \]

**Today’s goal:** an algorithm for CTL that does the following:

**Input:** \( M \) and \( \phi \)

**Output:** all states of \( M \) that satisfy \( \phi \)
Recall: Why so many operators?

AG EG AF EF AX EX AU EU
Recall: Why so many operators?

The acts of the mind, wherein it exerts its power over simple ideas, are chiefly these three: Combining several simple ideas into one compound one, and thus all complex ideas are made. The second is bringing two ideas, whether simple or complex, together, and setting them by one another so as to take a view of them at once, without uniting them into one, by which it gets all its ideas of relations. The third is separating them from all other ideas that accompany them in their real existence: this is called abstraction, and thus all its general ideas are made.

*SI*CP by Abelson, Sussman, and Sussman quoting John Locke from his *Essay Concerning Human Understanding*
Why so many operators?

We don’t actually need any of them if we are OK with always writing temporal properties using first order logic and quantifying over states and paths.
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However, they are useful to have on hand to state things concisely, like when writing \( \nu \text{SMV} \) specifications.
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However, they are useful to have on hand to state things concisely, like when writing νSMV specifications.

On the other hand, when performing meta-analysis of CTL, we need to examine each operator.

Hence, it is good to reduce everything down to a smallest set of sufficiently expressive operators.
Adequate set of operators for CTL

Let’s eliminate as many operators as possible by writing them in terms of other operators.

In fact, let’s try to write everything in terms of $E$-properties $EX$, $EU$, and $EG$, and Boolean operations $\neg$, $\land$, and $\lor$.

(On your HW, you wrote some operators in terms of $EX$, $EU$, and $AU$.)
Get rid of $AX \phi$
Get rid of $AX \phi$

$AX \phi \equiv \neg EX \neg \phi$
Get rid of $AG\phi$
Get rid of $AG\phi$

$AG\phi \equiv \neg EF\neg \phi$
Get rid of $AF\phi$
Get rid of $AF\phi$

$AF\phi \equiv \neg EG\neg \phi$
How to deal with $\phi AU \psi$?

$\phi AU \psi$ means that for all paths $\phi U \psi$.
How to deal with $\phi A U \psi$?

$\phi A U \psi$ means that for all paths $\phi U \psi$

$\phi A U \psi$ means that there is no path where $\neg (\phi U \psi)$

$\phi A U \psi \equiv \neg (\ )$
How to deal with $\phi AU \psi$?

$\phi AU \psi$ means that for all paths $\phi U \psi$
$\phi AU \psi$ means that there is no path where $\neg(\phi U \psi)$

What kinds of paths satisfy $\neg(\phi U \psi)$?

$\phi AU \psi \equiv \neg(\ )$
How to deal with $\phi AU \psi$?

$\phi AU \psi$ means that for all paths $\phi U \psi$
$\phi AU \psi$ means that there is no path where $\neg(\phi U \psi)$

What kinds of paths satisfy $\neg(\phi U \psi)$?

Either $\psi$ never holds:

Either $\neg \psi$ never holds:

$\phi AU \psi \equiv \neg ( EG \neg \psi )$
How to deal with $\phi \mathbf{AU} \psi$?

$\phi \mathbf{AU} \psi$ means that for all paths $\phi \mathbf{U} \psi$

$\phi \mathbf{AU} \psi$ means that there is no path where $\neg (\phi \mathbf{U} \psi)$

What kinds of paths satisfy $\neg (\phi \mathbf{U} \psi)$?

Either $\psi$ never holds:

\[ \neg \psi \quad \neg \psi \quad \neg \psi \quad \neg \psi \quad \neg \psi \quad \neg \psi \quad \neg \psi \]

Or $\phi$ stops holding sometime before $\psi$ holds:

\[ \phi, \neg \psi \quad \phi, \neg \psi \quad \phi, \neg \psi \quad \phi, \neg \psi \quad \neg \phi, \neg \psi \quad \phi, \neg \psi \quad \neg \phi, \psi \]

\[
\phi \mathbf{AU} \psi \equiv \neg (\mathbf{EG} \neg \psi)
\]
How to deal with $\phi \text{ AU} \psi$?

