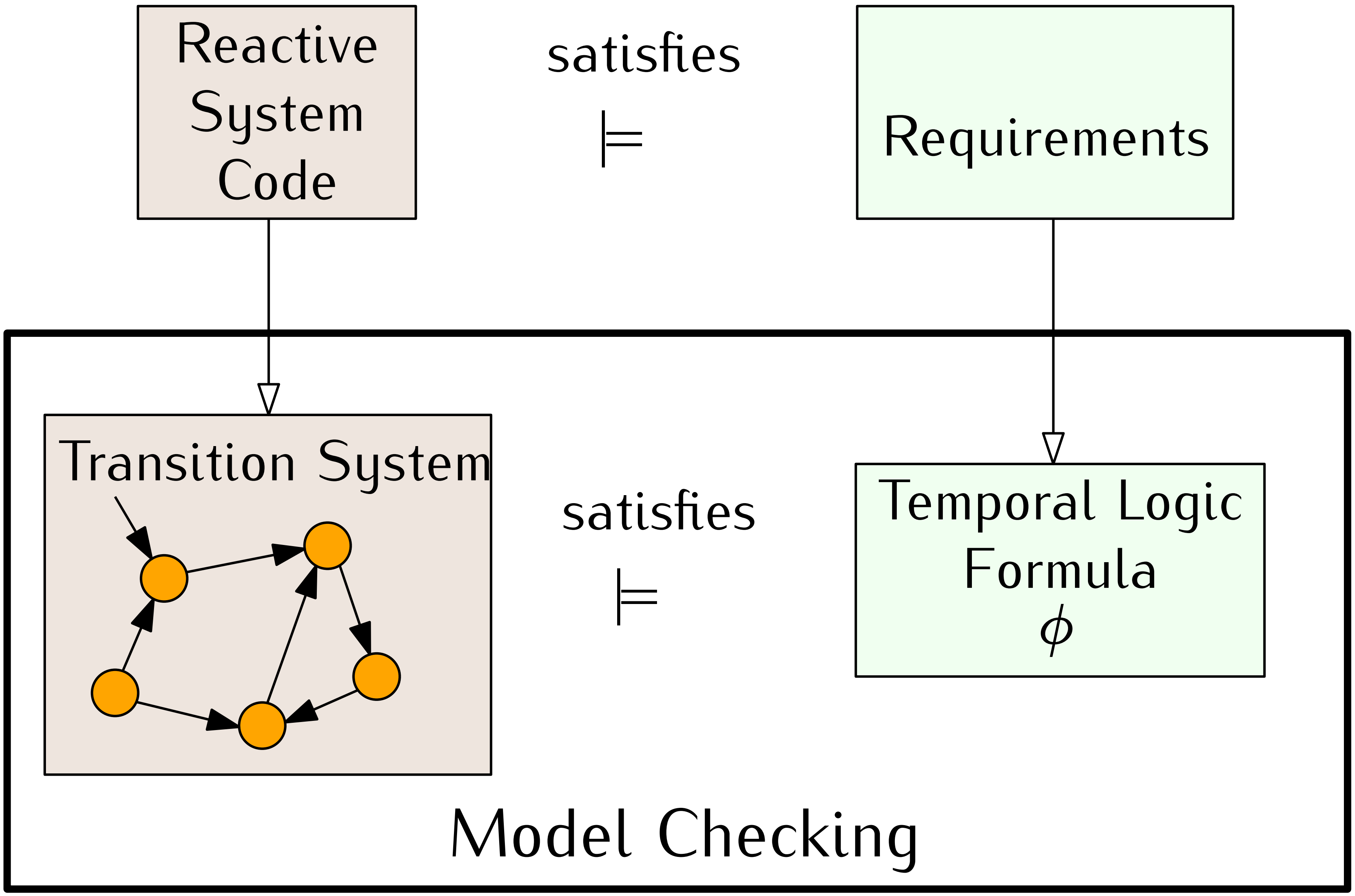
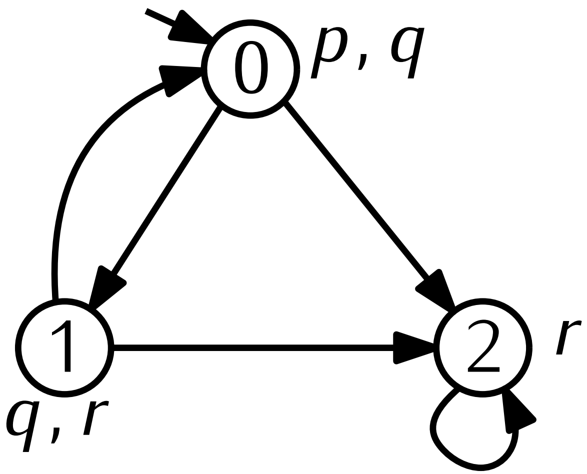
CS 181u Applied Logic

Lecture 12

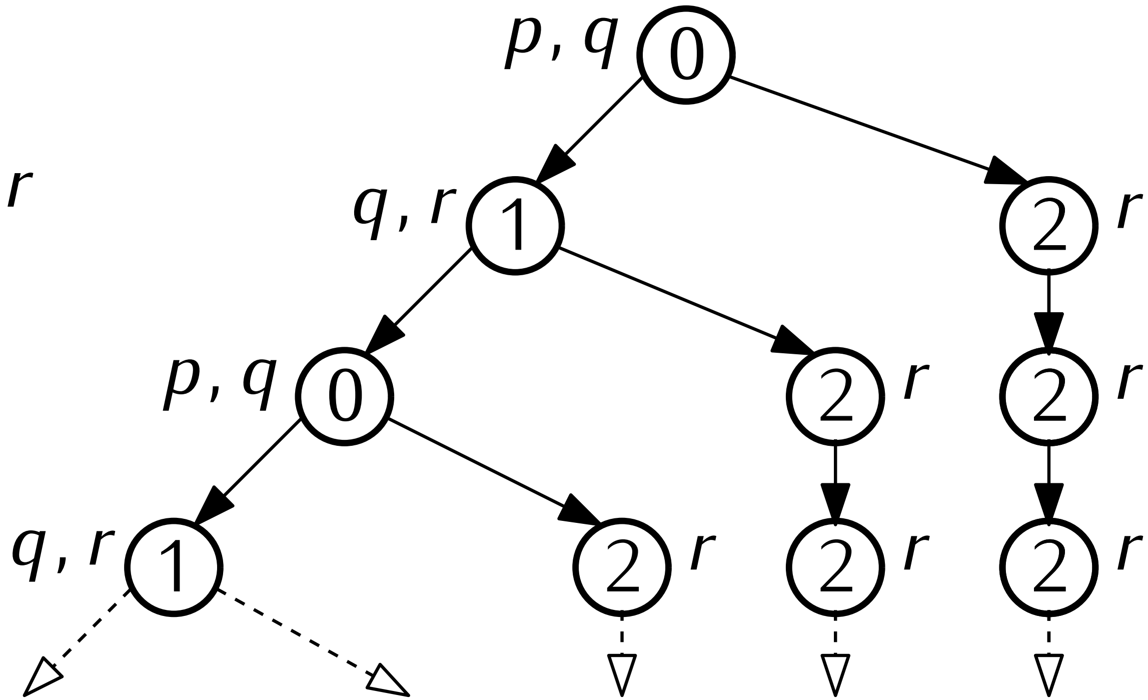
The Big Picture



Computation Tree Logic (CTL) Review



Computation tree for $\mathcal{M}$



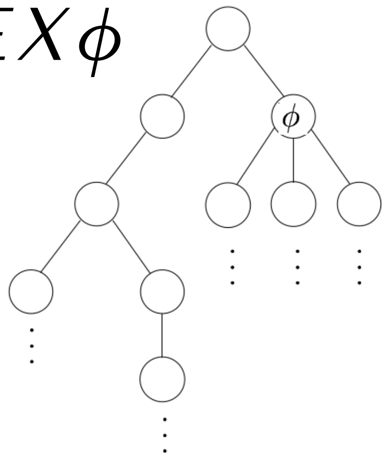
Computation Tree Logic (CTL) expresses properties of "alternative timelines".

$$\mathcal{M} \models \phi \Leftrightarrow \forall s \in I \;\; s \models \phi$$

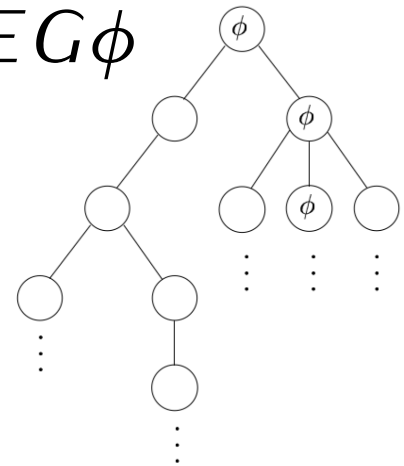
CTL Model Checking

CTL review $AG\phi$ $EG\phi$ $AF\phi$ $EF\phi$ $AX\phi$ $EX\phi$

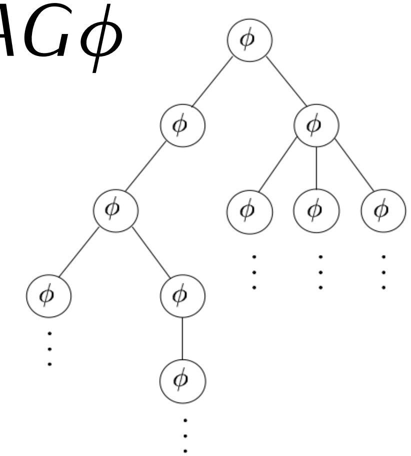
$EX\phi$



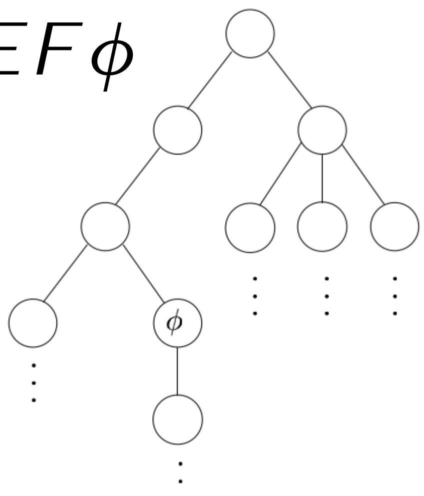
$EG\phi$



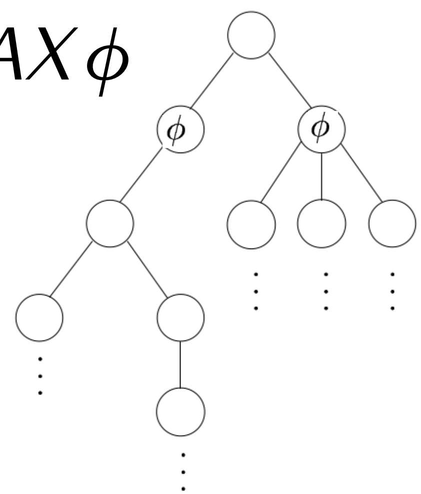
$AG\phi$



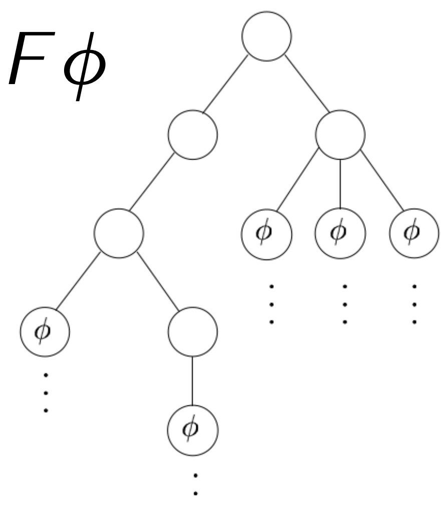
$EF\phi$



$AX\phi$



$AF\phi$



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A ***slightly*** **different idea:** figure out the set of states $S' \subseteq S$ such that for all $s \in S'$, $s \models \phi$. Then check if $I \subseteq S'$.

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This is easier.

CTL Model Checking

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Todays goal: an algorithm for CTL that does the following:

Input: $\mathcal{M}$ and ϕ
Output: all states of $\mathcal{M}$ that satisfy ϕ

Recall: Why so many operators?

AG EG AF EF AX EX AU EU

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The acts of the mind, wherein it exerts its power over simple ideas, are chiefly these three: Combining several simple ideas into one compound one, and thus all complex ideas are made. The second is bringing two ideas, whether simple or complex, together, and setting them by one another so as to take a view of them at once, without uniting them into one, by which it gets all its ideas of relations. The third is separating them from all other ideas that accompany them in their real existence: this is called abstraction, and thus all its general ideas are made.

SICP by Abelson, Sussman, and Sussman quoting John Locke from his *Essay Concerning Human Understanding*

Why so many operators?

We don't actually need any of them if we are OK with always writing temporal properties using first order logic and quantifying over states and paths.

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On the other hand, when performing meta-analysis of CTL, we need to examine each operator.

Hence, it is good to reduce everything down to a smallest set of sufficiently expressive operators.

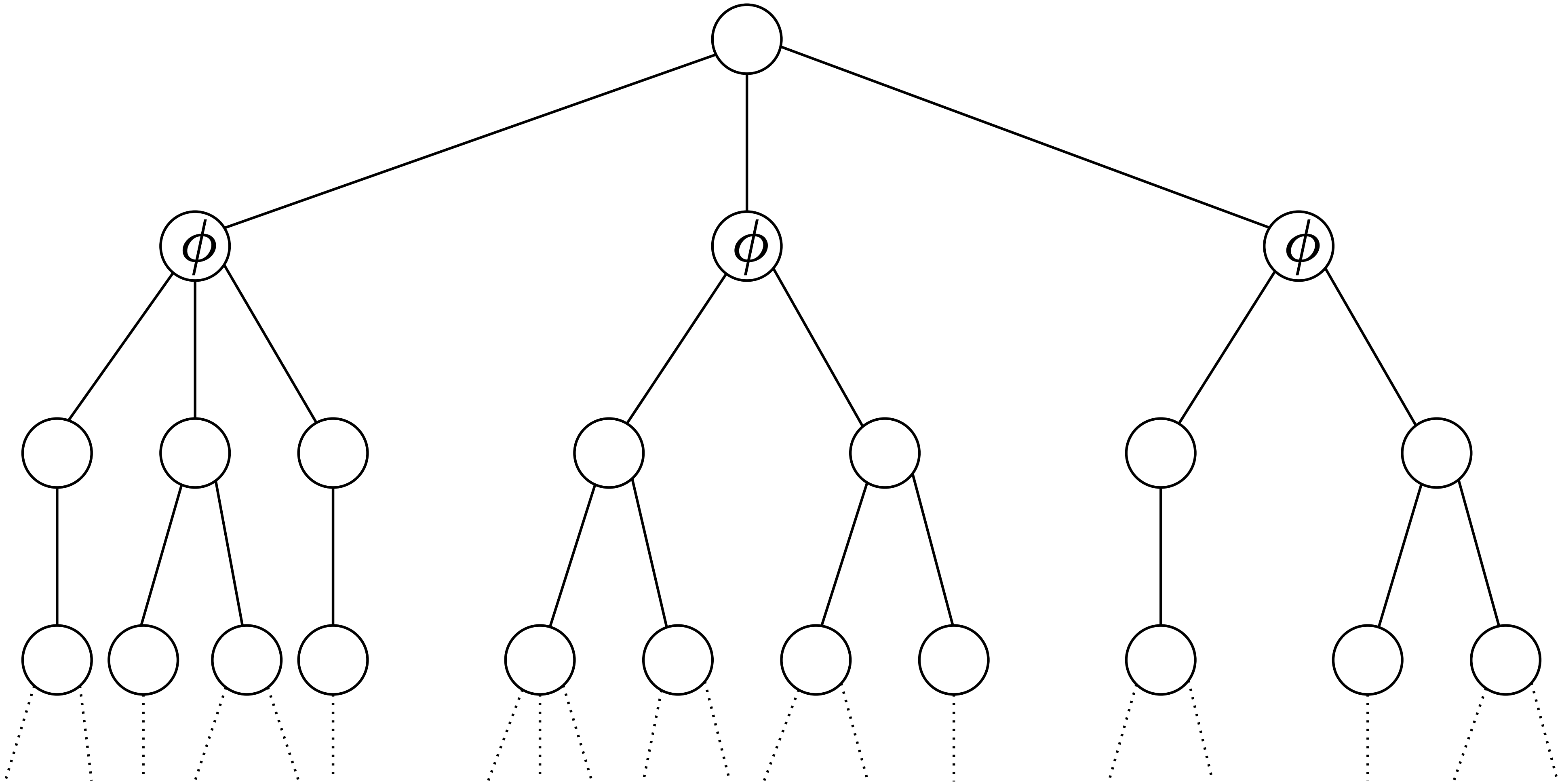
Adequate set of operators for CTL

Let's eliminate as many operators as possible by writing them in terms of other operators.

In fact, let's try to write everything in terms of E-properties EX, EU, and EG, and Boolean operations $\neg, \wedge$, and $\vee$.

(On your HW, you wrote some operators in terms of EX, EU, and AU.)

Get rid of $AX\phi$



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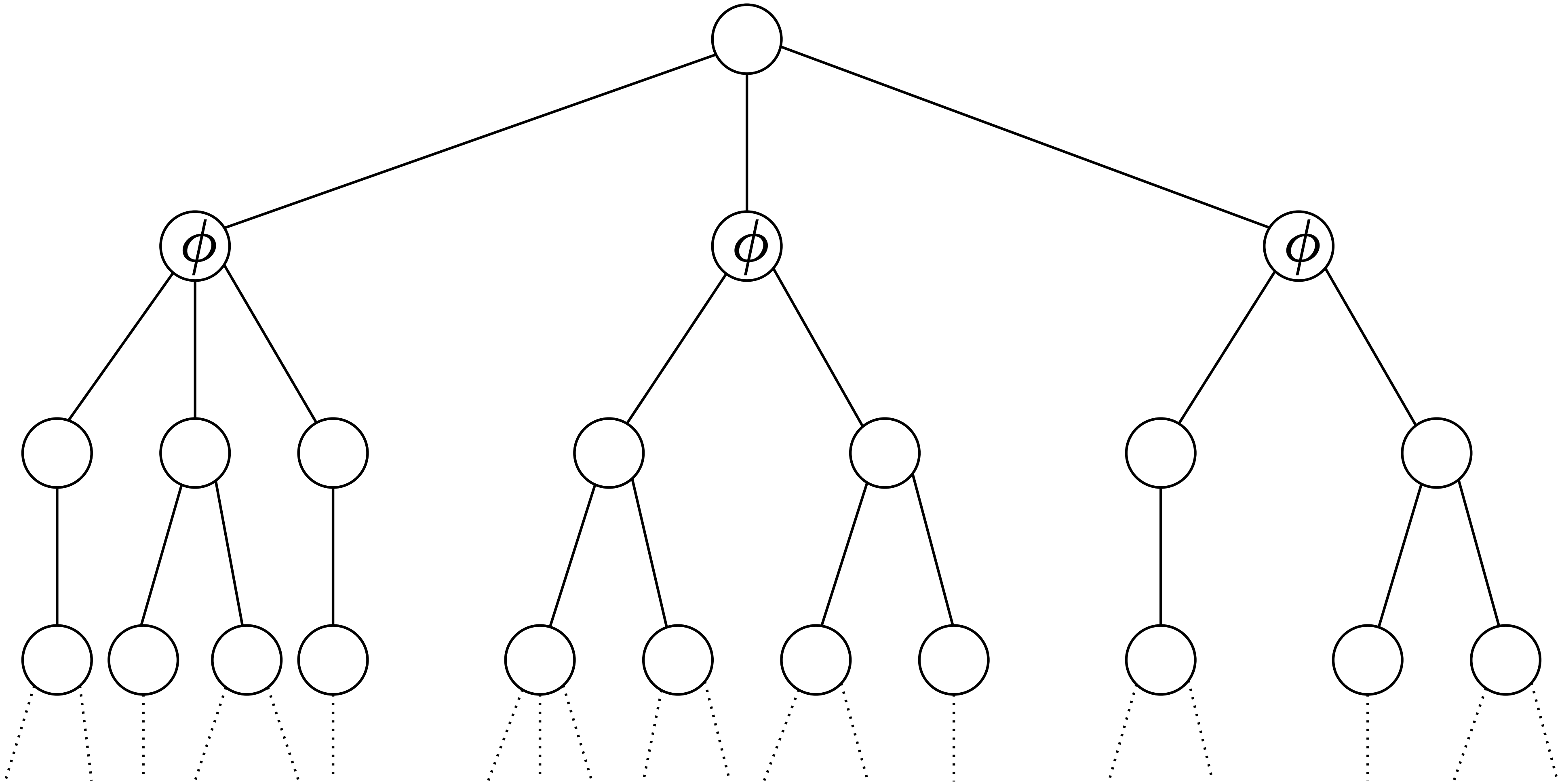

$$AX\phi \equiv \neg EX\neg\phi$$

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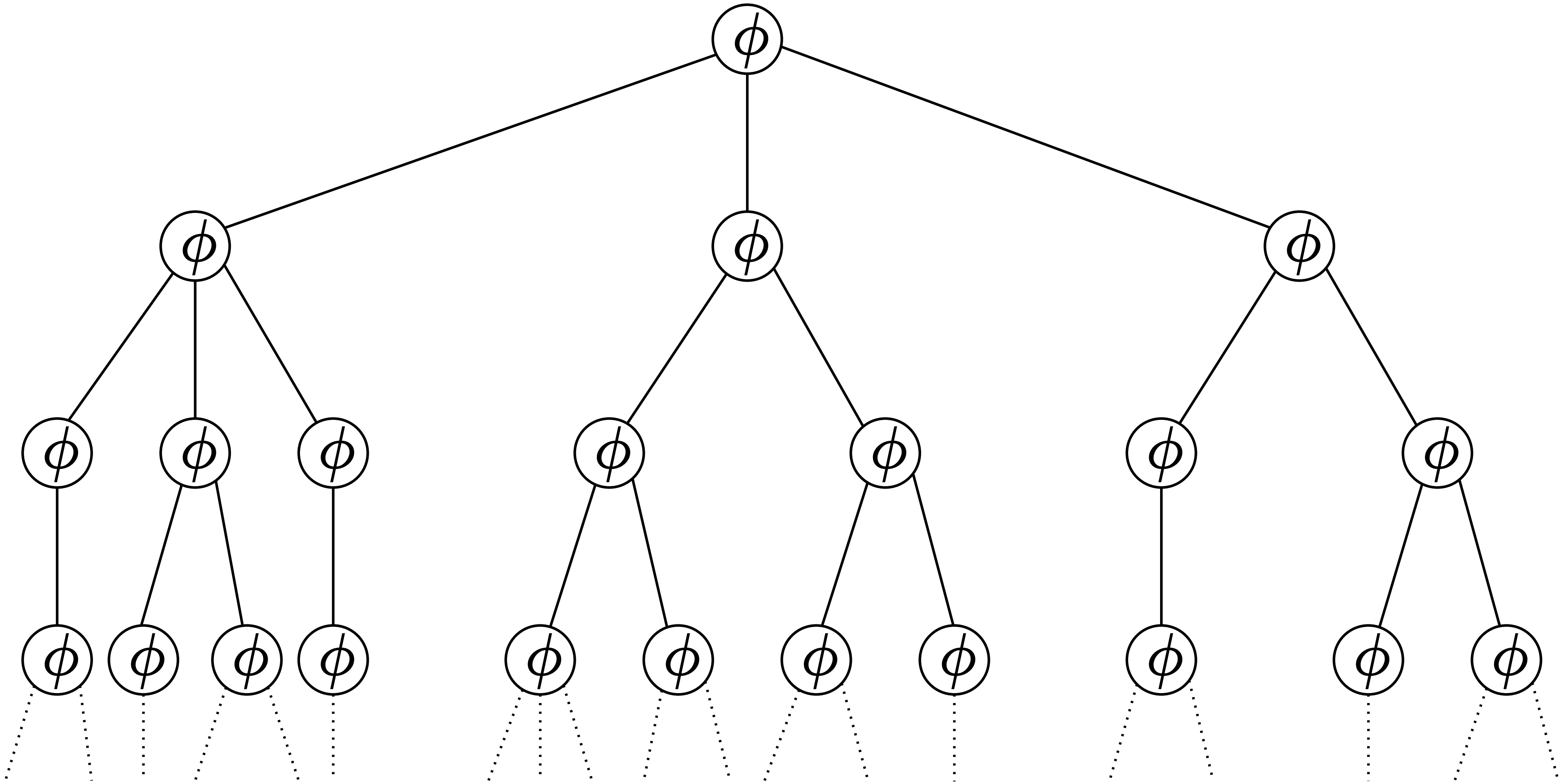