$\phi \text{ AU} \psi$ means that for all paths $\phi \text{ U} \psi$

$\phi \text{ AU} \psi$ means that there is no path where $\neg(\phi \text{ U} \psi)$

What kinds of paths satisfy $\neg(\phi \text{ U} \psi)$?

Either $\psi$ never holds:

- $\neg \psi$

Or $\phi$ stops holding sometime before $\psi$ holds

- $\phi, \neg \psi$
- $\phi, \neg \psi$
- $\phi, \neg \psi$
- $\neg \phi, \neg \psi$
- $\phi, \neg \psi$
- $\neg \phi, \psi$

$\phi \text{AU} \psi \equiv \neg ( \text{EG} \neg \psi \lor \neg \psi \text{ EU} (\neg \phi \land \neg \psi) )$
Summarizing:

\[ AX \phi \equiv \neg EX \neg \phi \]

\[ AG \phi \equiv \neg EF \neg \phi \]

\[ AF \phi \equiv \neg EG \neg \phi \]

\[ \phi AU \psi \equiv \neg ( EG \neg \phi \lor \neg \phi \ EU \ (\neg \phi \land \neg \psi)) \]
Summarizing:

<table>
<thead>
<tr>
<th>Property</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AX\phi$</td>
<td>$\neg EX \neg \phi$</td>
</tr>
<tr>
<td>$AG\phi$</td>
<td>$\neg EF \neg \phi$</td>
</tr>
<tr>
<td>$AF\phi$</td>
<td>$\neg EG \neg \phi$</td>
</tr>
<tr>
<td>$\phi AU \psi$</td>
<td>$\neg (\neg \neg \phi \lor \neg \phi \lor (\neg \phi \land \neg \psi))$</td>
</tr>
</tbody>
</table>

All of the $A$-properties can be written in terms of the $E$-properties and Boolean connectives.
Summarizing:

\[
\begin{align*}
AX \phi & \equiv \neg EX \neg \phi \\
AG \phi & \equiv \neg EF \neg \phi \\
AF \phi & \equiv \neg EG \neg \phi \\
\phi AU \psi & \equiv \neg (EG \neg \phi \lor \neg \phi EU (\neg \phi \land \neg \psi))
\end{align*}
\]

All of the \textit{A}-properties can be written in terms of the \textit{E}-properties and Boolean connectives. This is called \textbf{existential negation normal form} for CTL.
Summarizing:

\[
\begin{align*}
AX \phi & \equiv \neg EX \neg \phi \\
AG \phi & \equiv \neg EF \neg \phi \\
AF \phi & \equiv \neg EG \neg \phi \\
\phi AU \psi & \equiv \neg ( EG \neg \phi \lor \neg \phi EU (\neg \phi \land \neg \psi))
\end{align*}
\]

All of the $A$-properties can be written in terms of the $E$-properties and Boolean connectives. This is called **existential negation normal form** for CTL.

Furthermore, $EF \phi \equiv \top EU \phi$

We only need $EX, EU, EG$
The main idea

First convert everything into existential negation normal form using previous reductions, so that we have only formulas with $EX, EG, EU$.

For each of the operators $EX, EG, EU$, give a method to determine the corresponding set of states that satisfy the property.
The Algorithm for $EX \phi$

First, an example

$EX(p \land q)$

Diagram:

- $s_1 \rightarrow \emptyset$
- $s_1 \rightarrow s_2$
- $s_2 \rightarrow \{q\}$
- $s_2 \rightarrow s_5$
- $s_5 \rightarrow \{p\}$
- $s_3 \rightarrow \{p, q\}$
- $s_4 \rightarrow \{p, q\}$
- $s_4 \rightarrow s_5$
The Algorithm for $EX \phi$

First, an example $EX(p \land q)$

Diagram:
- Initial state $S_1$ with transition to $S_2$ on $q$
- $S_2$ with transition to $S_3$ on $p \land q$
- $S_3$ with transition to $S_1$ on $p \land q$
- $S_4$ with transition to $S_5$ on $p \land q$
- $S_5$ with self-loop on $p$
The Algorithm for $EX \ \phi$

First, an example

$$EX(p \land q)$$

\[ \begin{align*}
S_1 & \xrightarrow{\emptyset} S_2 \\
S_3 & \xrightarrow{p \land q} S_1 \\
S_4 & \xrightarrow{p \land q} S_3 \\
S_5 & \xrightarrow{\{p\}} S_4 \\
\end{align*} \]
The Algorithm for $EX \phi$

First, an example

$EX(p \land q)$
The Algorithm for $EX\ \phi$

First, an example

$EX(p \land q)$

$\emptyset$

$s_1$ $
\rightarrow$

$\{q\}$

$s_2$

$\{p\}$

$s_3$

$\{p, q\}$

$p \land q$

$s_4$

$\{p, q\}$

$p \land q$

$s_5$

$EX(p \land q)$

$EX(p \land q)$
The Algorithm for $EX \phi$

First, an example

$EX(p \land q)$

$\emptyset$

$s_1$

$p \land q$

$s_3$

$\{p, q\}$

$s_2$

$\{q\}$

$s_4$

$\{p, q\}$

$s_5$

$\{p\}$

$s_1, s_2, s_4 \models EX(p \land q)$
Another example

$EX(p \land \neg q)$

Diagram:

- States: $S_1$, $S_2$, $S_3$, $S_4$
- Transitions:
  - $S_1 \rightarrow S_2$ with label $\emptyset$
  - $S_2 \rightarrow S_3$ with label $\{q\}$
  - $S_3 \rightarrow S_4$ with label $\{p, q\}$
  - $S_4 \rightarrow S_1$ with label $\{p\}$
The Algorithm for $EX\phi$

Another example

$EX(p \land \neg q)$

\[\begin{array}{c}
S_1 \rightarrow \emptyset \\
S_2 \rightarrow \{q\} \\
S_3 \rightarrow \{p, q\} \\
S_4 \rightarrow \{p\}
\end{array}\]
The Algorithm for $EX \phi$

Another example

$EX(p \land \neg q)$

$EX(p \land \neg q)$

$\emptyset$

$\{q\}$

$\{p, q\}$

$\{p\}$

$p \land \neg q$

$p \land \neg q$
The Algorithm for \( EX \phi \)

Another example

\[ EX(p \land \neg q) \]

\[ \emptyset \]

\[ S_1 \]

\[ \{ p, q \} \]

\[ S_3 \]

\[ EX(p \land \neg q) \]

\[ S_2 \]

\[ \{ q \} \]

\[ S_4 \]

\[ \{ p \} \]

\[ EX(p \land \neg q) \]
The Algorithm for $EX \phi$

Another example

$EX(p \land \neg q)$

$s_2, s_3, s_4 \models EX(p \land q)$
The Algorithm for $EX \phi$

After labelling all states $s$ that satisfy $\phi$, label and state $s'$ with $EX\phi$ if there is a transition from $s'$ to $s$. 

\[ \text{Diagram:} \]

\[ s' \rightarrow \phi \]

\[ s' \rightarrow \phi \]

\[ s' \rightarrow \phi \]

\[ s' \rightarrow \phi \]
The Algorithm for $EX \phi$

After labelling all states $s$ that satisfy $\phi$, label and state $s'$ with $EX\phi$ if there is a transition from $s'$ to $s$. 
The Algorithm for $EX \phi$

After labelling all states $s$ that satisfy $\phi$, label and state $s'$ with $EX\phi$ if there is a transition from $s'$ to $s$.

Call this process $\text{SAT}_{EX}(\phi)$
The Algorithm for \( \phi \ E U \ \psi \)

First, an example \( p \ E U \ q \)
The Algorithm for $\phi \ EU \ \psi$

First, an example $p \ EU \ q$

![Diagram of the algorithm](image-url)
The Algorithm for $\phi \ EU \ \psi$

First, an example

$p \ EU \ q$

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Graphical representation of the algorithm with states $s_1$, $s_2$, $s_3$, $s_4$, $s_5$, $s_6$.
The Algorithm for $\phi \ EU \ \psi$

First, an example

$p \ EU \ q$

Diagram:

- **$s_1$** with set $\{p\}$
- **$s_2$** with set $\{p\}$
- **$s_3$** with set $\{p\}$
- **$s_4$** with set $\emptyset$
- **$s_5$** with set $\{p\}$
- **$s_6$** with set $\{q\}$

Transition paths:

- $s_1 \rightarrow s_2$
- $s_2 \rightarrow s_3$
- $s_3 \rightarrow s_4$
- $s_4 \rightarrow s_5$
- $s_5 \rightarrow s_6$
- $s_6 \rightarrow s_3$

**Note:** The diagram represents the algorithm's flow with states and transitions.
The Algorithm for $\phi E U \psi$

First, an example

$p E U q$
The Algorithm for $\phi \ E U \ \psi$

First, an example \[ p \ E U \ q \]
First, an example $p \text{ EU } q$

$s_1, s_2, s_3, s_5, s_6 \models (p \text{ EU } q)$
The Algorithm for $\phi \ EU \ \psi$

Another example

$\neg p \ EU \ \neg q$

![Diagram of the algorithm with states $S_1$ to $S_6$ and transitions labeled with sets $\{p\}$, $\{q\}$, and $\emptyset$.]
The Algorithm for $\phi \, EU \, \psi$

Another example: $\neg p \, EU \, \neg q$

Diagram:

- $s_1$ with $\neg p, \{q\}$
- $s_2$ with $\neg p, \{q\}$
- $s_3$ with $\neg p, \neg q$
- $s_4$ with $\{q\}, \neg p$
- $s_5$ with $\{p\}, \neg q$
- $s_6$ with $\{p\}, \neg q$
The Algorithm for $\phi \ E U \ \psi$

Another example

$\neg p \ E U \ \neg q$

Diagram:

- States: $s_1, s_2, s_3, s_4, s_5, s_6$
- Edges:
  - $s_1$ to $s_2$: $\neg p$, $\{q\}$
  - $s_2$ to $s_3$: $\neg p$
  - $s_3$ to $s_4$: $\emptyset$
  - $s_4$ to $s_5$: $\{q\}$
  - $s_5$ to $s_6$: $\{p\}$
  - $s_6$ to $s_1$: $\neg q$

Transitions:

- $s_1$ to $s_2$: $\neg p$, $\{q\}$
- $s_2$ to $s_3$: $\neg p$
- $s_3$ to $s_4$: $\emptyset$
- $s_4$ to $s_5$: $\{q\}$
- $s_5$ to $s_6$: $\{p\}$
- $s_6$ to $s_1$: $\neg q$

$\neg p \ E U \ \neg q$
The Algorithm for $\phi \ E U \ \psi$

Another example

$\neg p \ E U \ \neg q$

```
\begin{tikzpicture}
  \node[roundnode] (s1) at (0,0) {$\neg p$};
  \node[roundnode] (s2) at (2,1) {$\neg p \ \{q\}$};
  \node[roundnode] (s3) at (4,0) {$\neg p, \neg q$};
  \node[roundnode] (s4) at (6,1) {\{}; \node[roundnode] (s5) at (6,-1) {\{}; \node[roundnode] (s6) at (2,-1) {\{}; \node[roundnode] (s7) at (0,-1) {\{};
  \path[arrow] (s1) edge (s2);
  \path[arrow] (s2) edge (s3);
  \path[arrow] (s3) edge (s4);
  \path[arrow] (s4) edge (s2);
  \path[arrow] (s5) edge (s6);
  \path[arrow] (s6) edge (s1);
  \path[arrow] (s7) edge (s5);
  \path[arrow] (s5) edge (s1);
\end{tikzpicture}
```
The Algorithm for $\phi \ EU \ \psi$

Another example

$\neg p \ EU \ \neg q$

$\neg p, \neg q$

$\neg p \ EU \neg q$

$\emptyset$

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$

$\neg p$s

$\neg q$

$\neg p \ EU \neg q$

$\neg q$

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$
The Algorithm for $\phi \ EU \ \psi$