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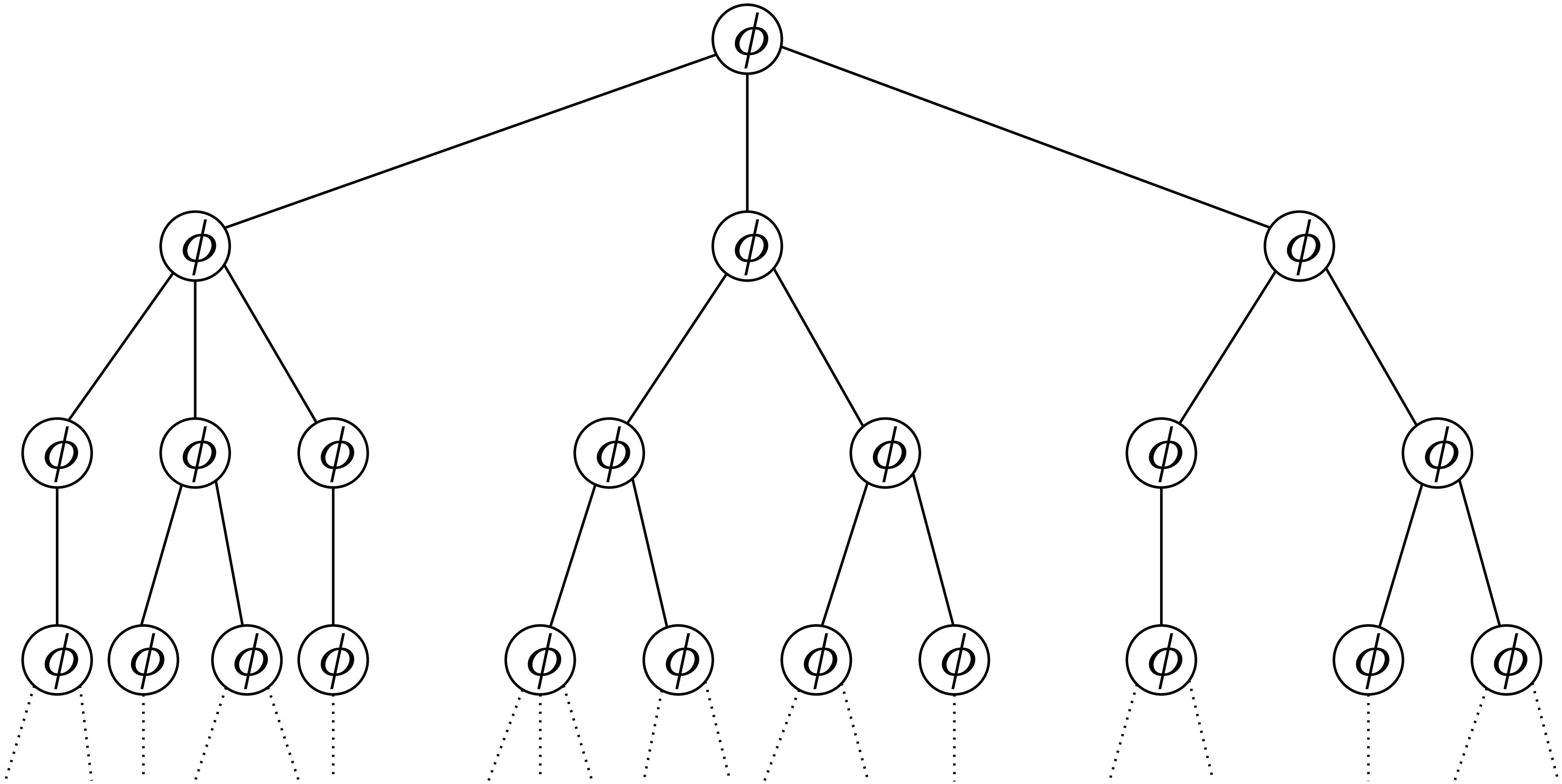

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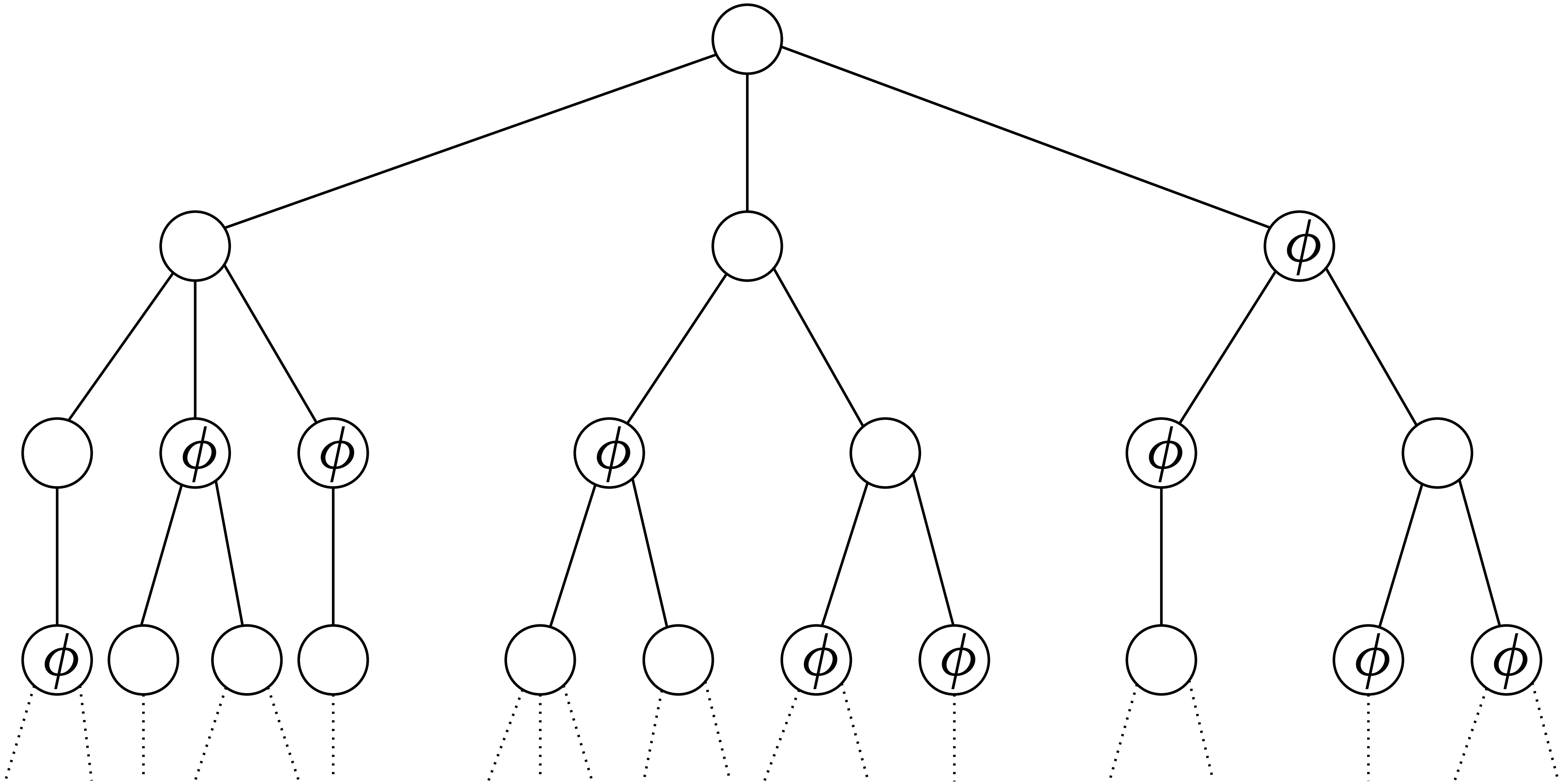

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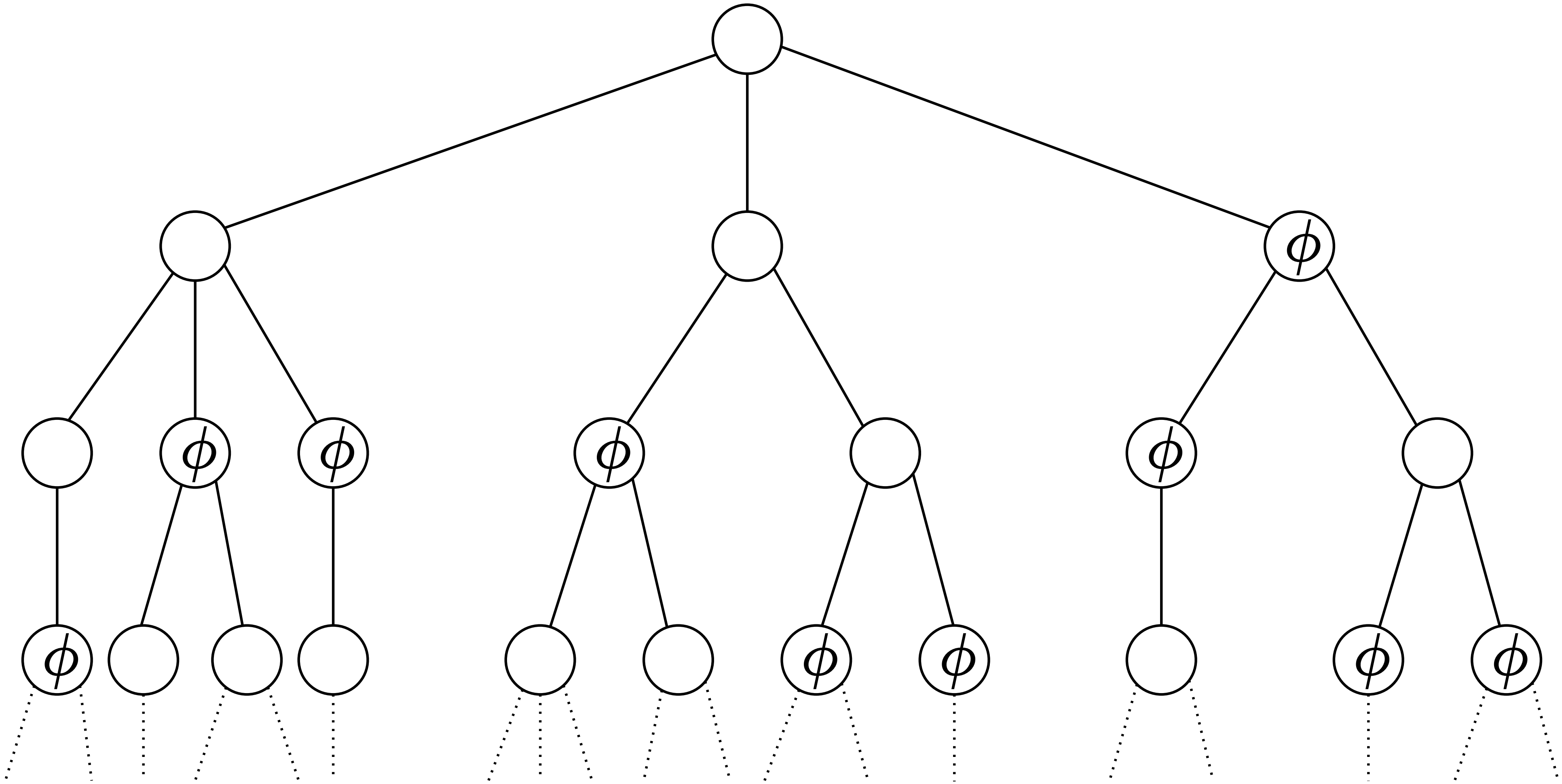

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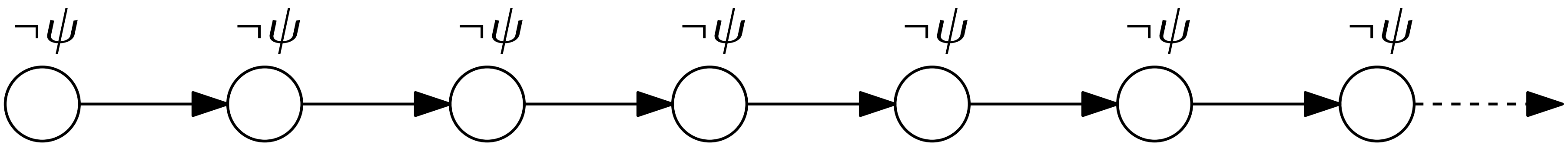

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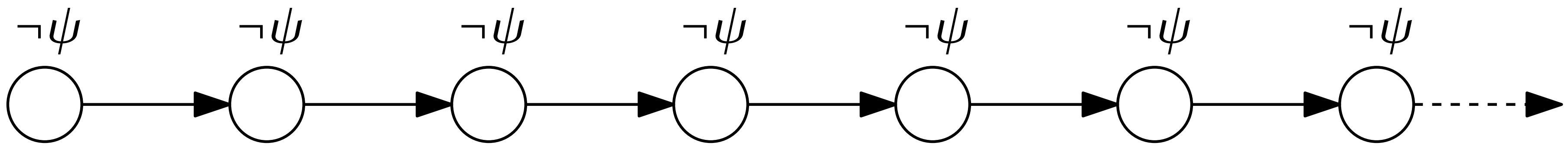

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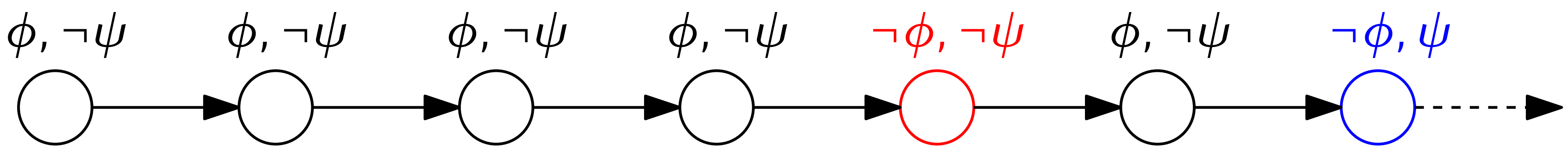

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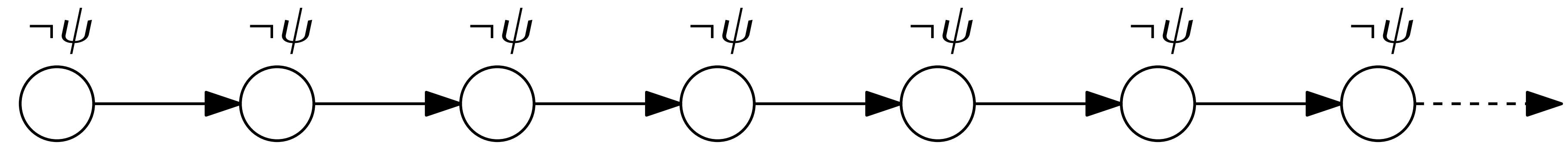

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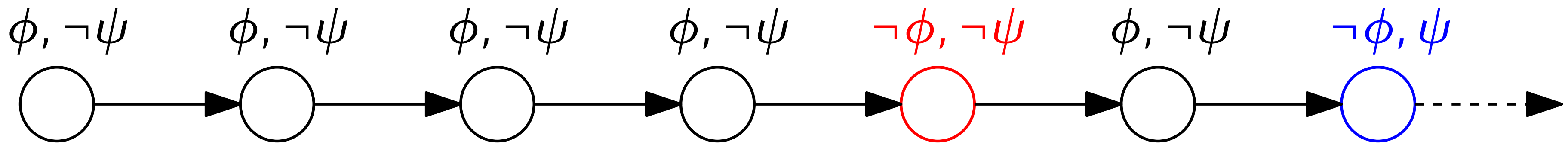

$$\phi AU\psi \equiv \neg(\ EG\neg\psi \quad \vee \quad \neg\psi\ EU\ (\neg\phi \wedge \neg\psi))$$

Summarizing:

$$AX\phi \equiv \neg EX\neg\phi$$

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This is called **existential negation normal form** for CTL.

Furthermore, $EF\phi \equiv \top\ EU\phi$

We only need EX, EU, EG

CTL model checking algorithm

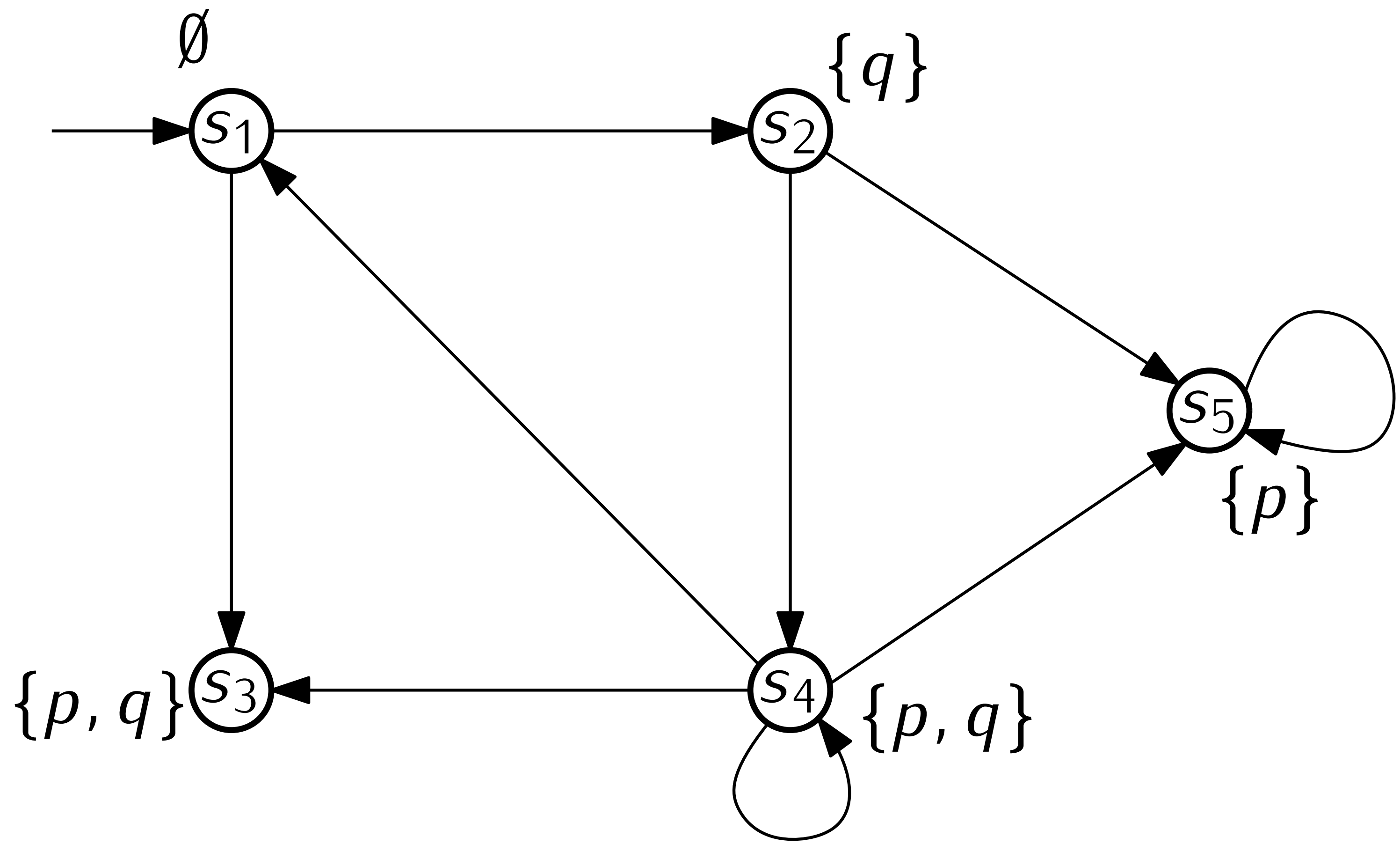
The main idea

First convert everything into existential negation normal form using previous reductions, so that we have only formulas with EX, EG, EU.

For each of the operators EX, EG, EU, give a method to determine the correposind set of states that satisfy the property.