Another example

$\neg p \ EU \ \neg q$

\[\neg p \ EU \ \neg q\]

\[\neg p \ E U \ \neg q\]

\[\neg p, \neg q\]

\[\emptyset\]

\[\{q\}\]

\[\{p\}\]

\[\neg q\]

\[\neg p \ EU \ \neg q\]

\[\neg p \ E U \ \neg q\]

\[\neg p \ E U \ \neg q\]

\[\neg p \ E U \ \neg q\]
The Algorithm for $\phi \ EU \ \psi$

Another example

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$

$\neg p$  

$\{q\}$

$s_2$

$\neg p \ EU \neg q$

$\neg p$  

$\{q\}$

$s_3$

$\neg p \ EU \neg q$

$\neg p, \neg q$

$\emptyset$

$s_4$

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$

$s_6$

$\neg q$

$s_5$

$\neg q$

$s_1$

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$

$s_2$

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$

$s_3$

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$

$s_4$

$\neg p \ EU \neg q$

$\neg p \ EU \neg q$

$s_5$

$\neg p \ EU \neg q$
The Algorithm for $\phi \text{EU} \psi$

Another example

$\neg p \text{EU} \neg q$

$\neg p \text{EU} \neg q$

$\neg p$

$\{q\}$

$s_2$

$s_3$

$s_4$

$s_1$

$s_6$

$s_5$

$\neg q$

$\neg p \text{EU} \neg q$

$\neg p \text{EU} \neg q$

$\neg p, \neg q$

$\emptyset$

$s_1, s_2, s_3, s_4, s_5, s_6 \models (\neg p \text{EU} \neg q)$
The Algorithm for $\phi \ EU \ \psi$
The Algorithm for $\phi \ EU \ \psi$

If a state is labelled with $\psi$ label it with $\phi \ EU \ \psi$. 
The Algorithm for $\phi \ EU \ \psi$

If a state is labelled with $\psi$ label it with $\phi \ EU \ \psi$. 
The Algorithm for $\phi \ EU \ \psi$

If a state is labelled with $\psi$ label it with $\phi \ EU \ \psi$.

For any state $s'$ labelled with $\phi$, if at least one successor state $s$ is labelled with $\phi \ EU \ \psi$, then label $s'$ with $\phi \ EU \ \psi$ as well. Repeat until labels stop changing.
If a state is labelled with $\psi$ label it with $\phi \ EU \ \psi$.

For any state $s'$ labelled with $\phi$, if at least one successor state $s$ is labelled with $\phi \ EU \ \psi$, then label $s'$ with $\phi \ EU \ \psi$ as well. Repeat until labels stop changing.
The Algorithm for $\phi \ EU \ \psi$

If a state is labelled with $\psi$ label it with $\phi \ EU \ \psi$.

For any state $s'$ labelled with $\phi$, if at least one successor state $s$ is labelled with $\phi \ EU \ \psi$, then label $s'$ with $\phi \ EU \ \psi$ as well. Repeat until labels stop changing.

Call this process $\text{SAT}_{EU}(\phi, \psi)$
The Algorithm for $EG \phi$

First, an example

$EG \ p$
The Algorithm for $EG \phi$

First, an example $EG p$

\[
\begin{array}{c}
S_1 \rightarrow S_2 \rightarrow S_3 \\
S_4 \rightarrow S_5 \rightarrow S_6
\end{array}
\]

$EG p$

$\{p\}$

$EG p$

$\{q\}$

$EG p$

$\emptyset$

$EG p$

$EG p$

$EG p$

$EG p$
The Algorithm for $EG \phi$

First, an example $EG p$

![Diagram]
The Algorithm for $EG \phi$

First, an example

$EG \ p$

\[
\begin{array}{c}
S_1 \xrightarrow{p} S_2 \\
S_2 \xrightarrow{p} S_3 \\
S_3 \xrightarrow{q} S_3 \\
S_3 \xrightarrow{} S_4 \\
S_4 \xrightarrow{p} S_5 \\
S_5 \xrightarrow{} S_6 \\
S_6 \xrightarrow{p} S_6
\end{array}
\]
The Algorithm for $EG \phi$