The Algorithm for $EX\ \phi$

First, an example $EX(p \wedge q)$



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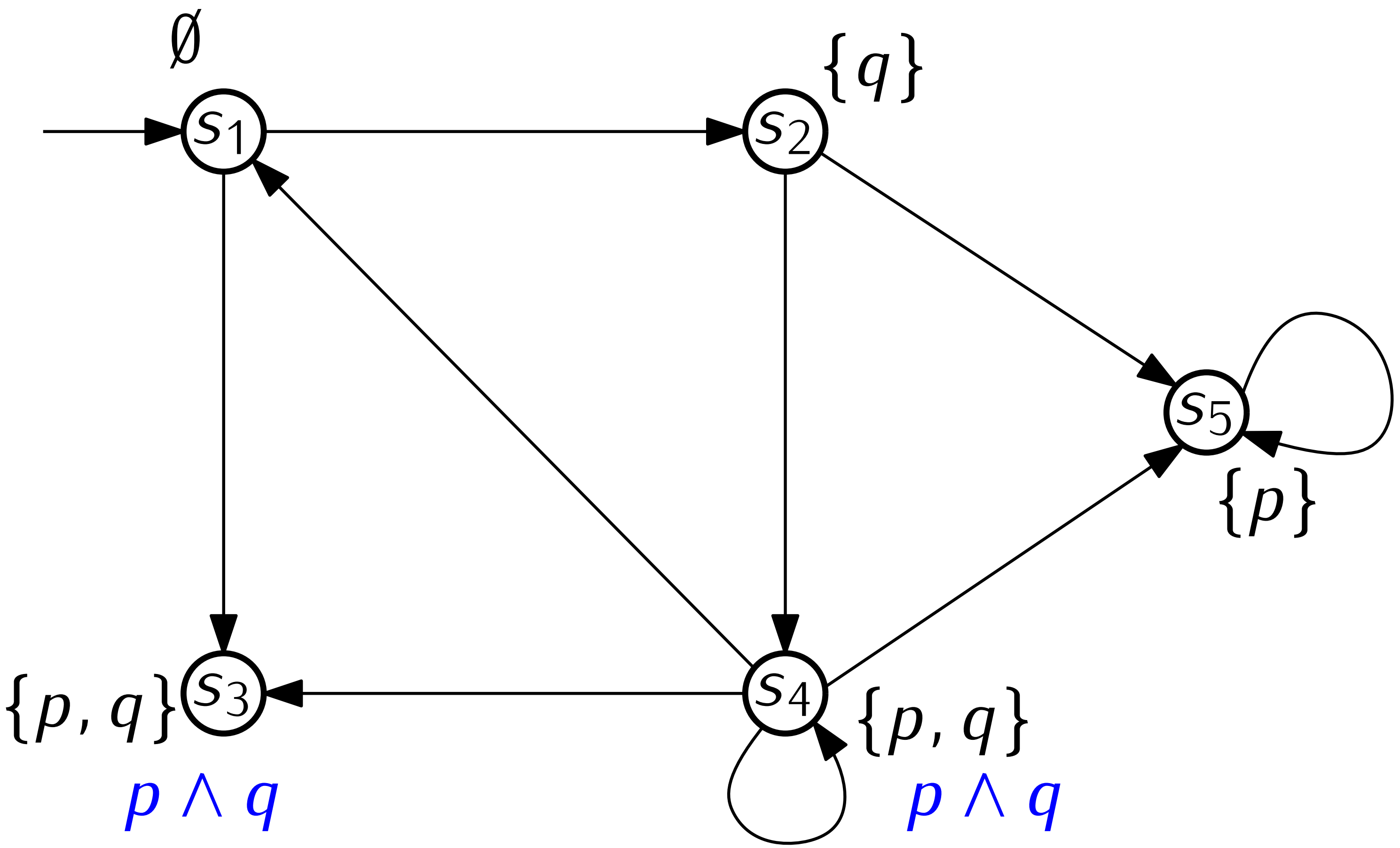

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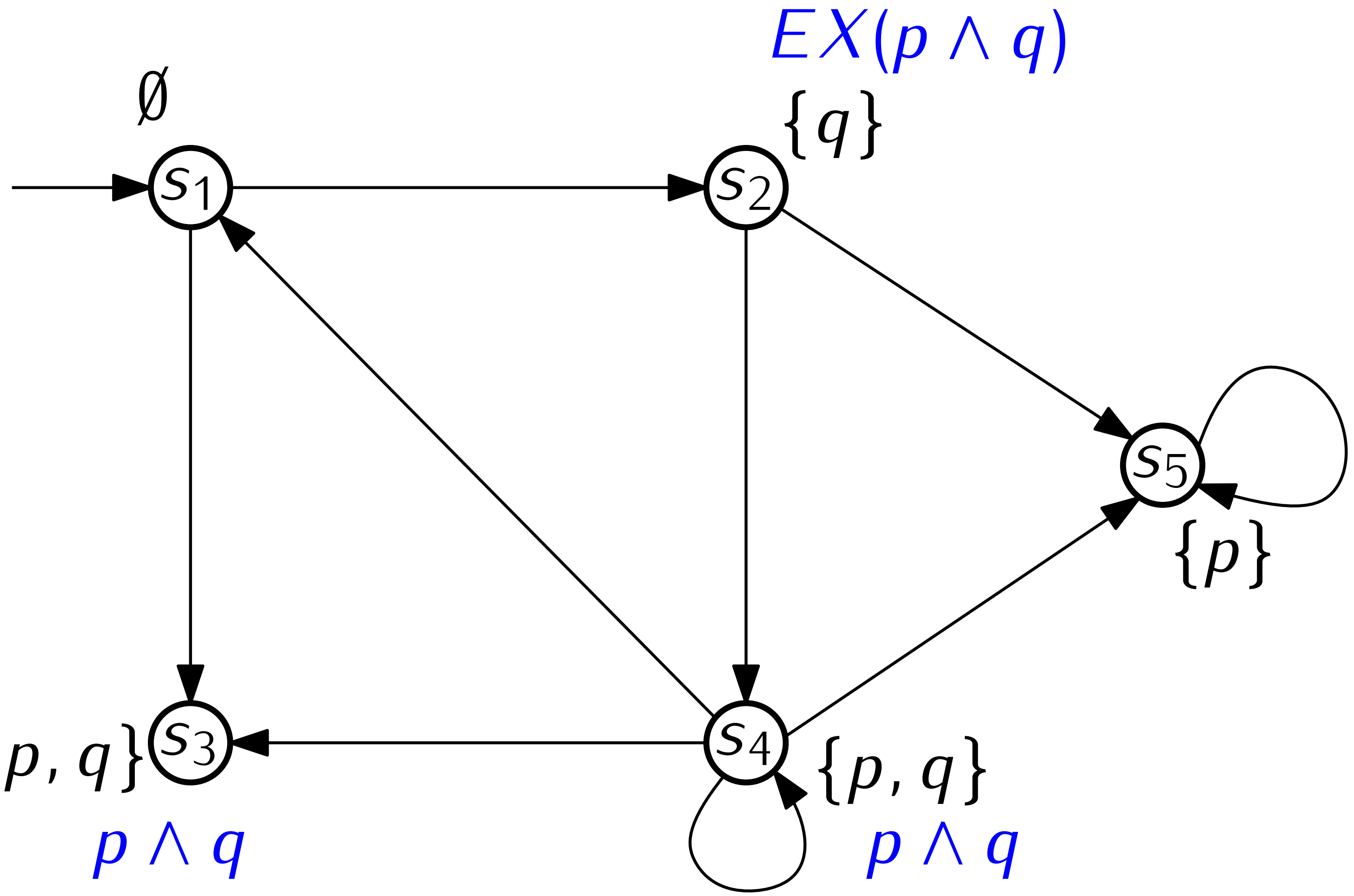

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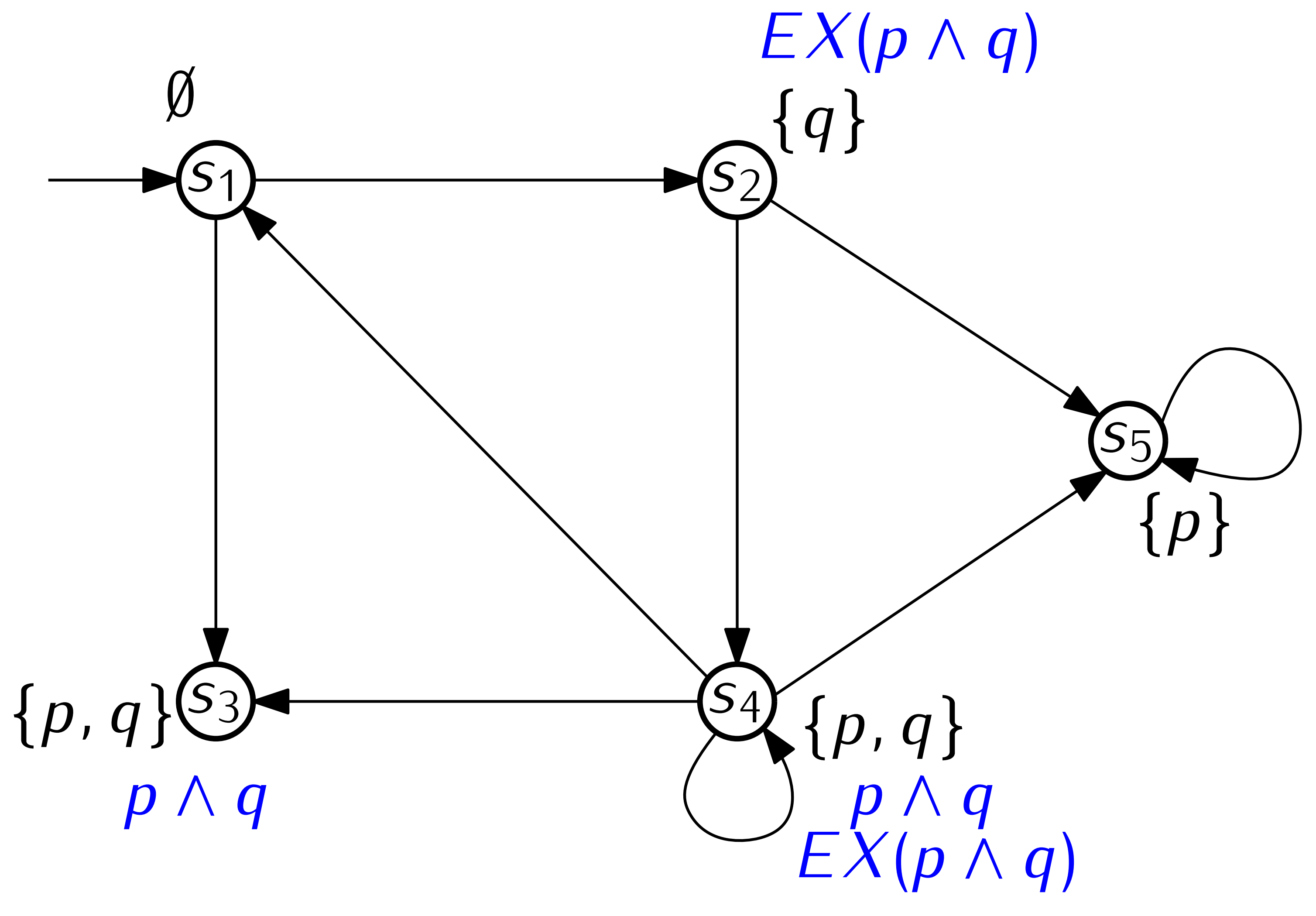

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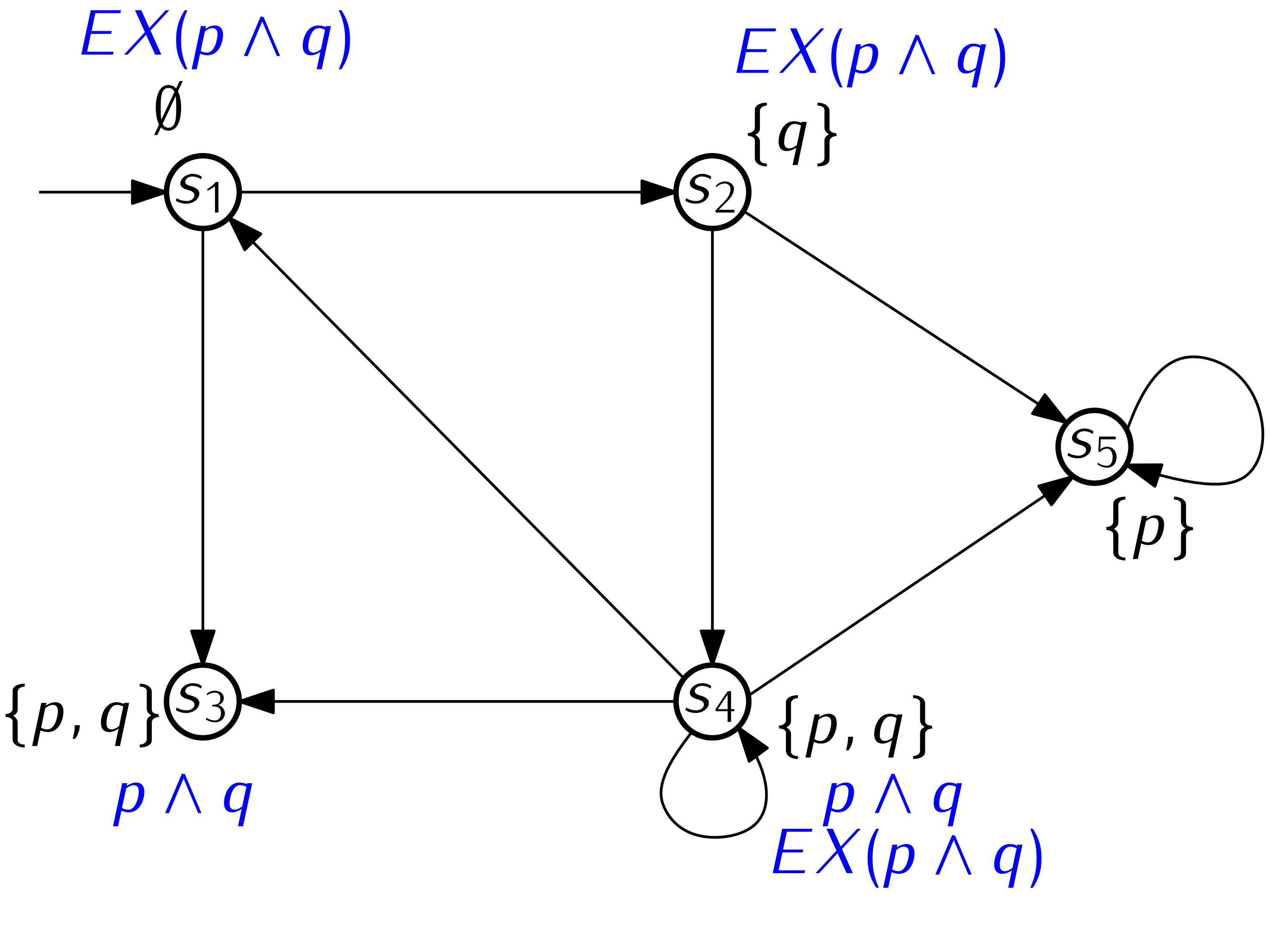

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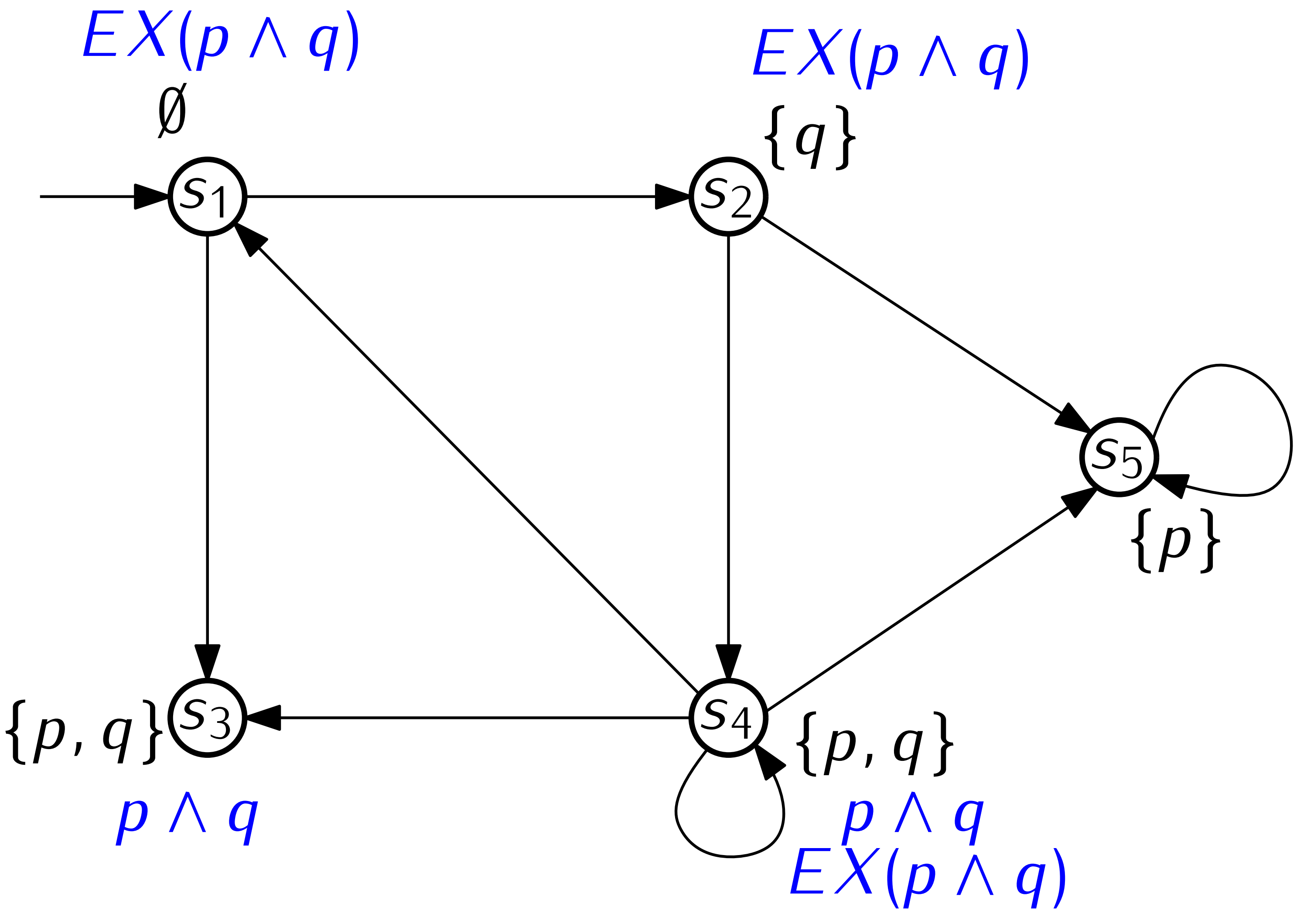
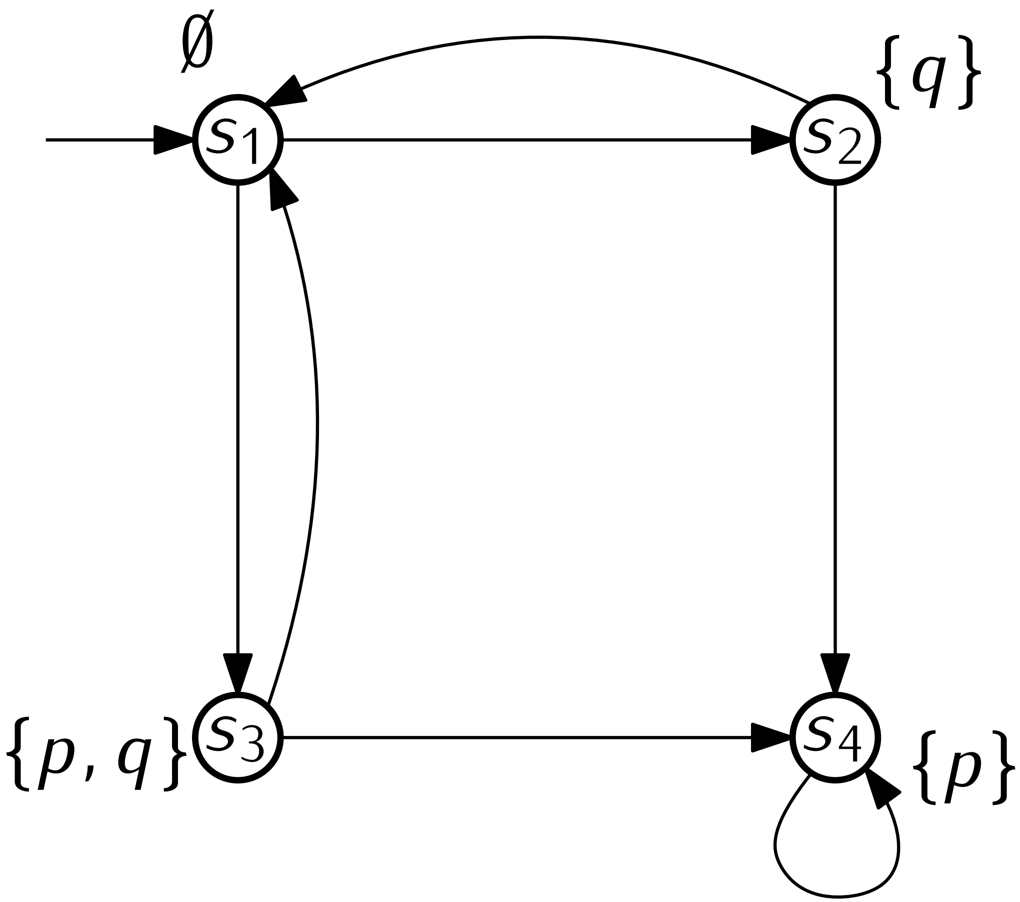


$$s_1, s_2, s_4 \models EX(p \wedge q)$$

The Algorithm for $EX\ \phi$

Another example $\qquad EX(p \wedge \neg q)$



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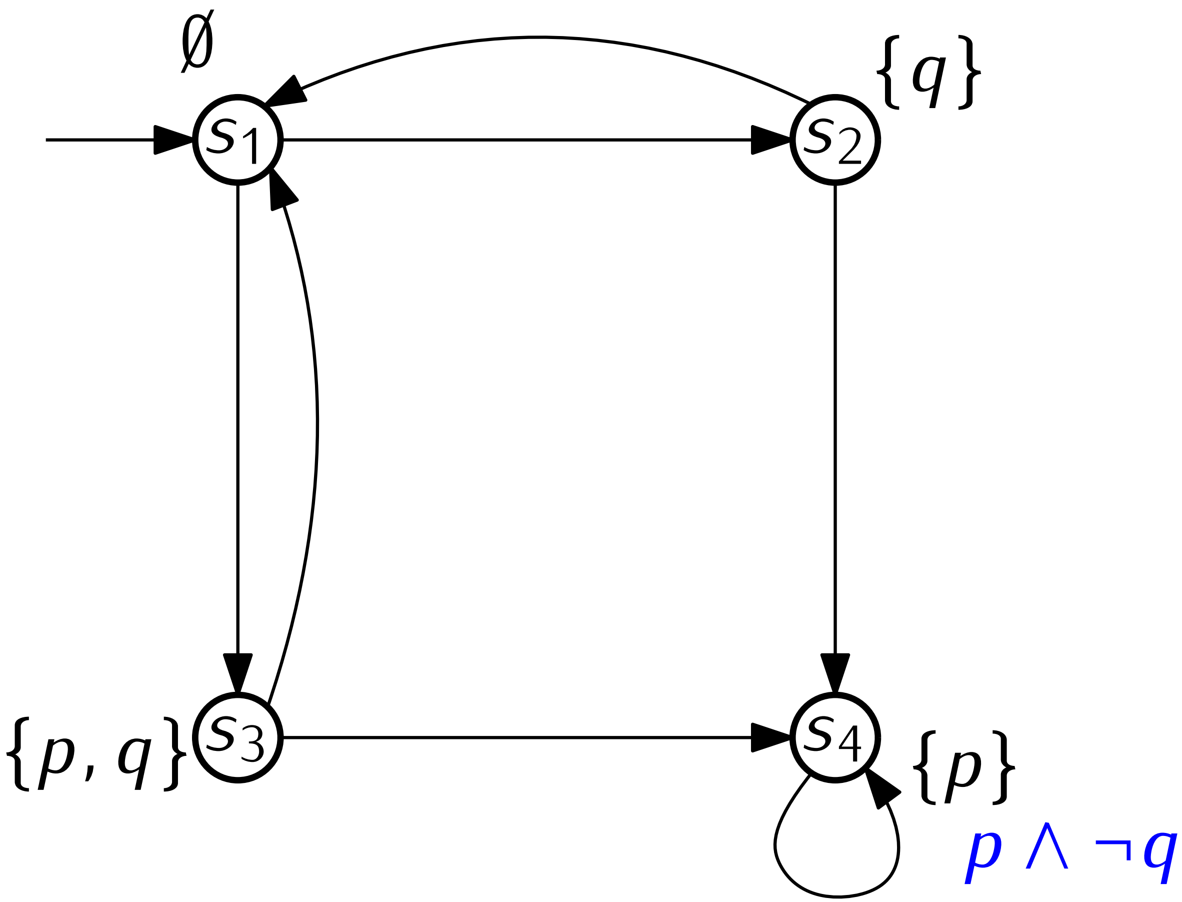

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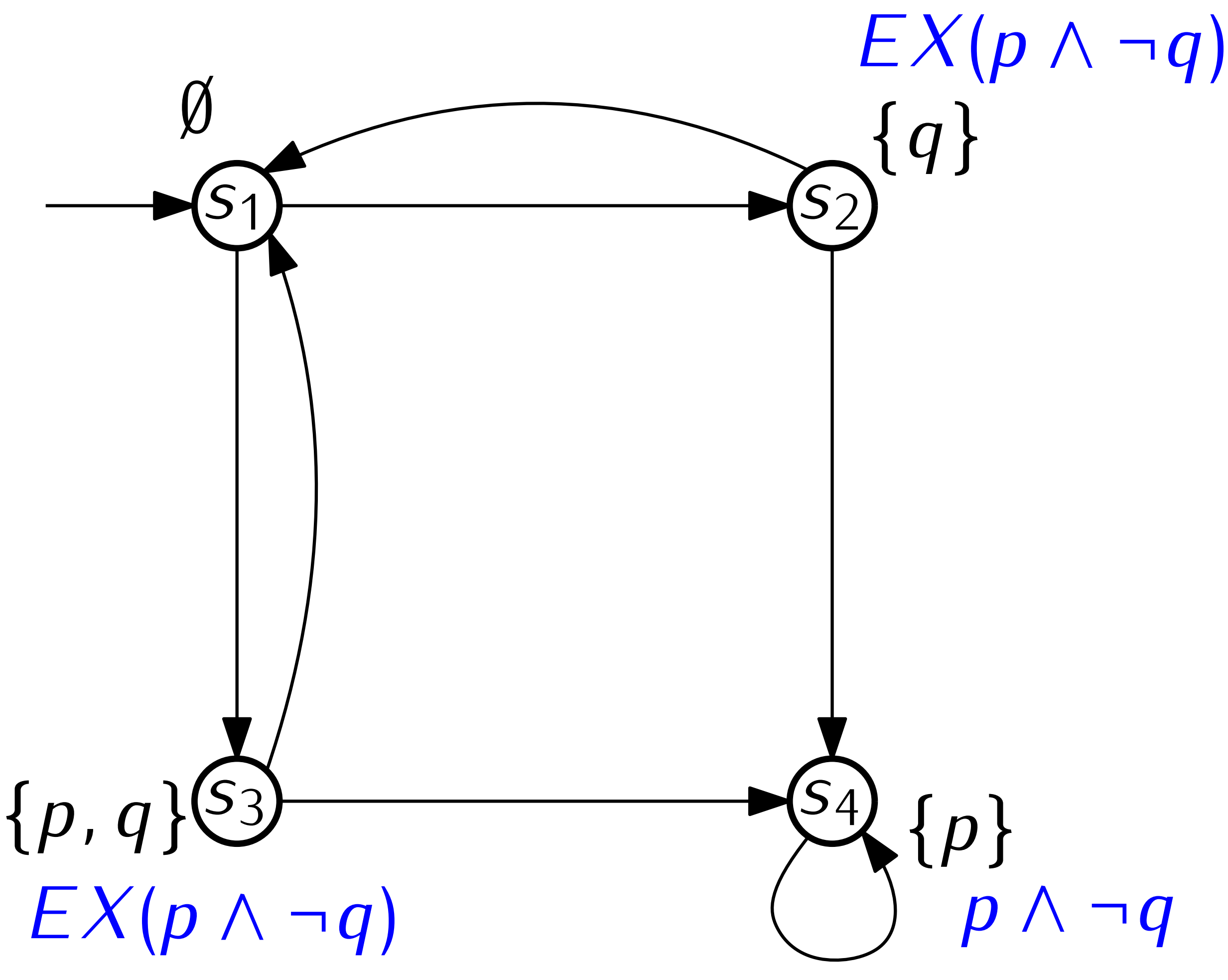

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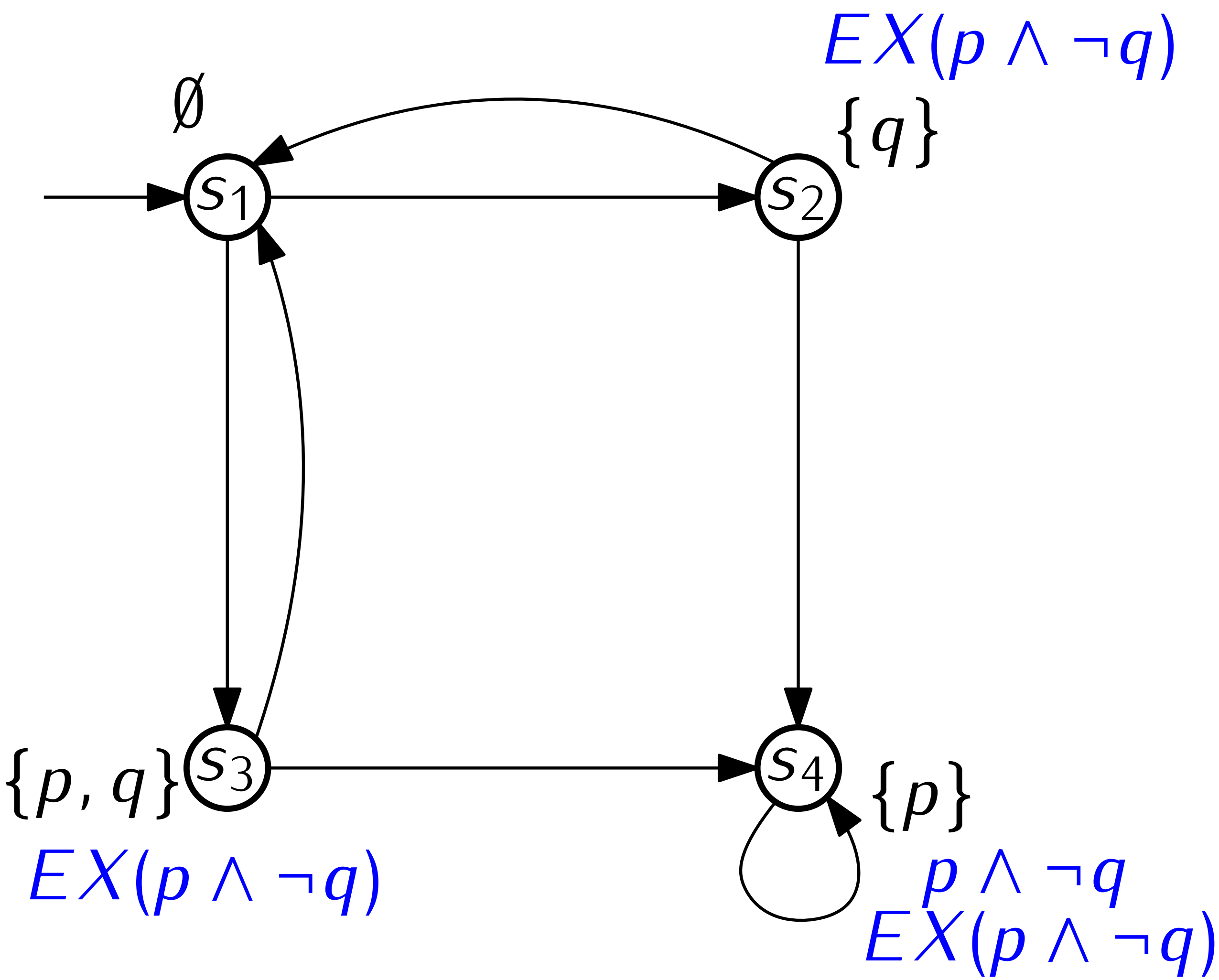
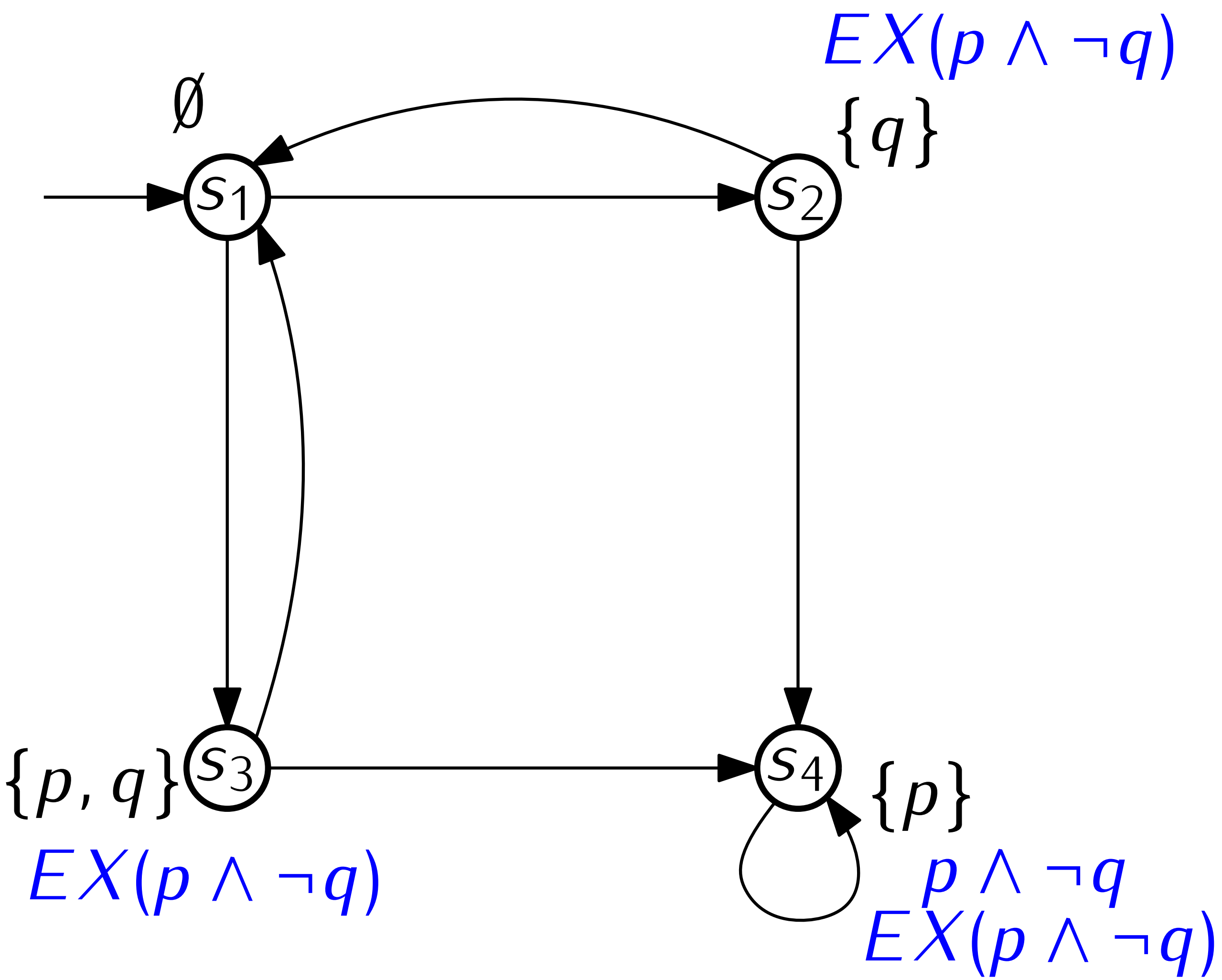


The Algorithm for $EX\ \phi$

Another example

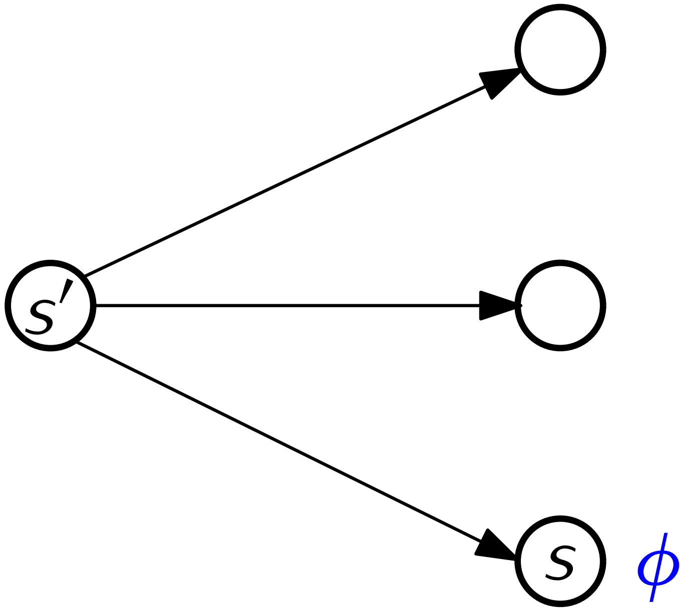
$EX(p \wedge \neg q)$



$s_2, s_3, s_4 \models EX(p \wedge q)$

The Algorithm for $EX\ \phi$

After labelling all states s that satisfy ϕ, label and state s' with $EX\phi$ if there is a transition from s' to s.



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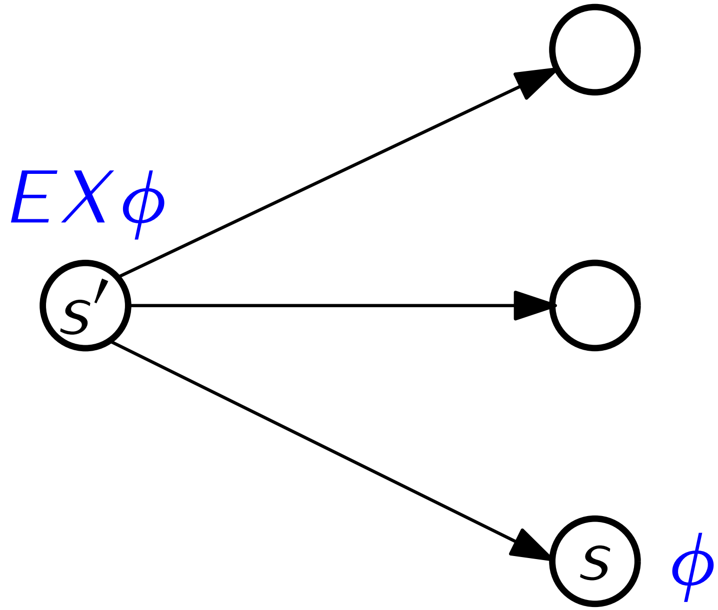

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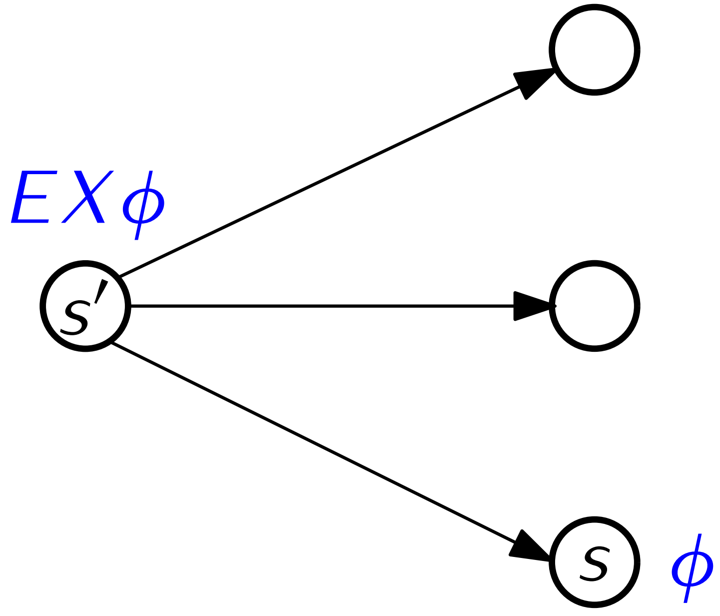
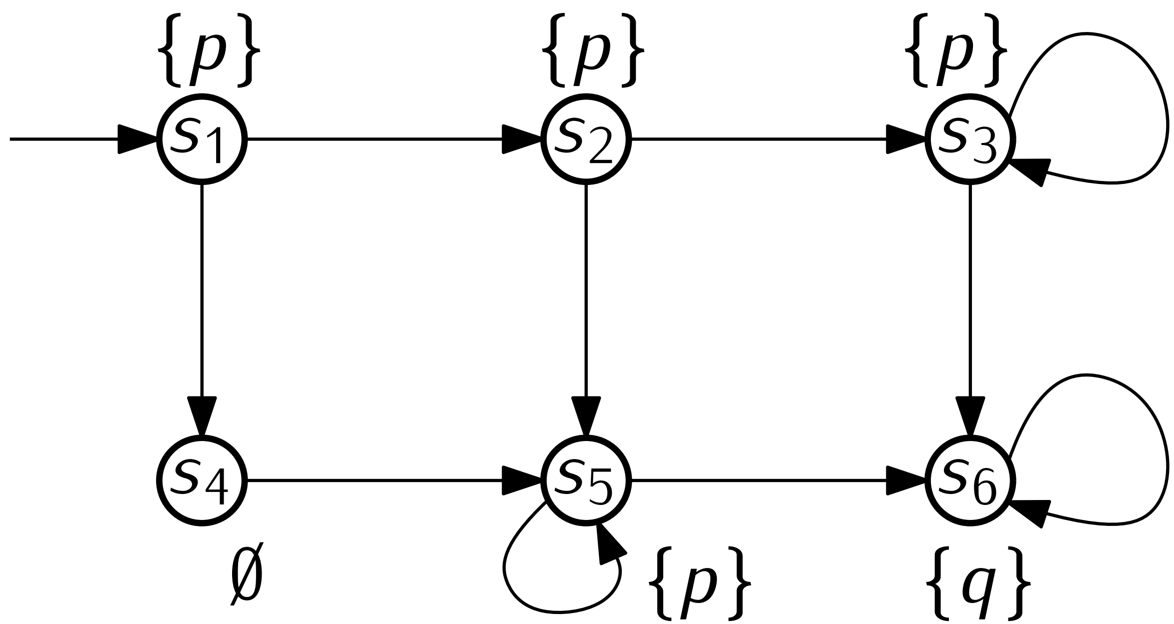


Call this process $\mathrm{SAT}_{EX}(\phi)$

The Algorithm for $\phi\ EU\ \psi$

First, an example $\quad p\ EU\ q$



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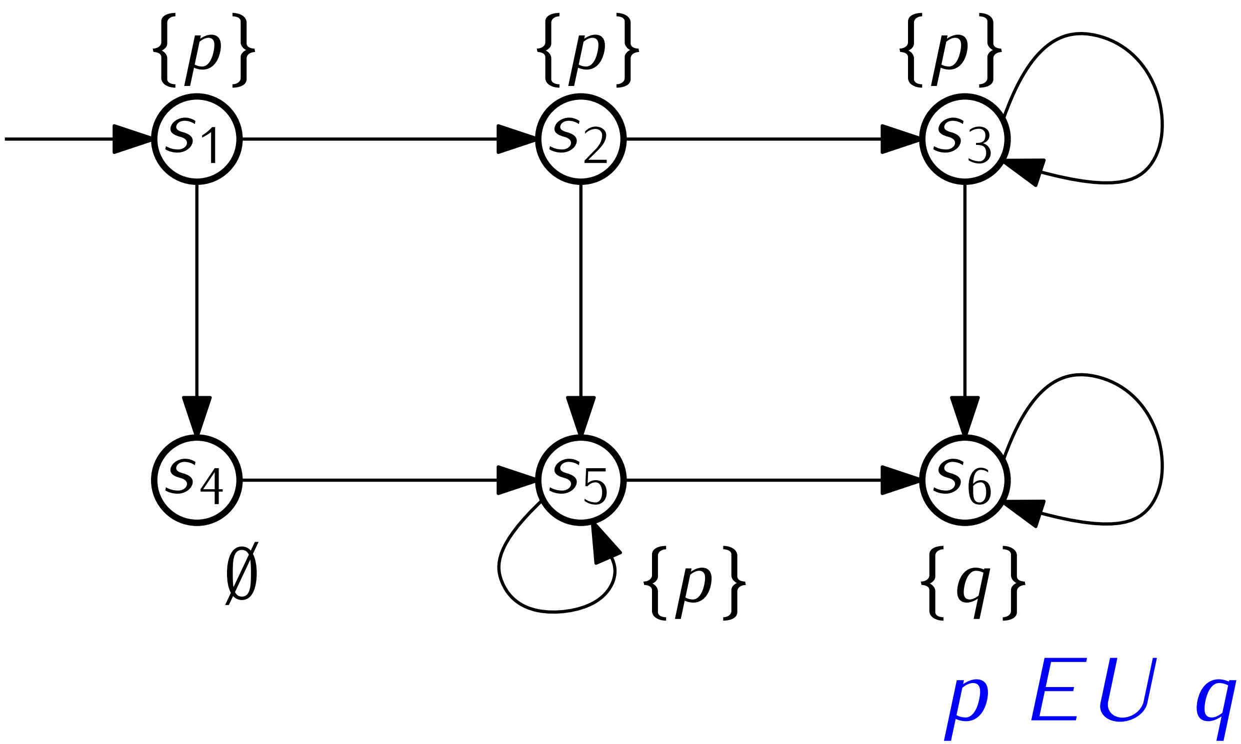

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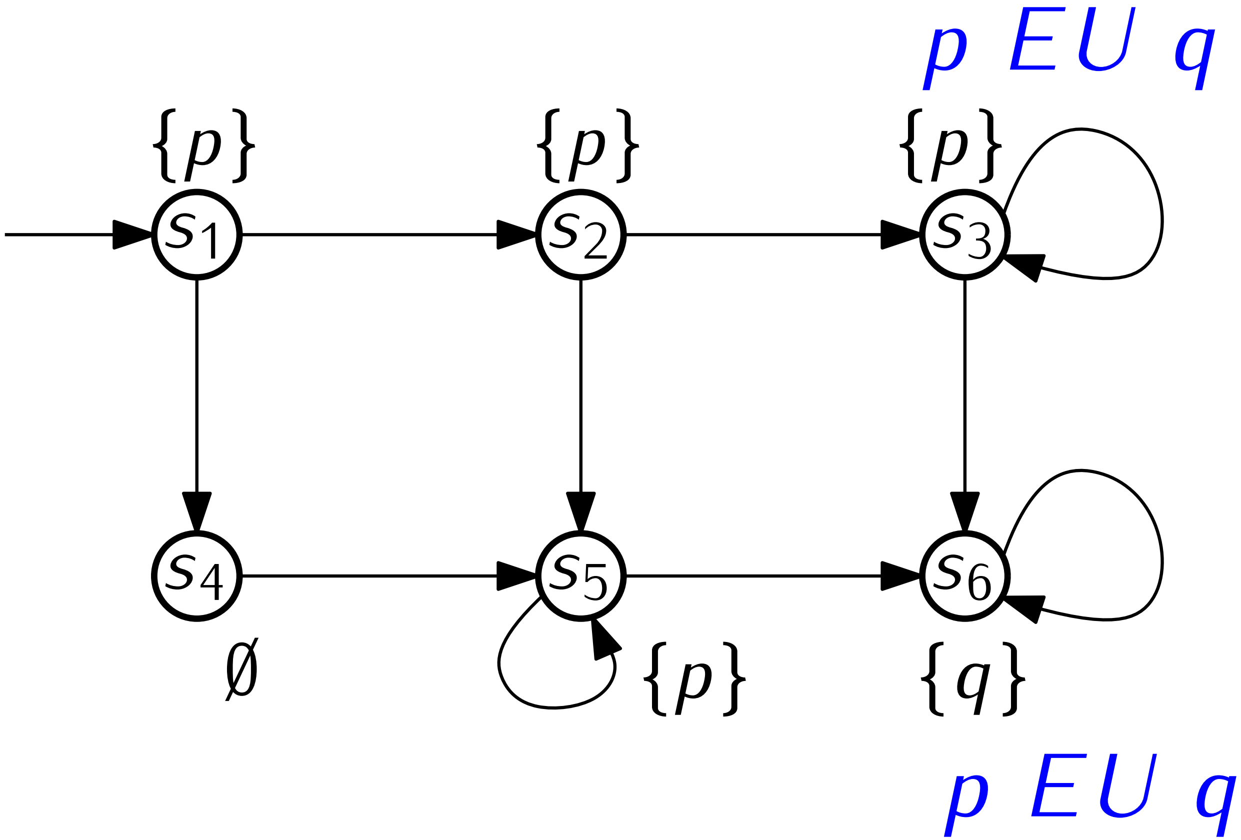