First, an example

$EG p$

\[\begin{array}{cccccc}
S_1 & \xleftarrow{s_1} & S_2 & \xrightarrow{s_2} & S_3 & \xrightarrow{s_3} \\
\{p\} & & \{p\} & & \{q\} \\
S_4 & \xleftarrow{s_4} & S_5 & \xleftarrow{s_5} & S_6 & \xleftarrow{s_6} \\
\{p\} & & \emptyset & & \{p\}
\end{array}\]
The Algorithm for $EG \phi$

First, an example: $EG \ p$

$EG \ p$

$\{p\}$

$s_1$

$EG \ p$

$s_2$

$\{p\}$

$s_3$

$\{q\}$

$s_6$

$\emptyset$

$s_5$

$s_4$

$\{p\}$

$s_1, s_2, s_4 \models EG \ p$
The Algorithm for $EG \phi$
The Algorithm for $EG \phi$

Label all states with $EG \phi$
The Algorithm for $EG \phi$

Label all states with $EG \phi$

Delete $EG \phi$ from any state not labelled with $\phi$. 

Diagram:

![Diagram showing states labeled with $EG \phi$ and transitions](#)
The Algorithm for $EG \phi$

Label all states with $EG \phi$

Delete $EG \phi$ from any state not labelled with $\phi$.

Delete $EG \phi$ from any state where none of its successors is labelled with $EG \phi$. Repeat until no more labels can be deleted.
The Algorithm for $EG \phi$

Label all states with $EG \phi$

Delete $EG \phi$ from any state not labelled with $\phi$.

Delete $EG \phi$ from any state where none of its successors is labelled with $EG \phi$. Repeat until no more labels can be deleted.

Call this process $SAT_{EG}(\phi)$
Summary so far:

Rewrite everything in terms of $EX, EG, EU$.

$$AX\phi \equiv \neg EX\neg\phi$$

$$AG\phi \equiv \neg EF\neg\phi$$

$$AF\phi \equiv \neg EG\neg\phi$$

$$\phi AU\psi \equiv \neg (EG\neg\phi \lor \neg\phi \ EU (\neg\phi \land \neg\psi))$$

Procedures for determining the set of satisfied states

$$SAT_{EX}(\phi), SAT_{EG}(\phi), SAT_{EU}(\phi, \psi)$$
The General CTL Model Checking Algorithm

Example  \( \text{EX} \ EG \ (p \land q) \)

\[
\begin{array}{c}
\{p, q\} \\
S_4 \\
\{p, q\} \\
S_5 \\
\{p, q\} \\
S_3 \\
\{p, q\} \\
S_2 \\
\{q\} \\
S_6 \\
\{p, q\} \\
S_7 \\
\{p, q\} \\
S_8
\end{array}
\]
The General CTL Model Checking Algorithm

Example

$$EX\ EG\ (p \land q)$$
The General CTL Model Checking Algorithm

Example

$$EX\ EG\ (p \land q)$$

---

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_4$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_5$$

$$p \land q$$

$$EG\ (p \land q)$$

$$\{p, q\}$$

$$s_3$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_2$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{q\}$$

$$s_1$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_6$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_7$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_8$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_6$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_7$$

$$EG\ (p \land q)$$

$$p \land q$$

$$\{p, q\}$$

$$s_8$$
The General CTL Model Checking Algorithm

Example

\( EX \ EG (p \land q) \)
The General CTL Model Checking Algorithm

Example

$EX\ EG\ (p \land q)$

Diagram:

- $s_4$: $\{p, q\}$
  - $EG\ (p \land q)$
  - $p \land q$

- $s_5$: $\{p, q\}$
  - $p \land q$
  - $EG\ (p \land q)$

- $s_3$: $\{p, q\}$
  - $p \land q$

- $s_2$: $\{p, q\}$
  - $p \land q$
  - $EG\ (p \land q)$

- $s_1$: $\{p\}$
  - $p \land q$

- $s_6$: $\{q\}$
  - $EG\ (p \land q)$

- $s_7$: $\{p, q\}$
  - $EG\ (p \land q)$

- $s_8$: $\{p, q\}$
  - $p \land q$
The General CTL Model Checking Algorithm

Example

\[ \text{EX } \text{EG } (p \land q) \]

\[
\begin{align*}
  S_1 & \{p\} \\
  S_4 & \{p, q\} \\
  S_5 & \{p, q\} \\
  S_2 & \{p, q\} \\
  S_3 & \{p, q\} \\
  S_6 & \{q\} \\
  S_7 & \{p, q\} \\
  S_8 & \{p, q\}
\end{align*}
\]
The General CTL Model Checking Algorithm

Example

$\text{EX } \text{EG } (p \land q)$
The General CTL Model Checking Algorithm

Example

\[ EX \ EG (p \land q) \]

\[ EX \ EG (p \land q) \]

\[ EG (p \land q) \]

\[ p \land q \]

\[ \{p, q\} \]

\[ s_4 \]

\[ \{p, q\} \]

\[ s_5 \]

\[ p \land q \]

\[ EG (p \land q) \]

\[ \{p, q\} \]

\[ s_6 \]

\[ p \land q \]

\[ EG (p \land q) \]

\[ \{p, q\} \]

\[ s_7 \]

\[ p \land q \]

\[ EX \ EG (p \land q) \]

\[ s_1 \]

\[ \{p\} \]

\[ \{p\} \]

\[ s_2 \]

\[ \{p\} \]

\[ s_3 \]

\[ \{p\} \]

\[ s_4 \]

\[ \{q\} \]

\[ s_5 \]

\[ s_6 \]

\[ s_7 \]

\[ s_8 \]
The General CTL Model Checking Algorithm

Example

\[ \text{EX EG } (p \land q) \]

\[ \text{EX EG } (p \land q) \]
\[ \text{EG } (p \land q) \]
\[ p \land q \]
\[ \{p, q\} \]

\[ \text{EX EG } (p \land q) \]
\[ \text{EG } (p \land q) \]
\[ p \land q \]
\[ \{p, q\} \]

\[ s_1 \]
\[ \{p\} \]

\[ \text{EX EG } (p \land q) \]
\[ \text{EG } (p \land q) \]
\[ p \land q \]
\[ \{p, q\} \]

\[ s_7 \]
\[ \{p, q\} \]

\[ s_8 \]
\[ \{p, q\} \]

\[ s_1, s_2, s_3, s_4, s_5, s_6 \models \text{EX EG } (p \land q) \land s_1 \in I \]
\[ \Rightarrow M \models \text{EX EG } (p \land q) \]
The General CTL Model Checking Algorithm

Example

$p \; EU(EG \; q)$
Example: \[ p \text{ EU}(\text{EG } q) \]
The General CTL Model Checking Algorithm

Example

$p \ EU(EG\ q)$
The General CTL Model Checking Algorithm

Example

\[ p \ EU(EG \ q) \]
The General CTL Model Checking Algorithm

Example

\[ p \text{ EU}(\text{EG } q) \]
The General CTL Model Checking Algorithm

Example

$p \ EU(EG \ q)$
The General CTL Model Checking Algorithm

Example

\[ p \ EU(EG \ q) \]
The General CTL Model Checking Algorithm

Example

$p \ EU(EG\ q)$
Example

\[ p \text{ EU}(\text{EG } q) \]
The General CTL Model Checking Algorithm

Example: $p \text{ EU}(\text{EG } q)$
The General CTL Model Checking Algorithm

Example

\[ p \ EU(EG \ q) \]
The General CTL Model Checking Algorithm

Example

\( p \text{ EU}(EG \ q) \)

\[ s_1, s_2, s_3, s_4 \models p \text{ EU}(EG \ q) \land s_1 \in I \Rightarrow \mathcal{M} \models p \text{ EU}(EG \ q) \]
The General CTL Model Checking Algorithm