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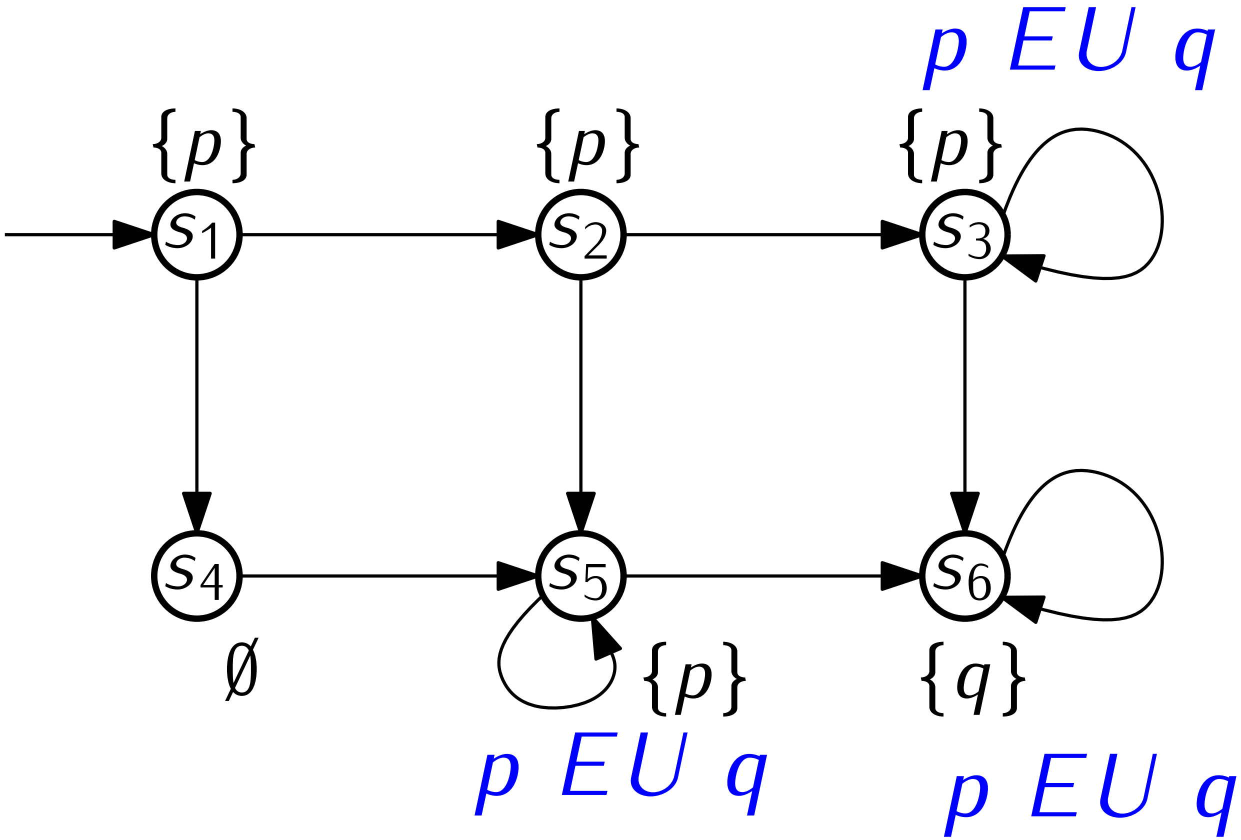

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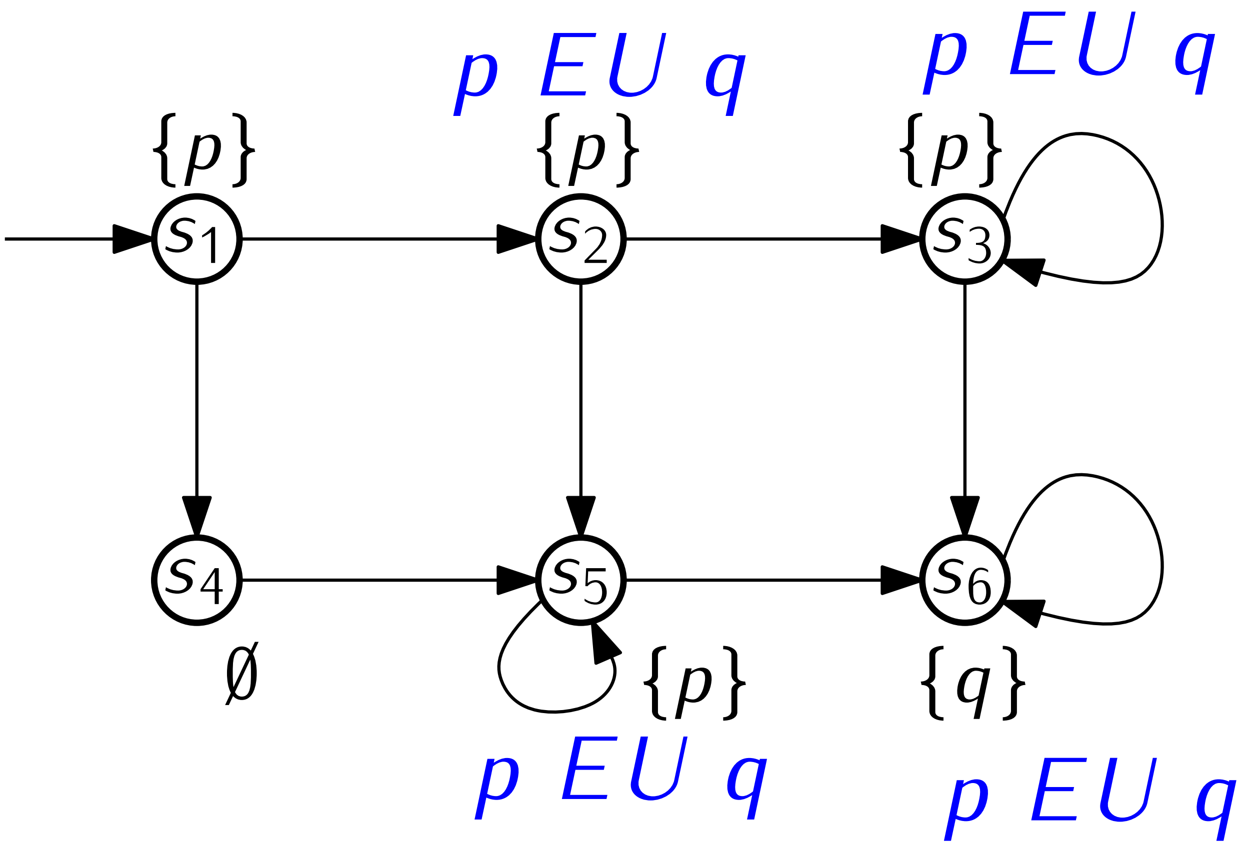

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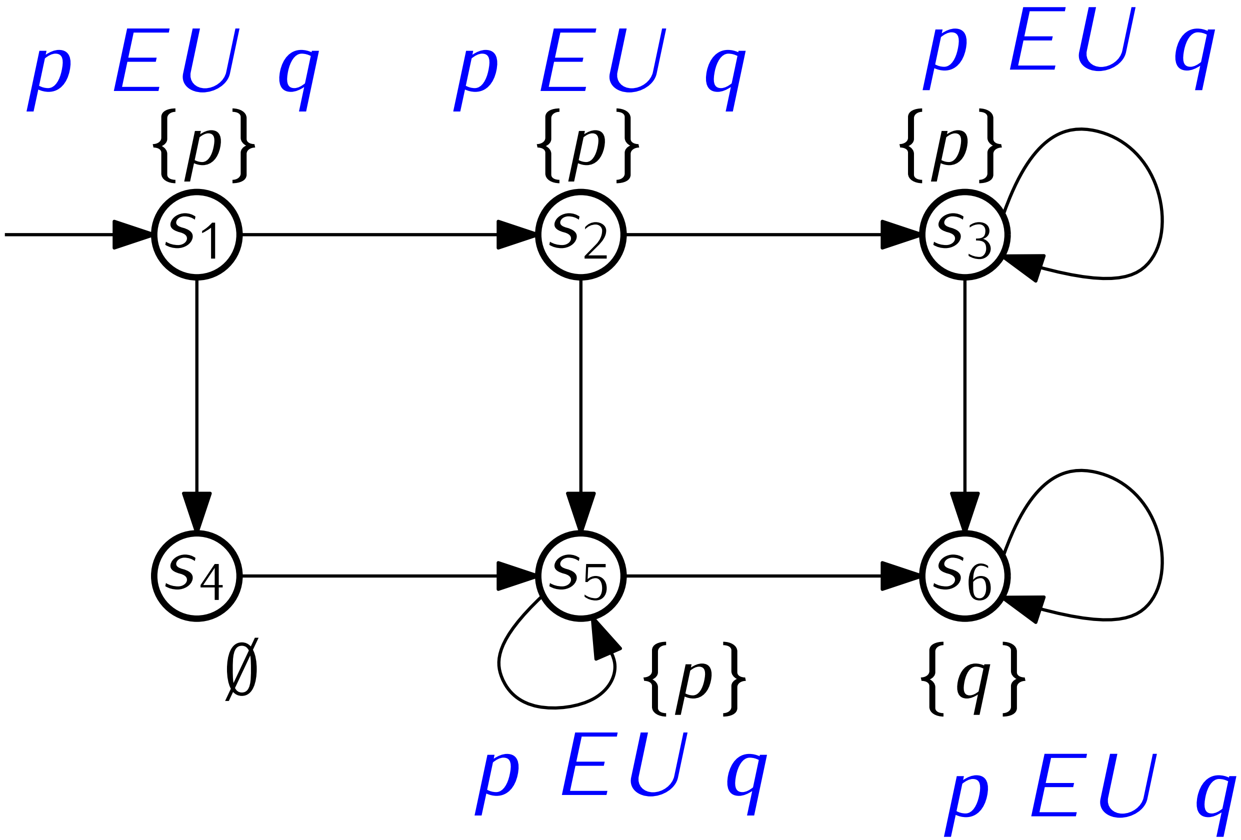

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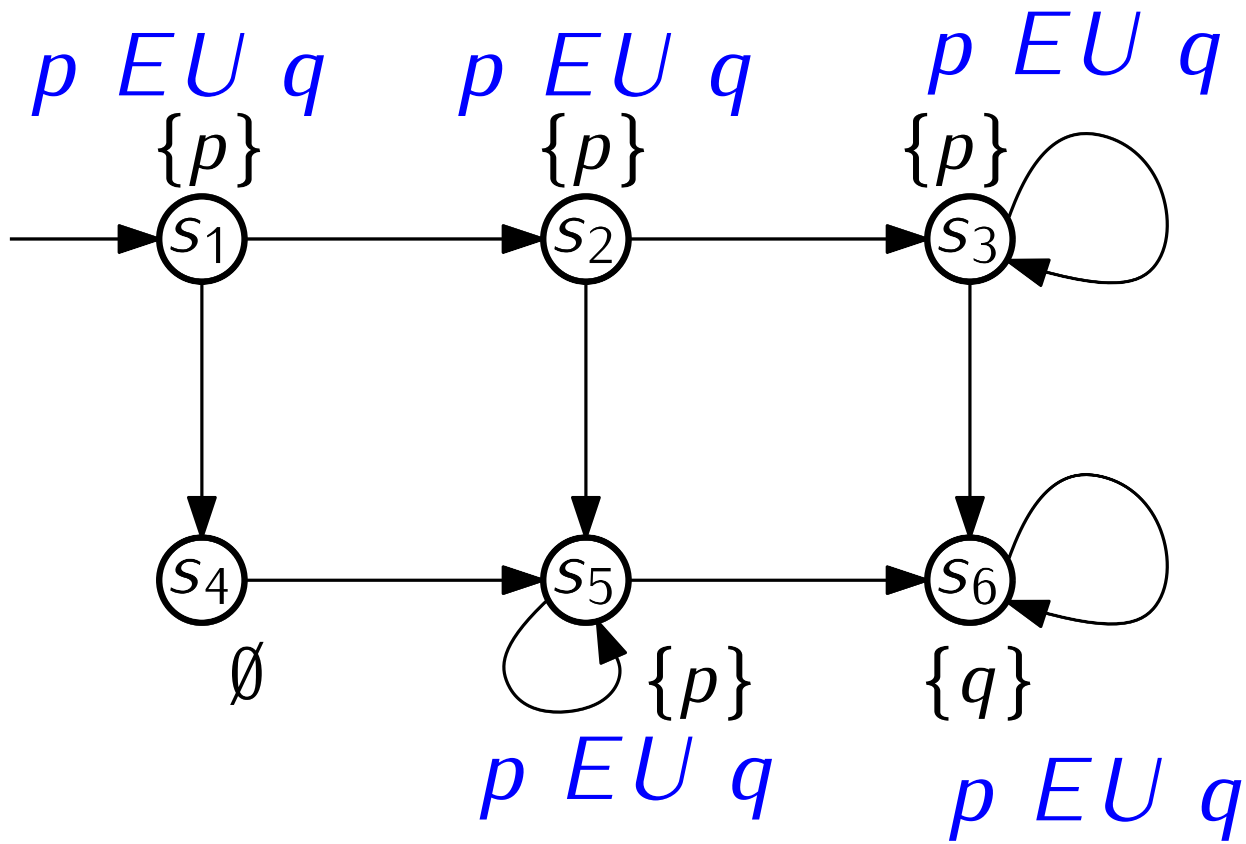
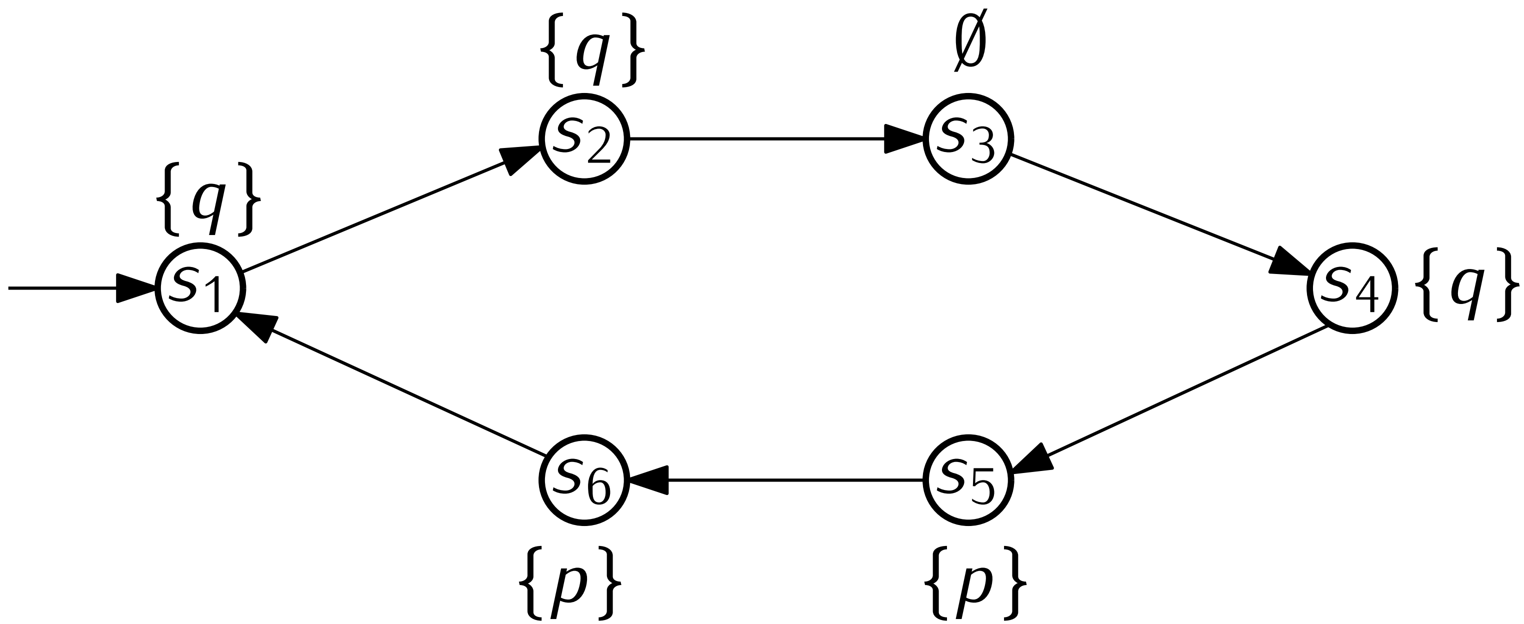


$$s_1, s_2, s_3, s_5, s_6 \models (p\ EU\ q)$$

The Algorithm for $\phi\ EU\ \psi$

Another example $\neg p\ EU\ \neg q$



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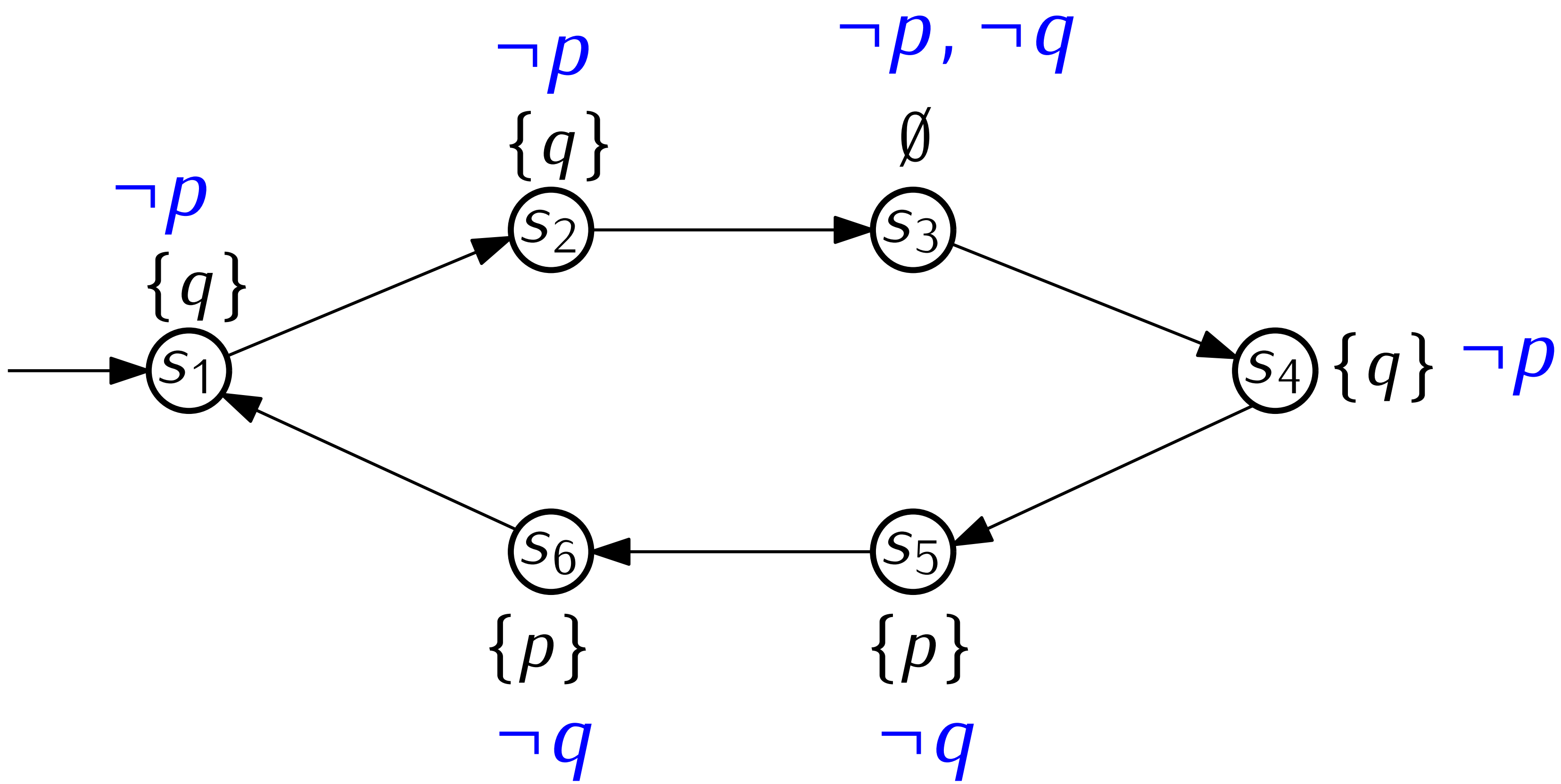

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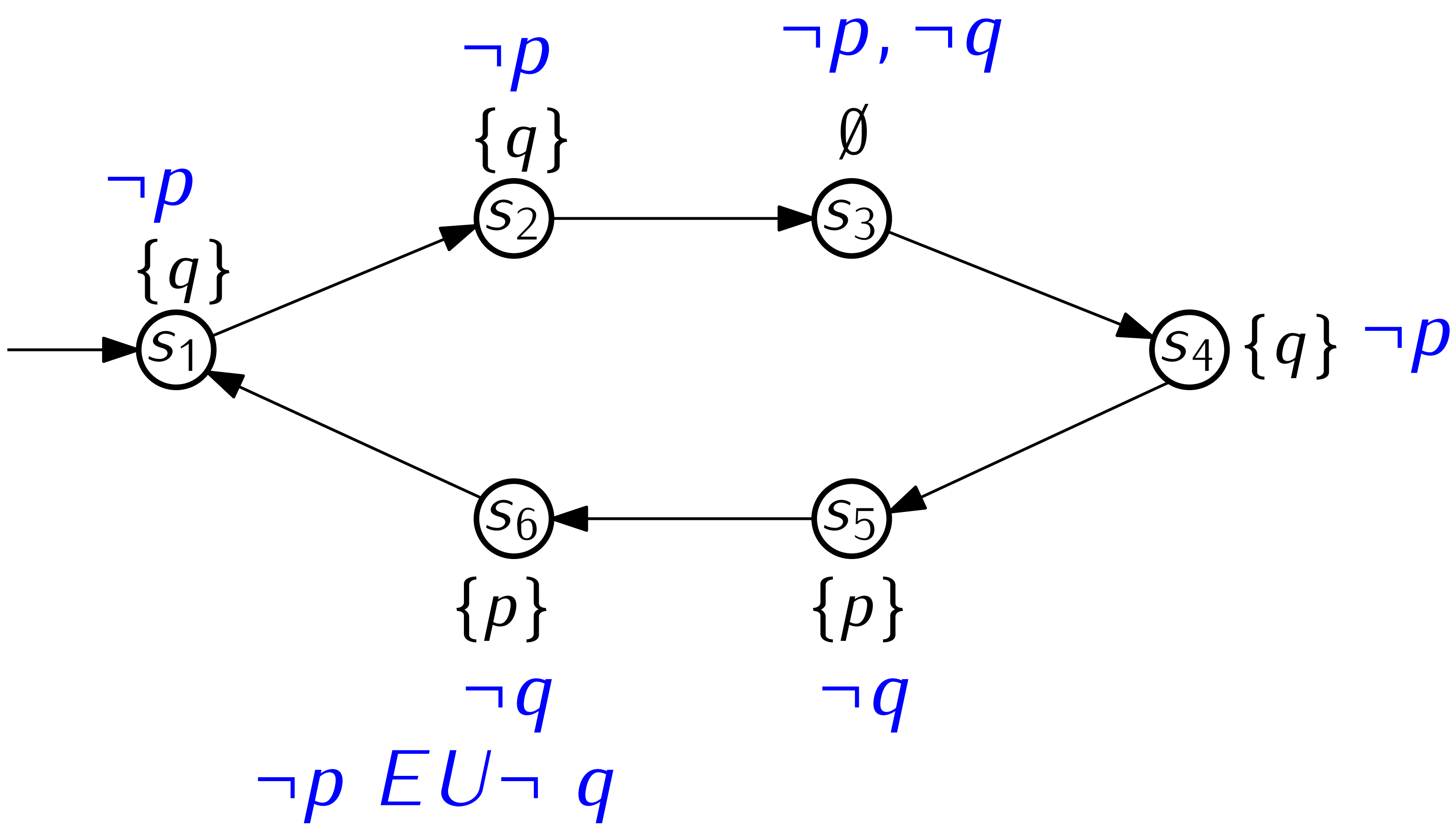

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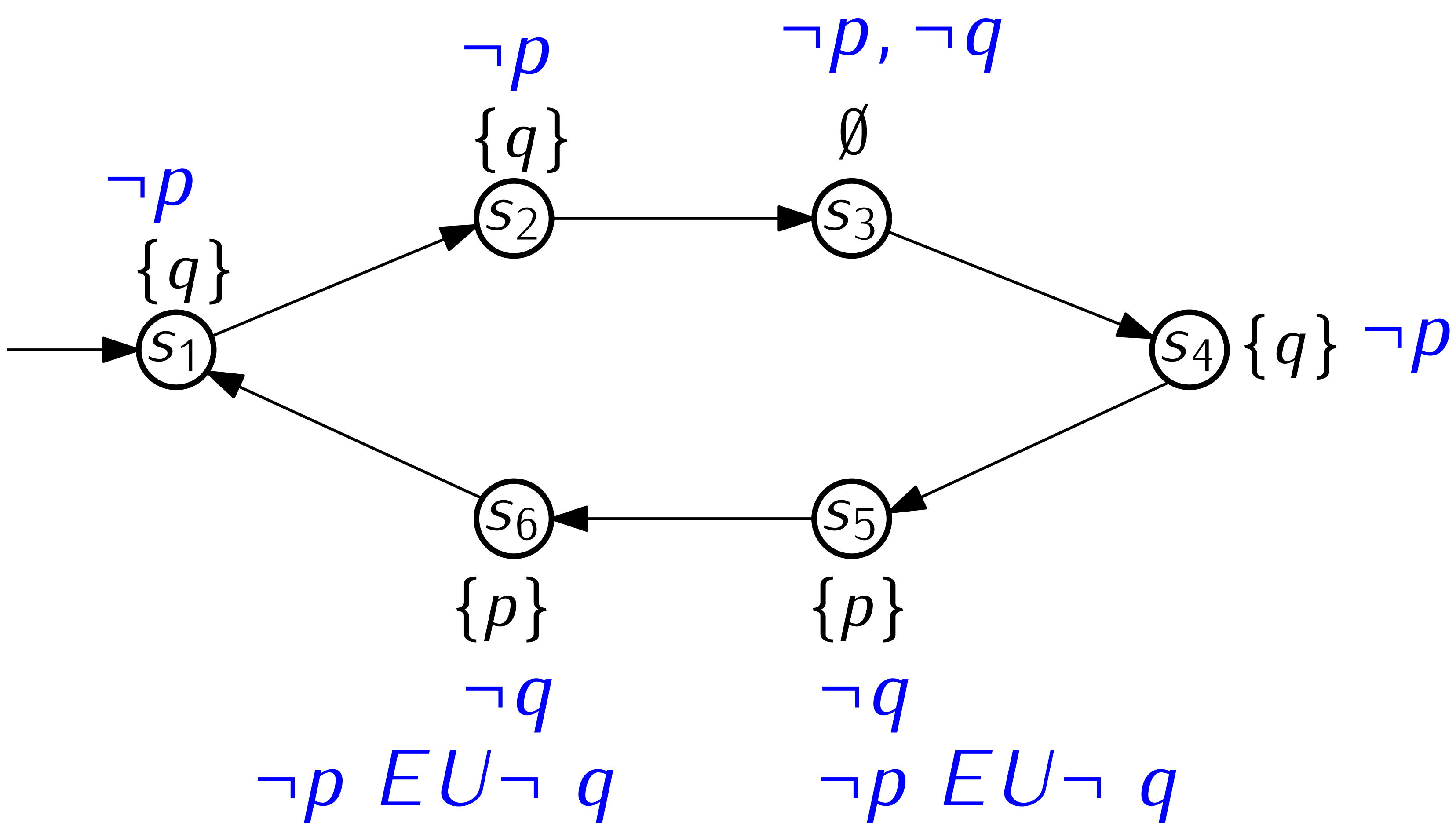

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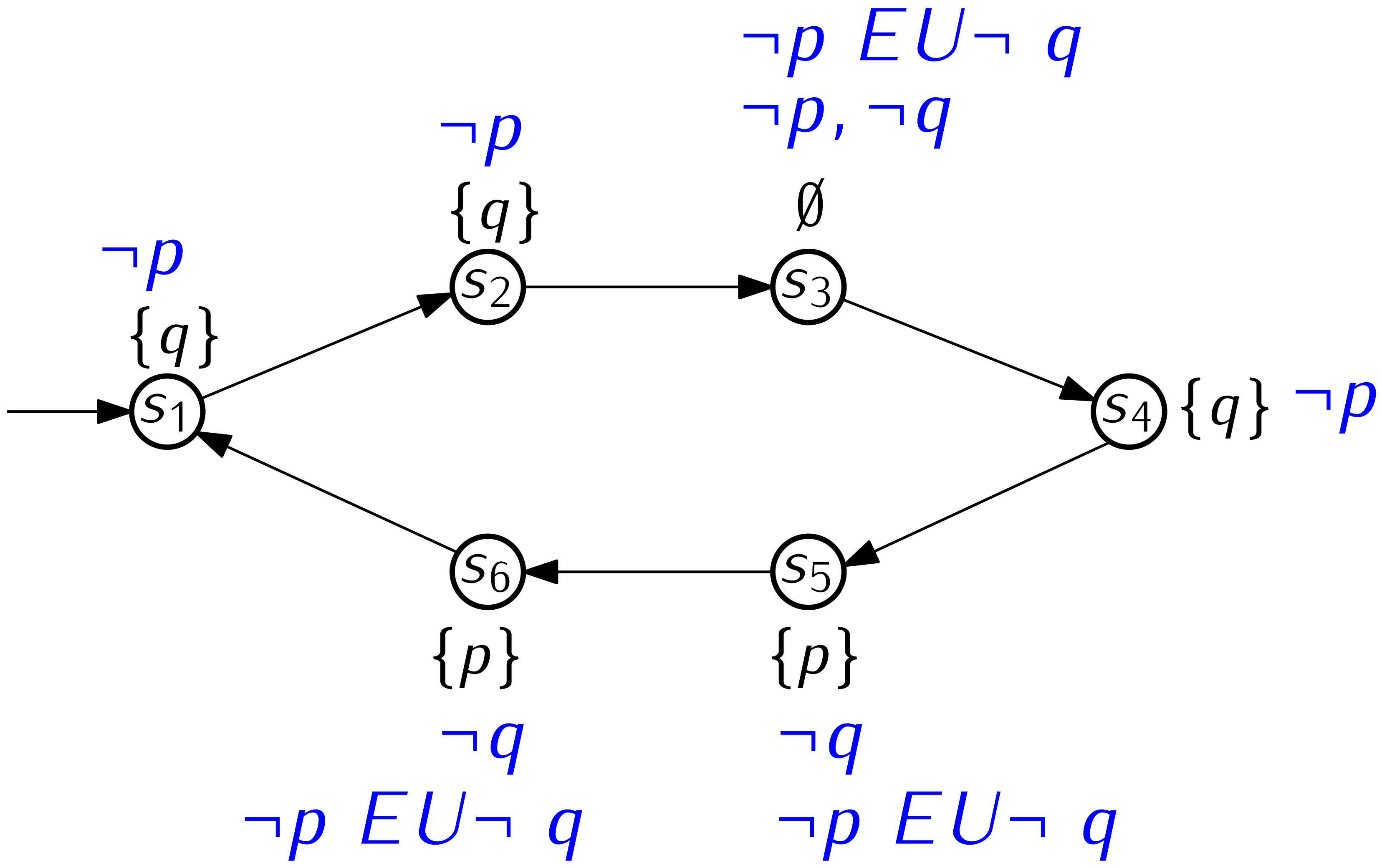

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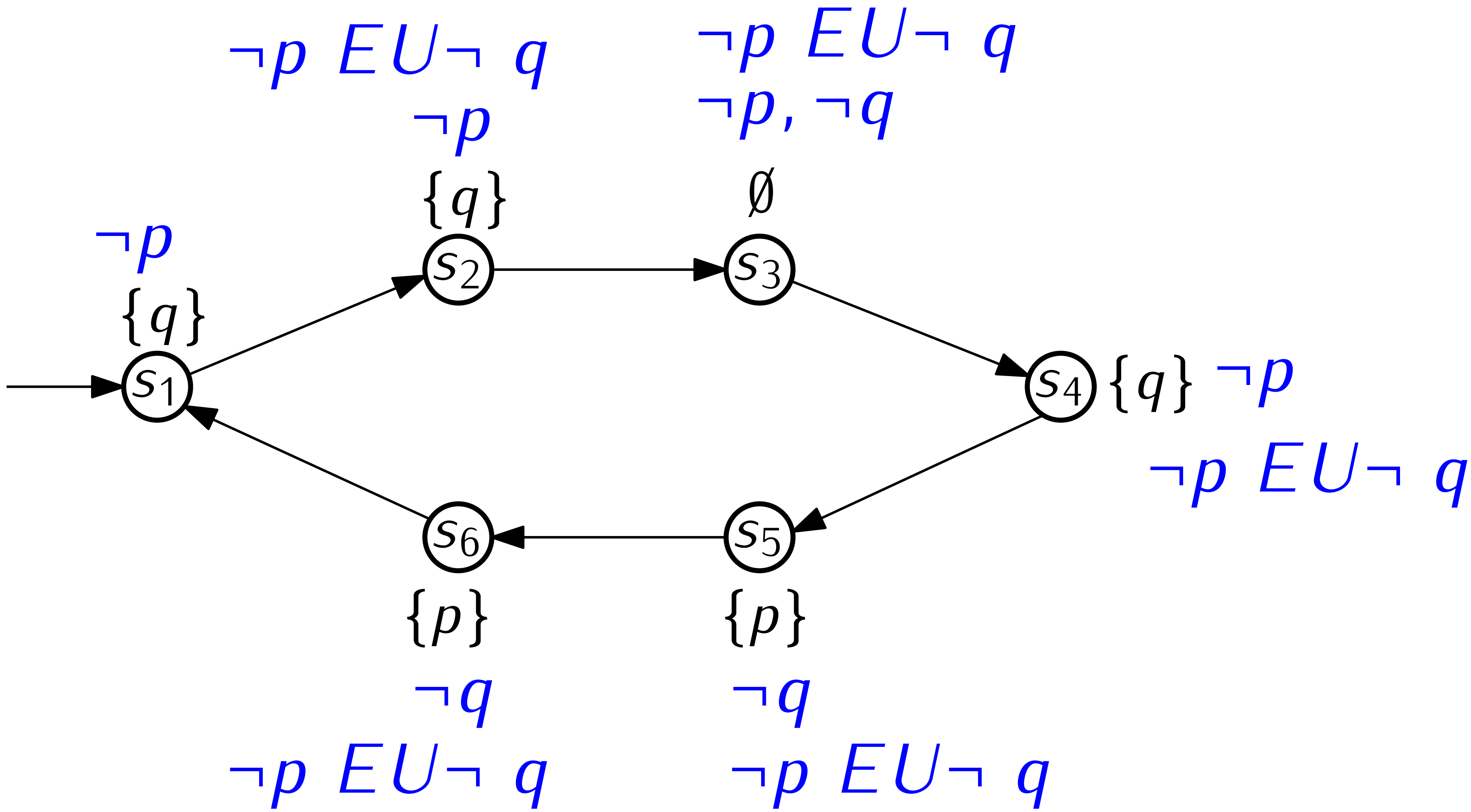

The Algorithm for $\phi\ EU\ \psi$

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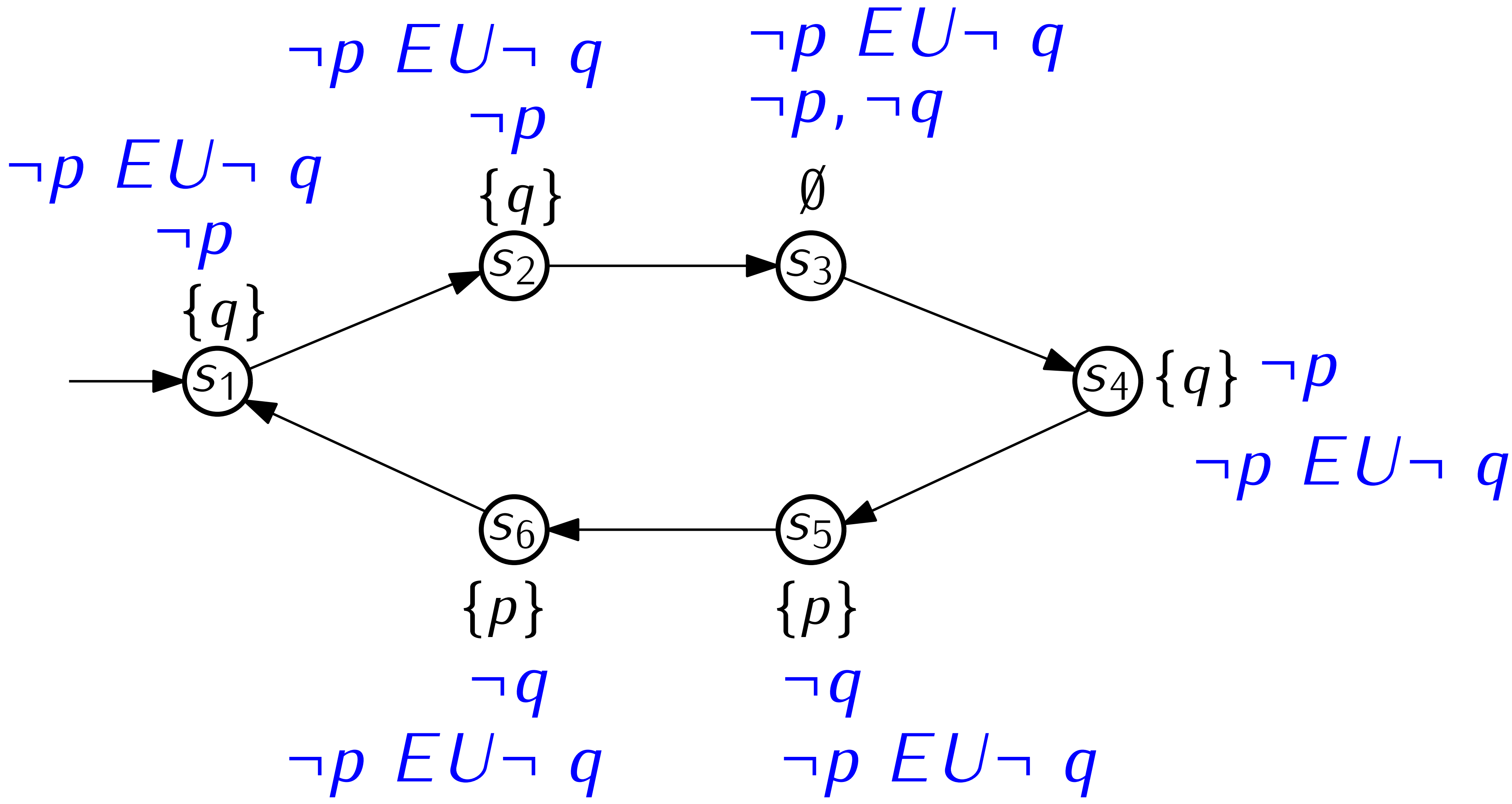
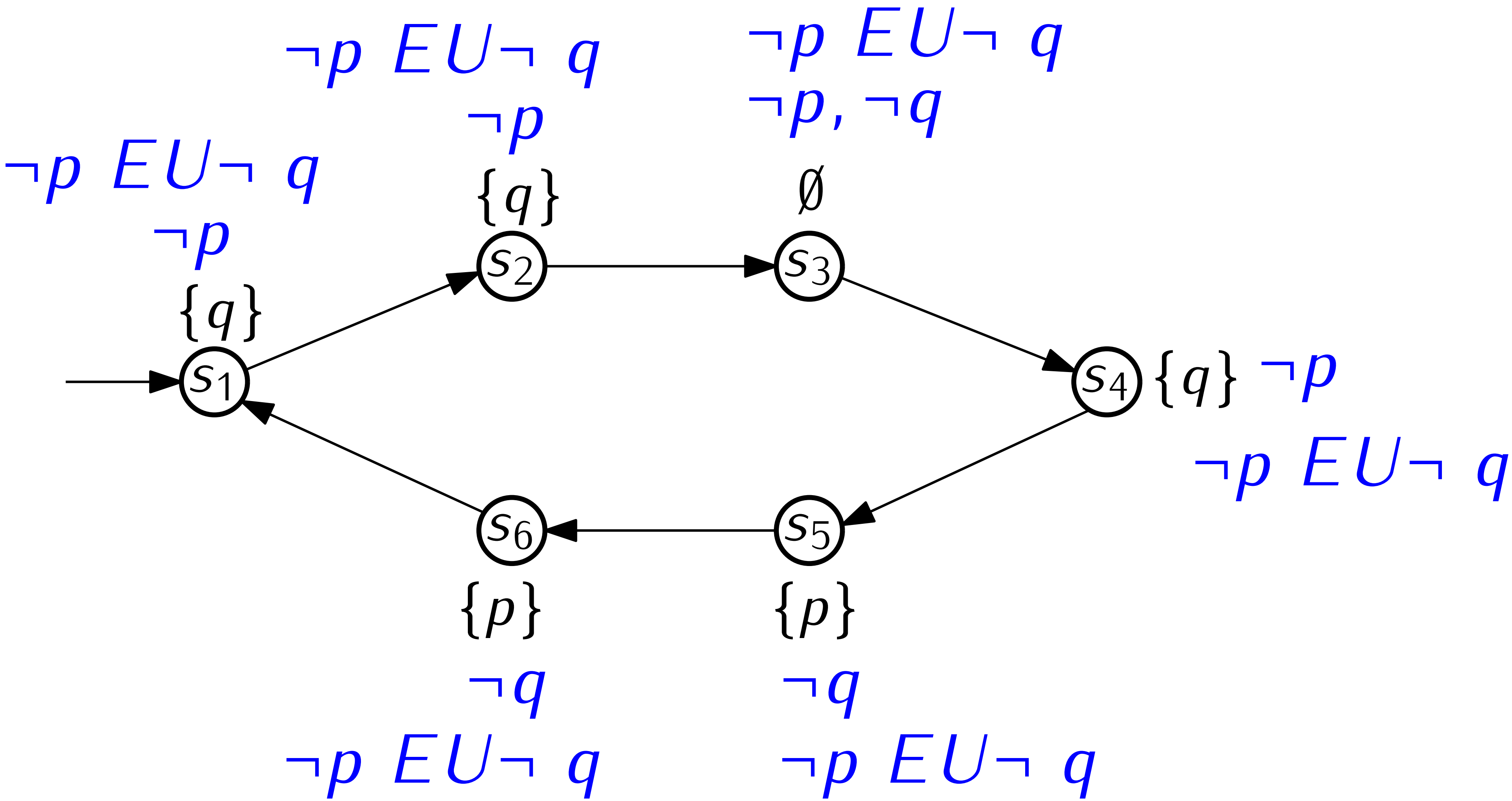