$\text{SAT}(\phi) =$
The General CTL Model Checking Algorithm

\[ SAT(\phi) = \]

\[ \text{case} \]
The General CTL Model Checking Algorithm

\[ \text{SAT}(\phi) = \]

\[
\text{case} \quad \phi \text{ is } T : \text{return } S
\]
\textbf{The General CTL Model Checking Algorithm}

\[ \text{SAT}(\phi) = \]

\[
\text{case } \quad \\
\phi \text{ is } T : \text{ return } S \\
\phi \text{ is } p_i : \text{ return } \{ s : p \in L(S) \}
\]
The General CTL Model Checking Algorithm

\[
\text{SAT}(\phi) =
\]
\[
\text{case}
\]
\[
\phi \text{ is } \top : \text{return } S
\]
\[
\phi \text{ is } p_i : \text{return } \{s : p \in L(S)\}
\]
\[
\phi \text{ is } \phi \land \psi : \text{return } \text{SAT}(\phi) \cap \text{SAT}(\psi)
\]
The General CTL Model Checking Algorithm

\[
\text{SAT}(\phi) = \\
\text{case} \\
\phi \text{ is } \top : \text{return } S \\
\phi \text{ is } p_i : \text{return } \{s : p \in L(S)\} \\
\phi \text{ is } \phi \land \psi : \text{return } \text{SAT}(\phi) \cap \text{SAT}(\psi) \\
\phi \text{ is } \phi \land \psi : \text{return } \text{SAT}(\phi) \cup \text{SAT}(\psi)
\]
The General CTL Model Checking Algorithm

\[
\text{SAT}(\phi) =
\begin{align*}
\text{case} & \\
\phi \text{ is } \top : & \text{return } S \\
\phi \text{ is } p_i : & \text{return } \{s : p \in L(S)\} \\
\phi \text{ is } \phi \land \psi : & \text{return } \text{SAT}(\phi) \cap \text{SAT}(\psi) \\
\phi \text{ is } \phi \land \psi : & \text{return } \text{SAT}(\phi) \cup \text{SAT}(\psi) \\
\phi \text{ is } \neg \phi : & \text{return } S \setminus \text{SAT}(\phi)
\end{align*}
\]
SAT(\(\phi\)) =

    \begin{align*}
    \text{case} \quad & \\
    \phi \text{ is } \top : \ & \text{return } S \\
    \phi \text{ is } p_i : \ & \text{return } \{s : p \in L(S)\} \\
    \phi \text{ is } \phi \land \psi : \ & \text{return } SAT(\phi) \cap SAT(\psi) \\
    \phi \text{ is } \phi \land \psi : \ & \text{return } SAT(\phi) \cup SAT(\psi) \\
    \phi \text{ is } \neg \phi : \ & \text{return } S - SAT(\phi) \\
    \phi \text{ is } EX \phi : \ & \text{return } SAT_{EX}(\phi)
    \end{align*}
The General CTL Model Checking Algorithm

\[
\text{SAT}(\phi) = \\
\quad \text{case} \\
\quad \phi \text{ is } T : \text{ return } S \\
\quad \phi \text{ is } p_i : \text{ return } \{s : p \in L(S)\} \\
\quad \phi \text{ is } \phi \land \psi : \text{ return } \text{SAT}(\phi) \cap \text{SAT}(\psi) \\
\quad \phi \text{ is } \phi \land \psi : \text{ return } \text{SAT}(\phi) \cup \text{SAT}(\psi) \\
\quad \phi \text{ is } \neg \phi : \text{ return } S - \text{SAT}(\phi) \\
\quad \phi \text{ is } EX\phi : \text{ return } \text{SAT}_{EX}(\phi) \\
\quad \phi \text{ is } EU\phi : \text{ return } \text{SAT}_{EU}(\phi)
\]
The General CTL Model Checking Algorithm

\[ SAT(\phi) = \]

\begin{align*}
\text{case} \\
\phi \text{ is } \top & : \text{return } S \\
\phi \text{ is } p_i & : \text{return } \{s : p \in L(S)\} \\
\phi \text{ is } \phi \land \psi & : \text{return } SAT(\phi) \cap SAT(\psi) \\
\phi \text{ is } \phi \land \psi & : \text{return } SAT(\phi) \cup SAT(\psi) \\
\phi \text{ is } \neg \phi & : \text{return } S - SAT(\phi) \\
\phi \text{ is } EX\phi & : \text{return } SAT_{EX}(\phi) \\
\phi \text{ is } EU\phi & : \text{return } SAT_{EU}(\phi) \\
\phi \text{ is } EG\phi & : \text{return } SAT_{EG}(\phi)
\end{align*}

esac