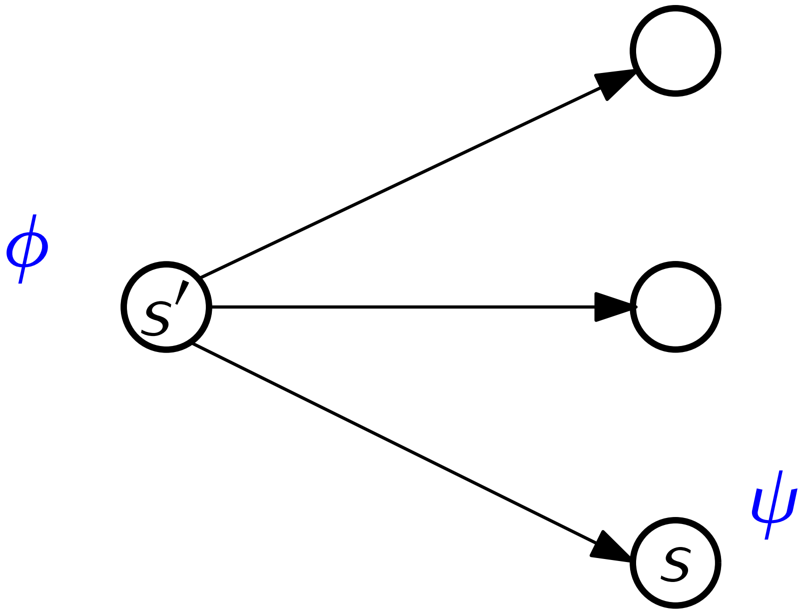
The Algorithm for $\phi\ EU\ \psi$

Another example $\neg p\ EU\ \neg q$

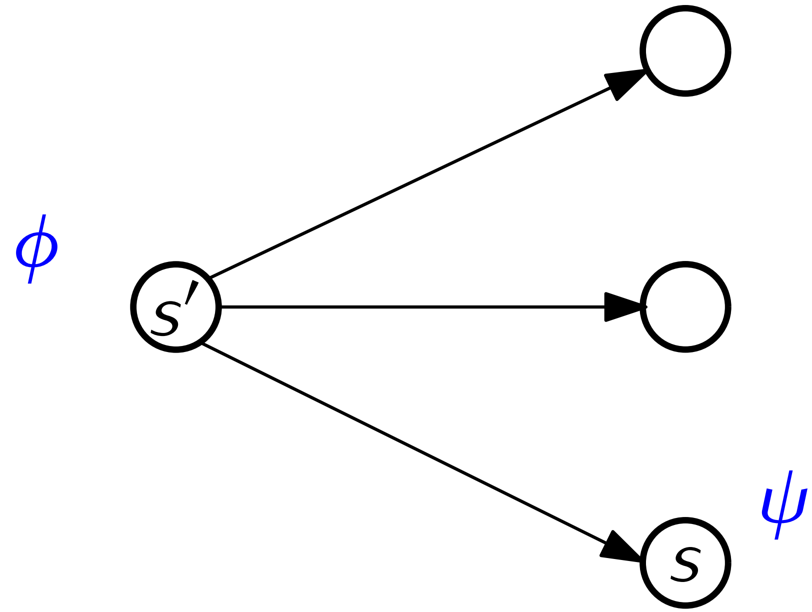


$s_1, s_2, s_3, s_4, s_5, s_6 \models (\neg p\ EU\ \neg q)$

The Algorithm for $\phi \; EU \; \psi$

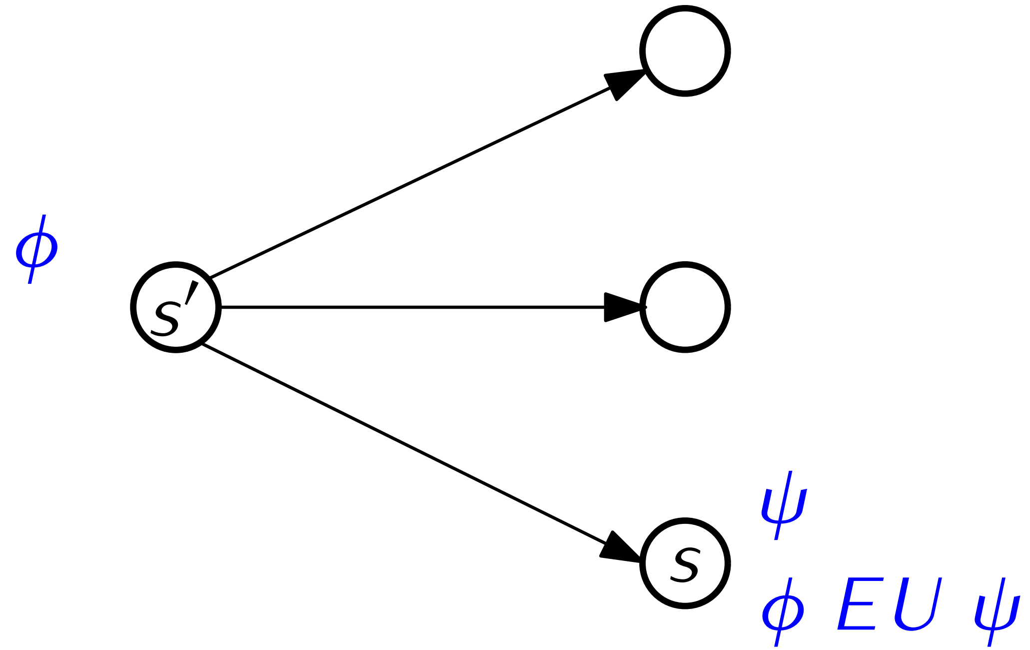


The Algorithm for $\phi \; EU \; \psi$



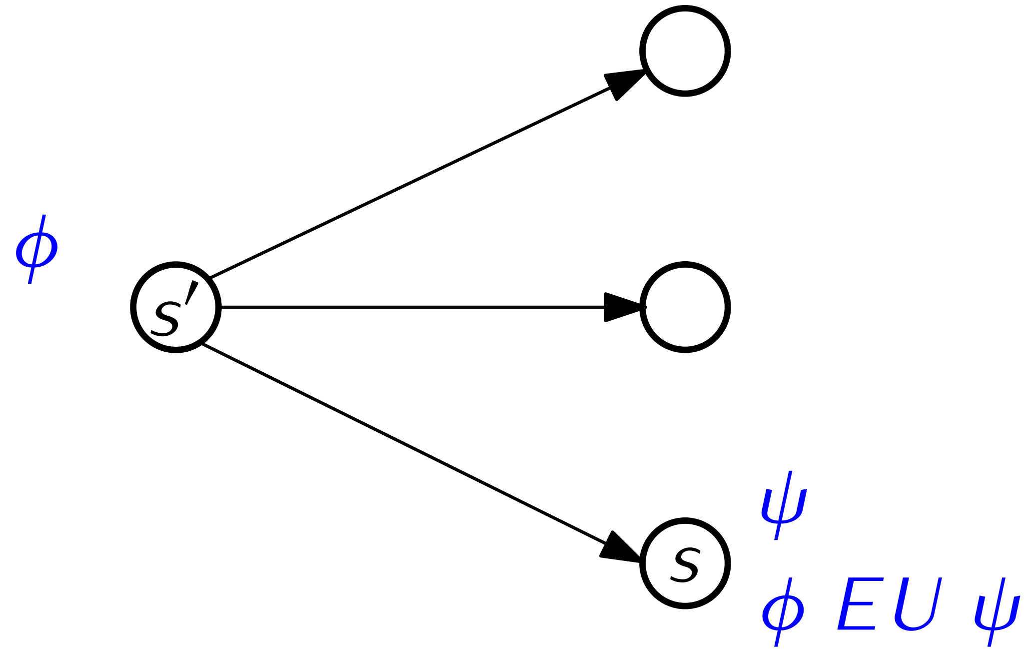
If a state is labelled with ψ label it with $\phi \; EU \; \psi$.

The Algorithm for $\phi\ EU\ \psi$



If a state is labelled with ψ label it with $\phi\ EU\ \psi$.

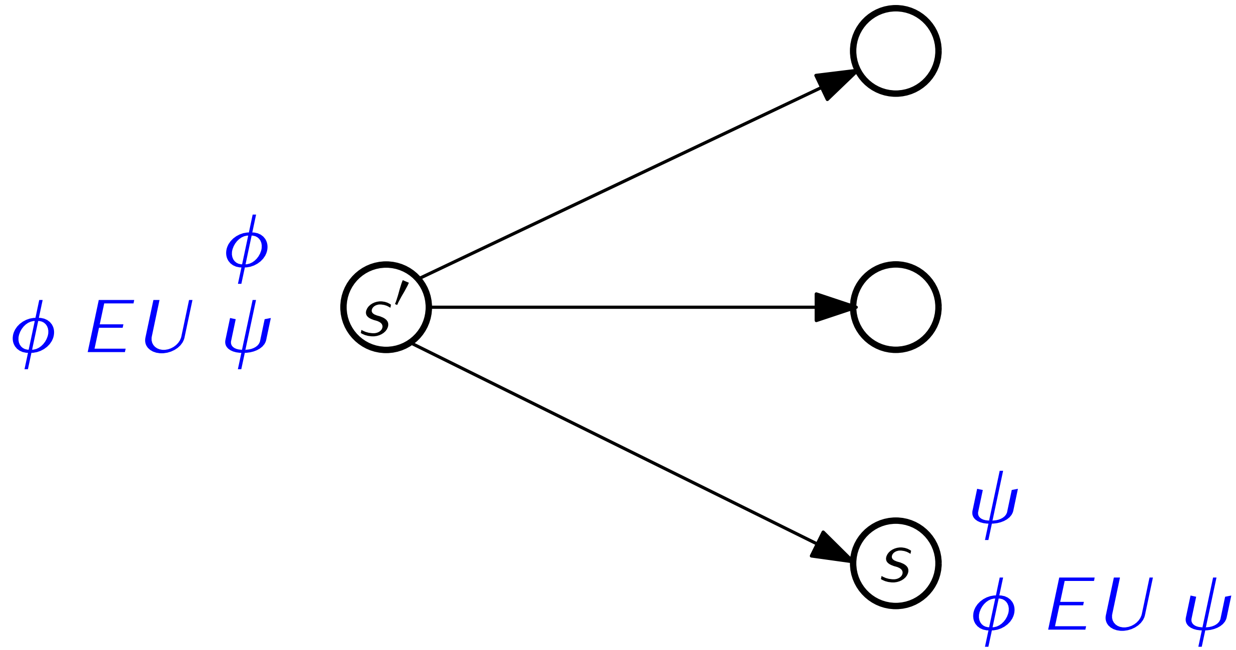
The Algorithm for $\phi\ EU\ \psi$



If a state is labelled with ψ label it with $\phi\ EU\ \psi$.

For any state s' labelled with ϕ, if at least one successor state s is labelled with $\phi\ EU\ \psi$, then label s' with $\phi\ EU\ \psi$ as well. Repeat until labels stop changing.

The Algorithm for $\phi\ EU\ \psi$



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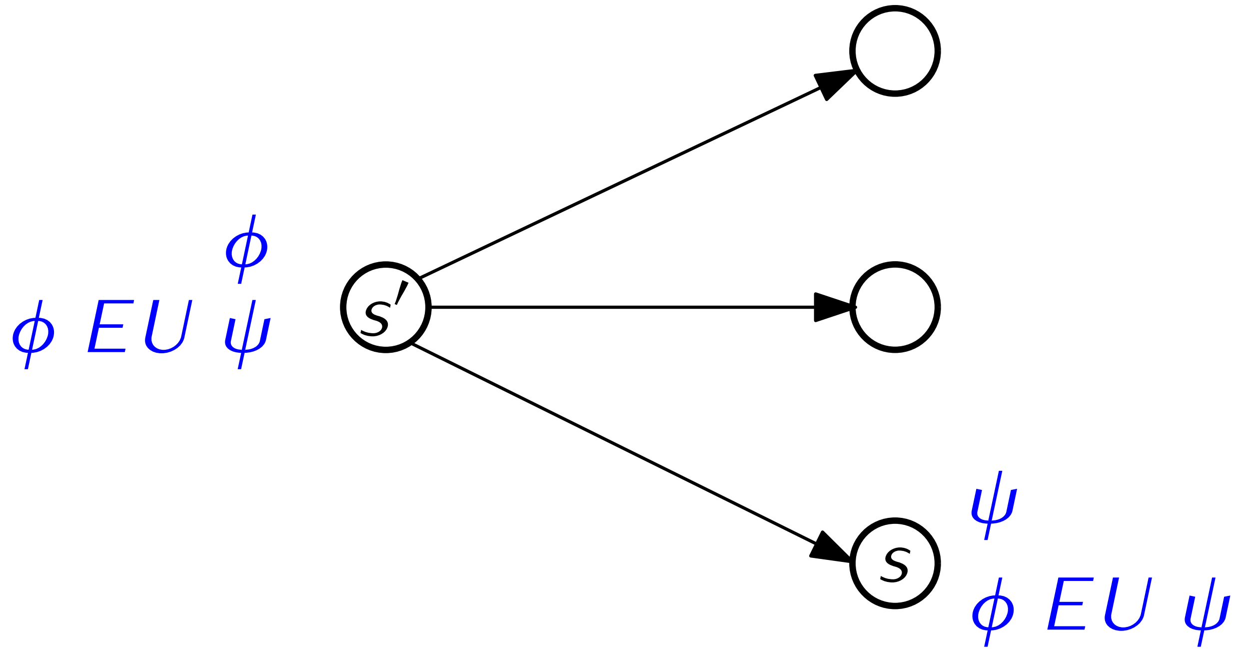


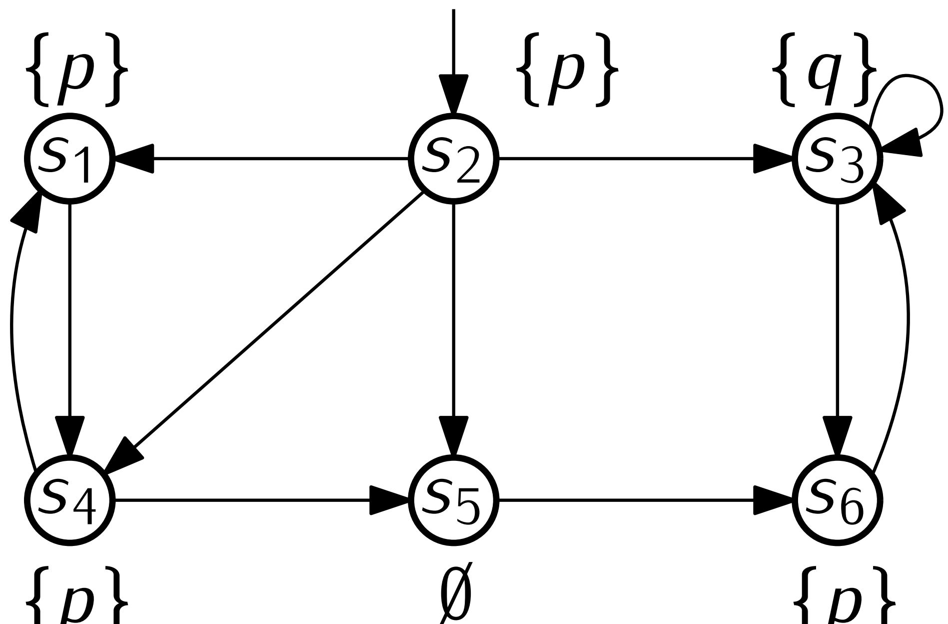
If a state is labelled with ψ label it with $\phi\ EU\ \psi$.

For any state s' labelled with ϕ, if at least one successor state s is labelled with $\phi\ EU\ \psi$, then label s' with $\phi\ EU\ \psi$ as well. Repeat until labels stop changing.

Call this process $\text{SAT}_{EU}(\phi, \psi)$

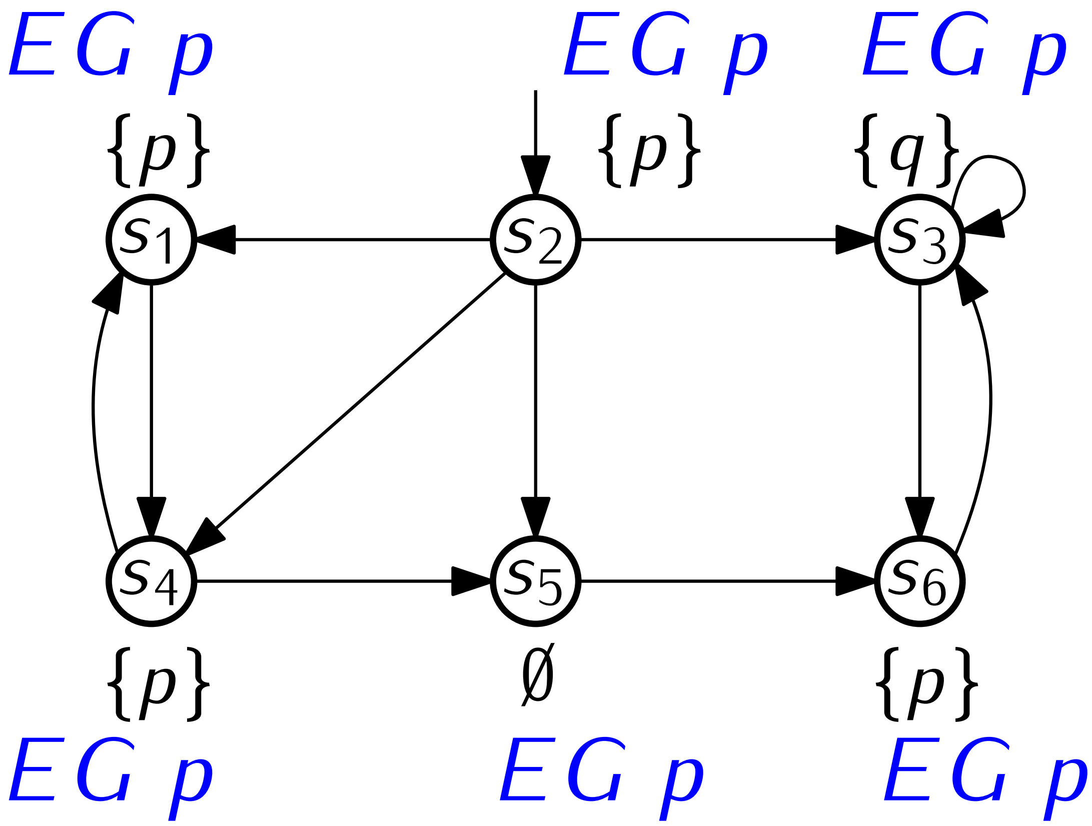
The Algorithm for $EG\ \phi$

First, an example $EG\ p$



The Algorithm for $EG\ \phi$

First, an example $\qquad EG\ p$



The Algorithm for $EG\ \phi$

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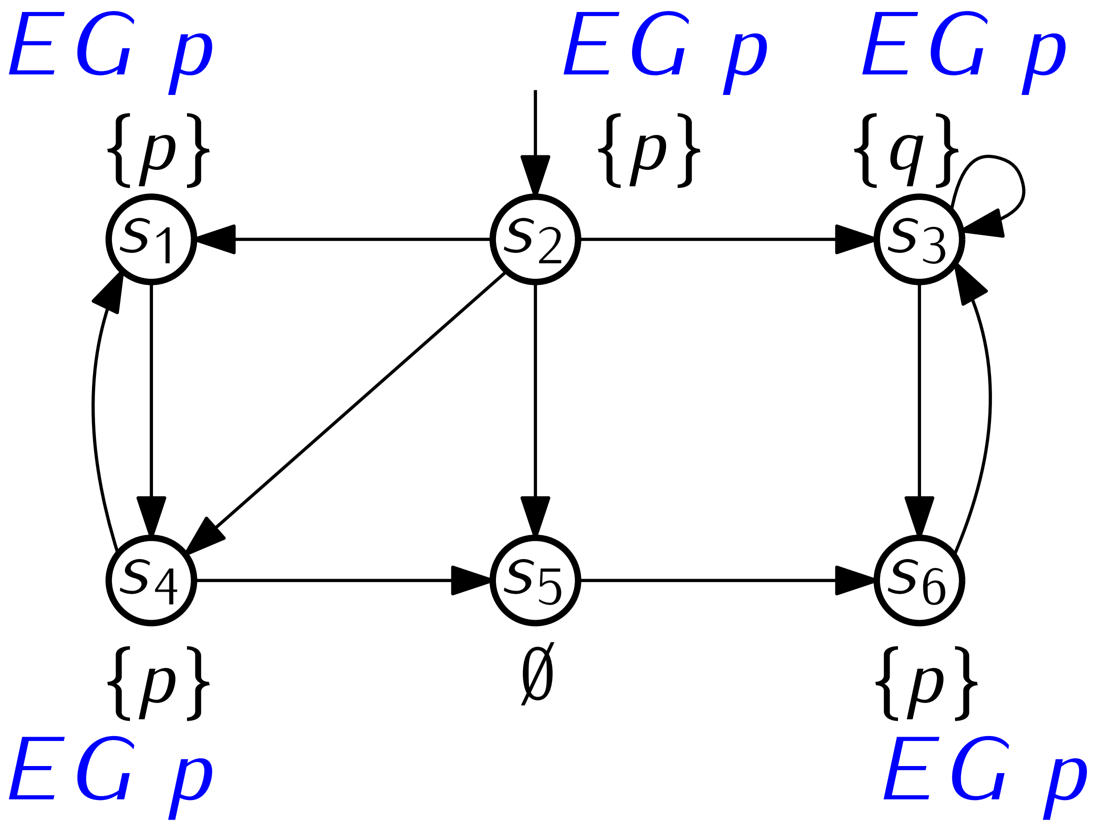

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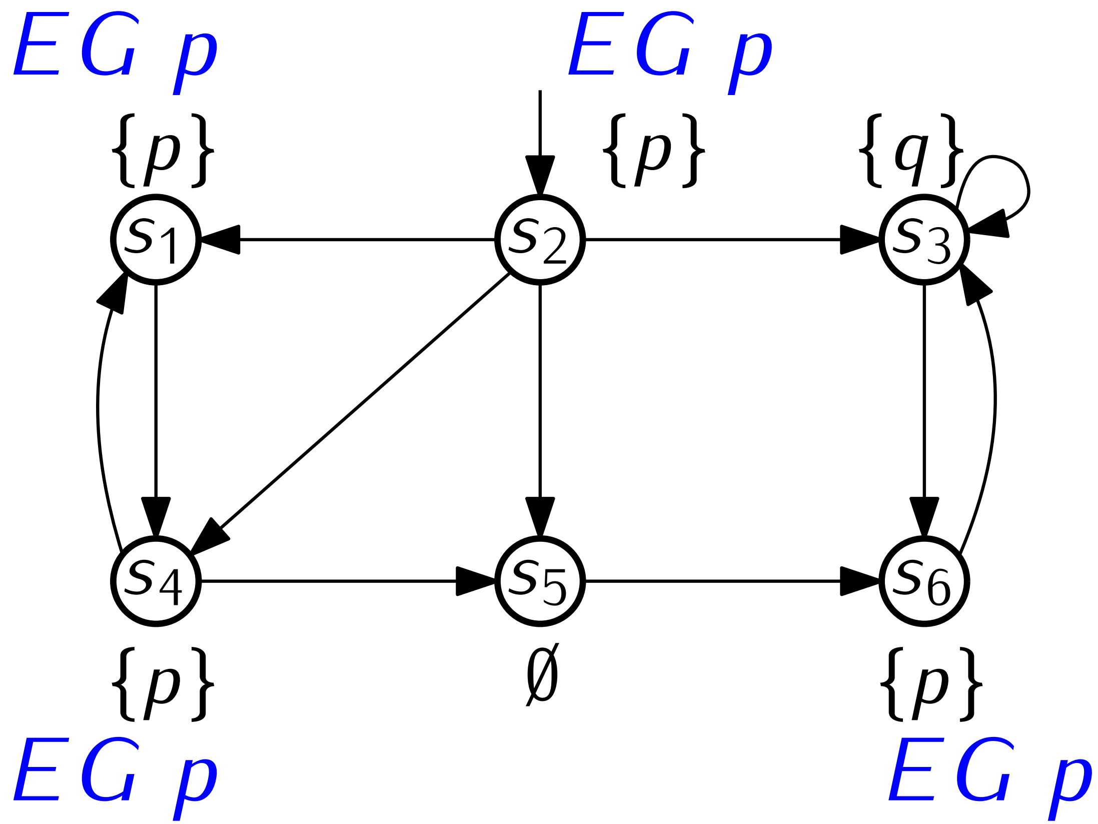

The Algorithm for $EG\ \phi$

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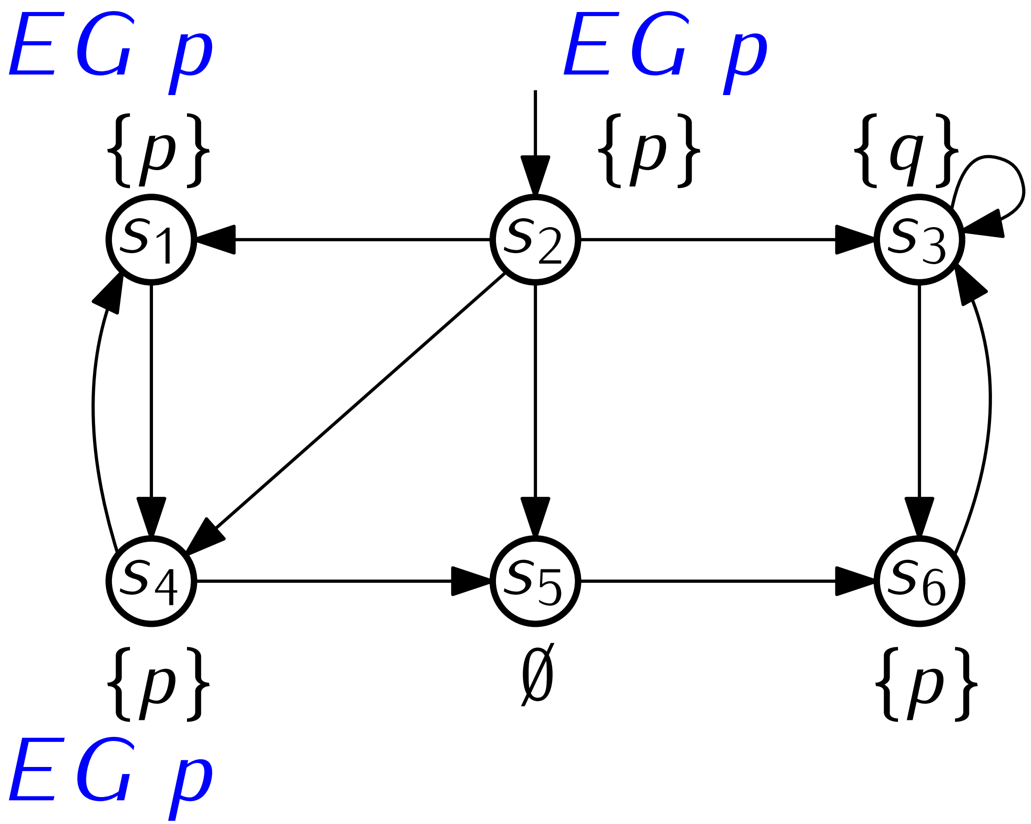
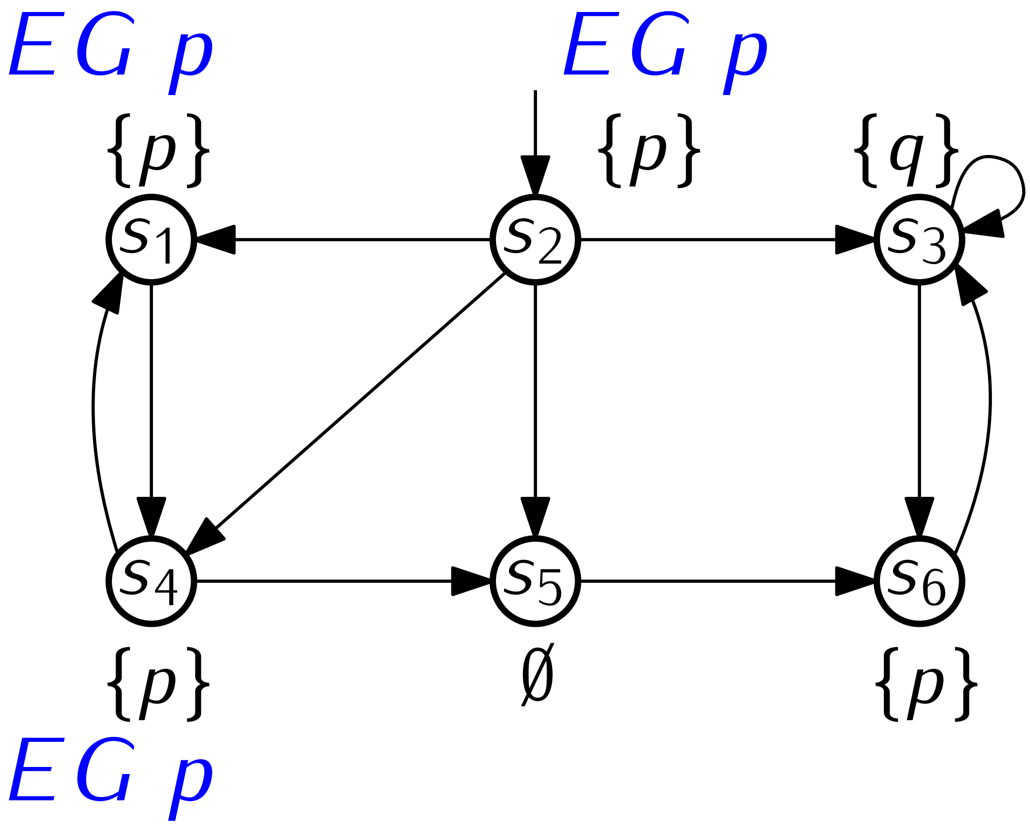


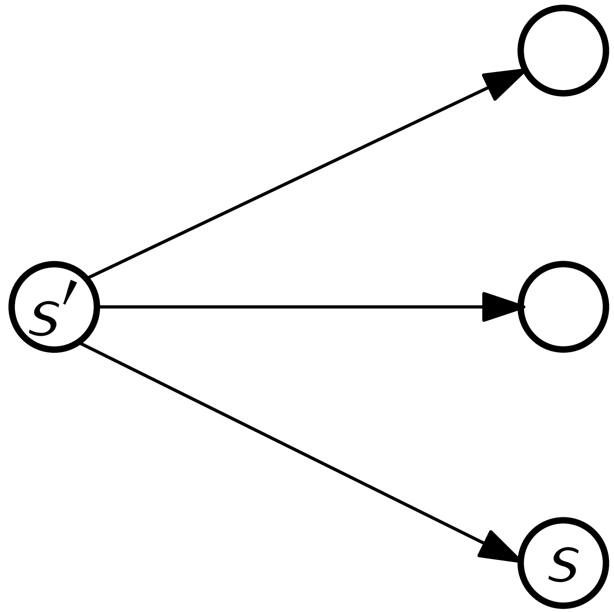
The Algorithm for $EG\ \phi$

First, an example $\quad EG\ p$

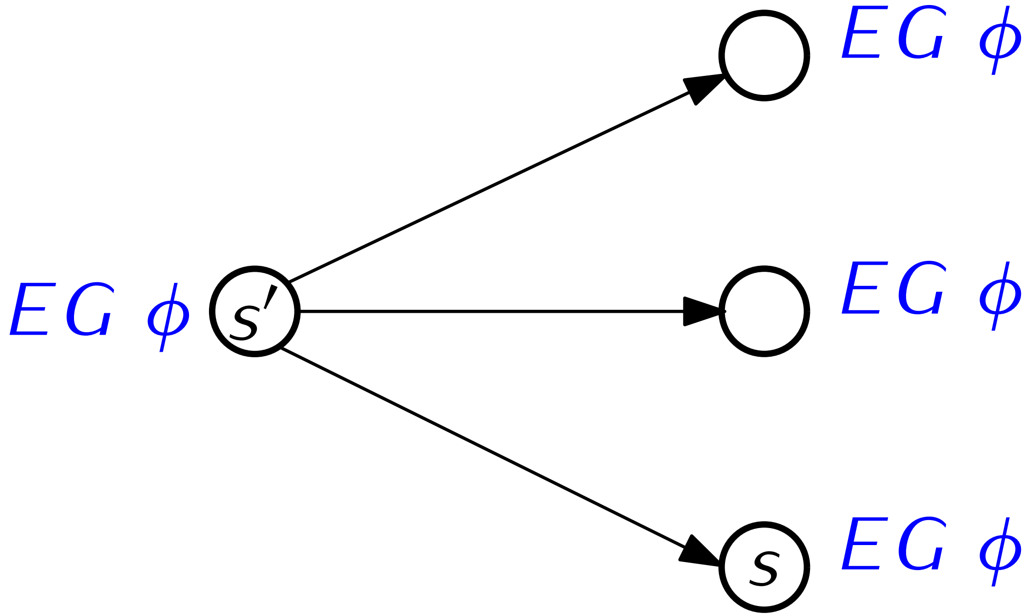


$$s_1, s_2, s_4 \models EG\ p$$

The Algorithm for $EG\ \phi$

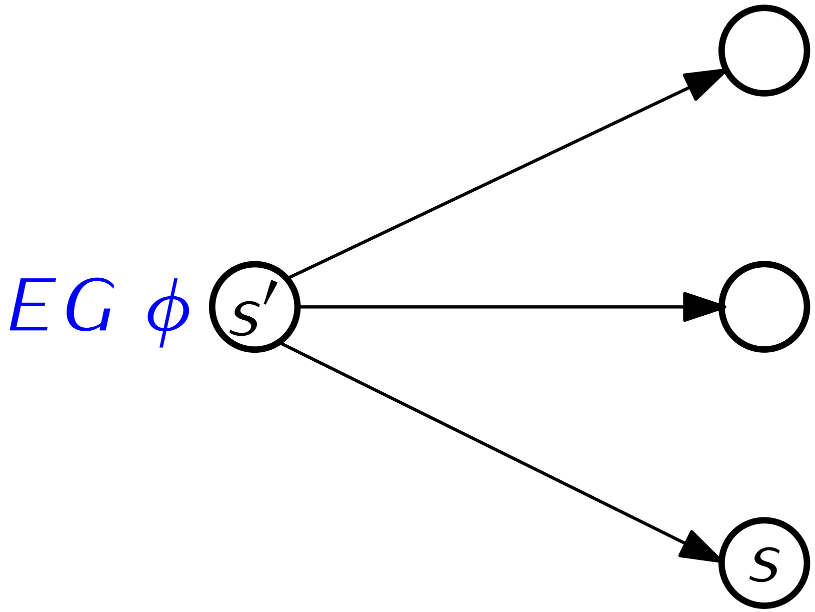


The Algorithm for $EG\ \phi$



Label all states with $EG\ \phi$

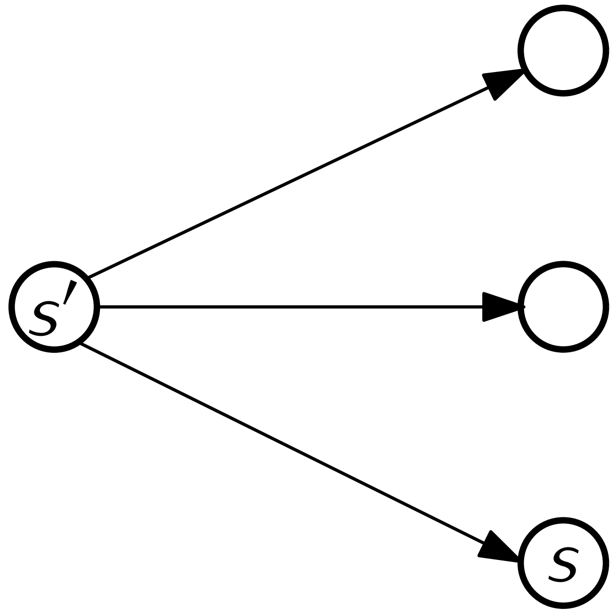
The Algorithm for $EG\ \phi$



Label all states with $EG\ \phi$

Delete $EG\ \phi$ from any state not labelled with ϕ.

The Algorithm for $EG\ \phi$

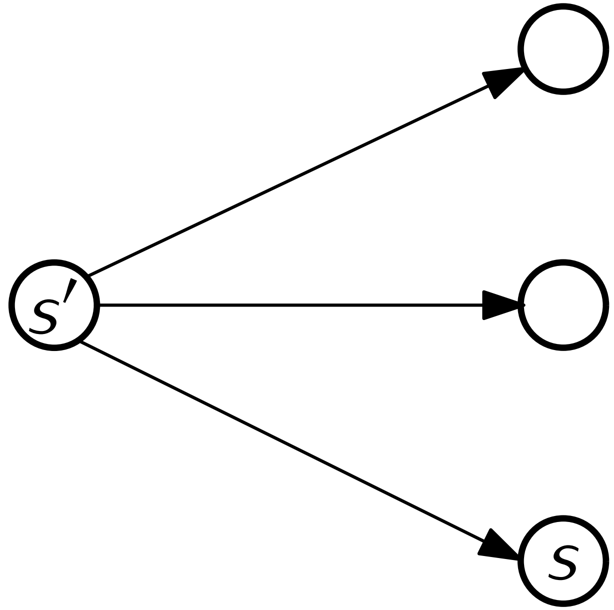


Label all states with $EG\ \phi$

Delete $EG\ \phi$ from any state not labelled with ϕ.

Delete $EG\ \phi$ from any state where none of its successors is labelled with $EG\ \phi$. Repeat until no more labels can be deleted.

The Algorithm for $EG\ \phi$



Label all states with $EG\ \phi$

Delete $EG\ \phi$ from any state not labelled with ϕ.

Delete $EG\ \phi$ from any state where none of its successors is labelled with $EG\ \phi$. Repeat until no more labels can be deleted.

Call this process $\mathrm{SAT}_{EG}(\phi)$

Summary so far:

Rewrite everything in terms of EX, EG, EU.

$$AX\phi \equiv \neg EX \neg \phi$$

$$AG\phi \equiv \neg EF \neg \phi$$

$$AF\phi \equiv \neg EG \neg \phi$$

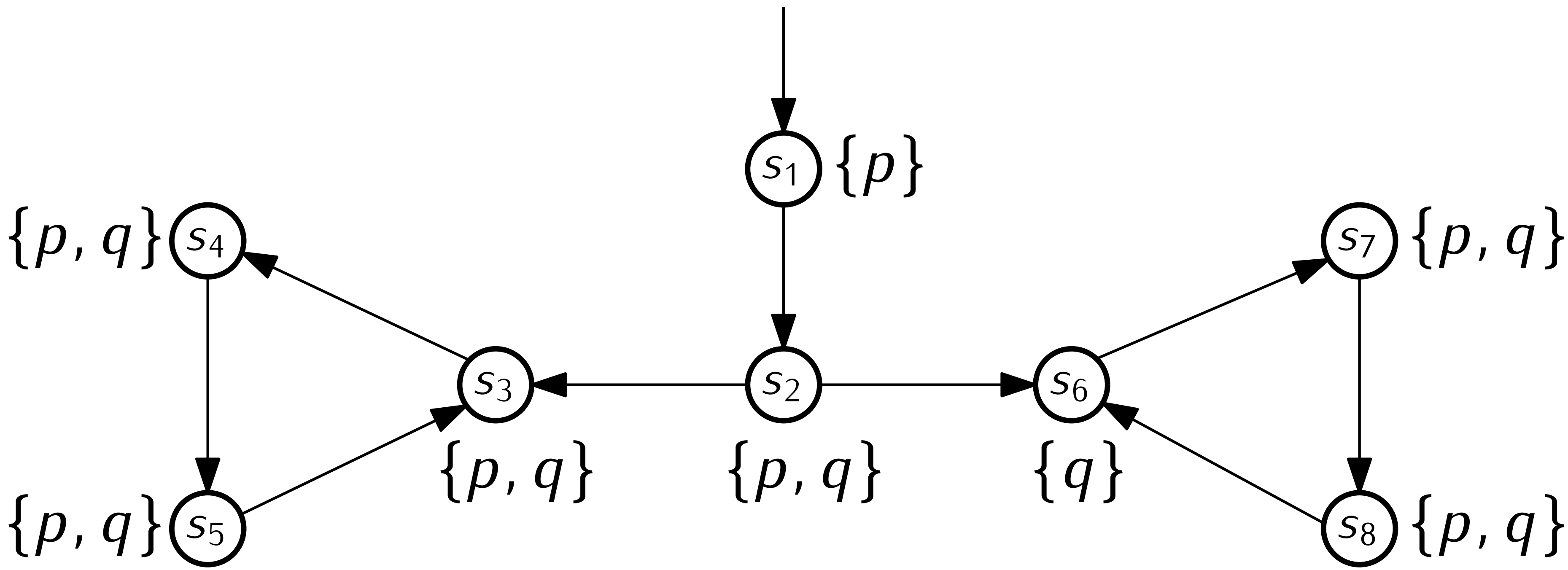
$$\phi AU \psi \equiv \neg(EG \neg \phi \quad \vee \quad \neg \phi \; EU \; (\neg \phi \wedge \neg \psi))$$

Procedures for determining the set of satisfied states

$\mathrm{SAT}_{EX}(\phi)$, $\mathrm{SAT}_{EG}(\phi)$, $\mathrm{SAT}_{EU}(\phi, \psi)$

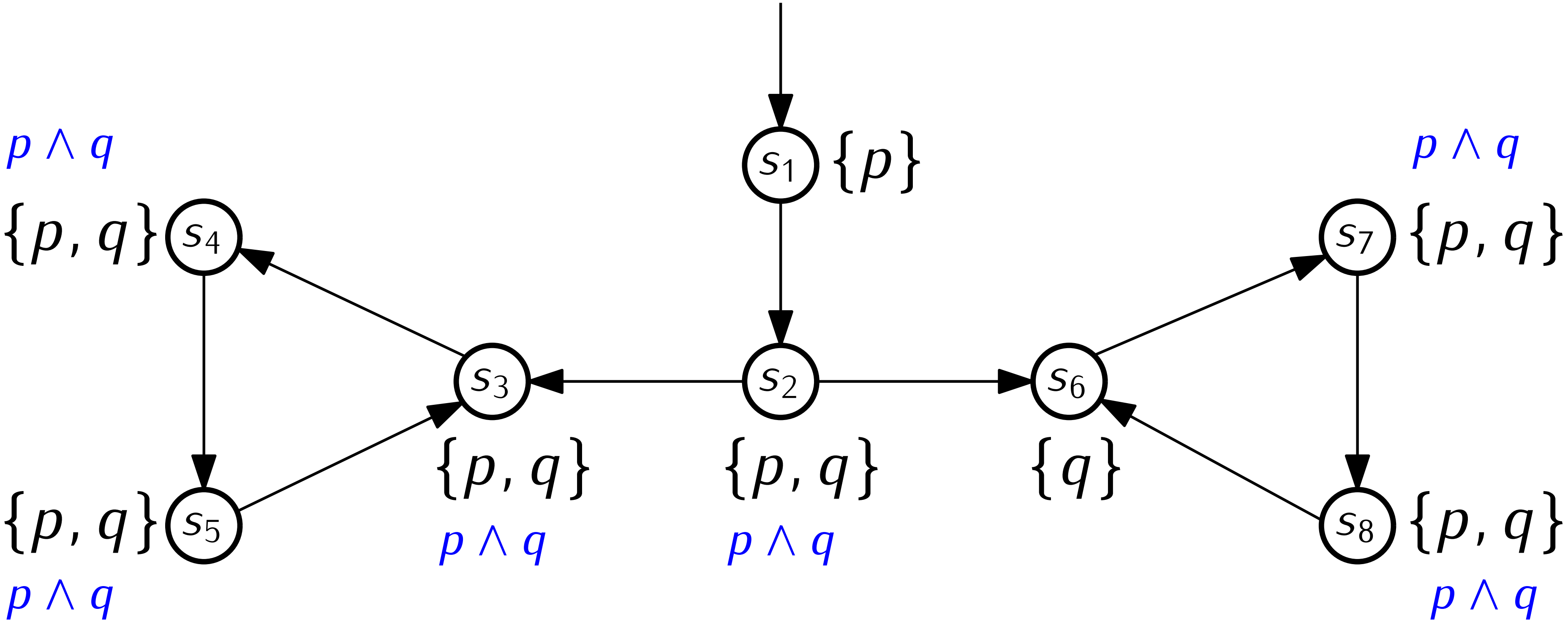
The General CTL Model Checking Algorithm

Example $EX\ EG\ (p \wedge q)$



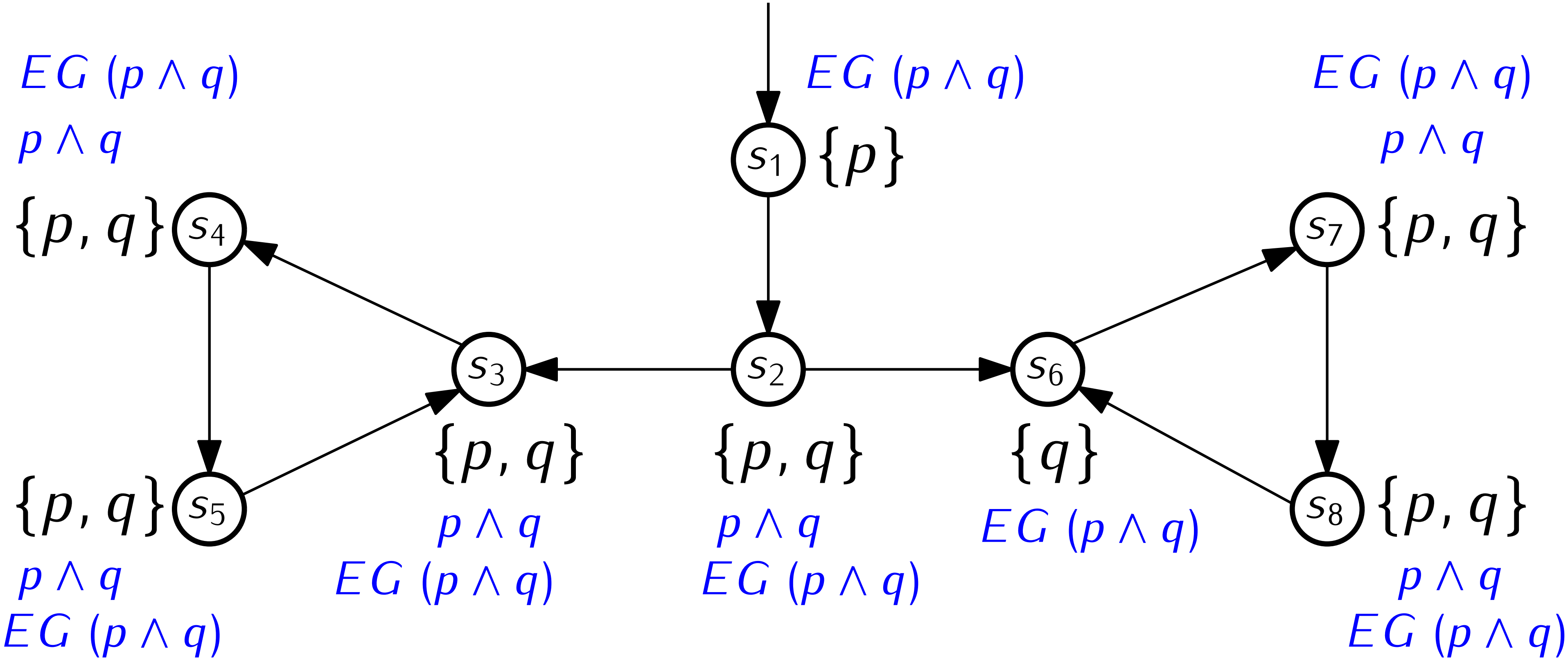
The General CTL Model Checking Algorithm

Example $EX\ EG\ (p \land q)$



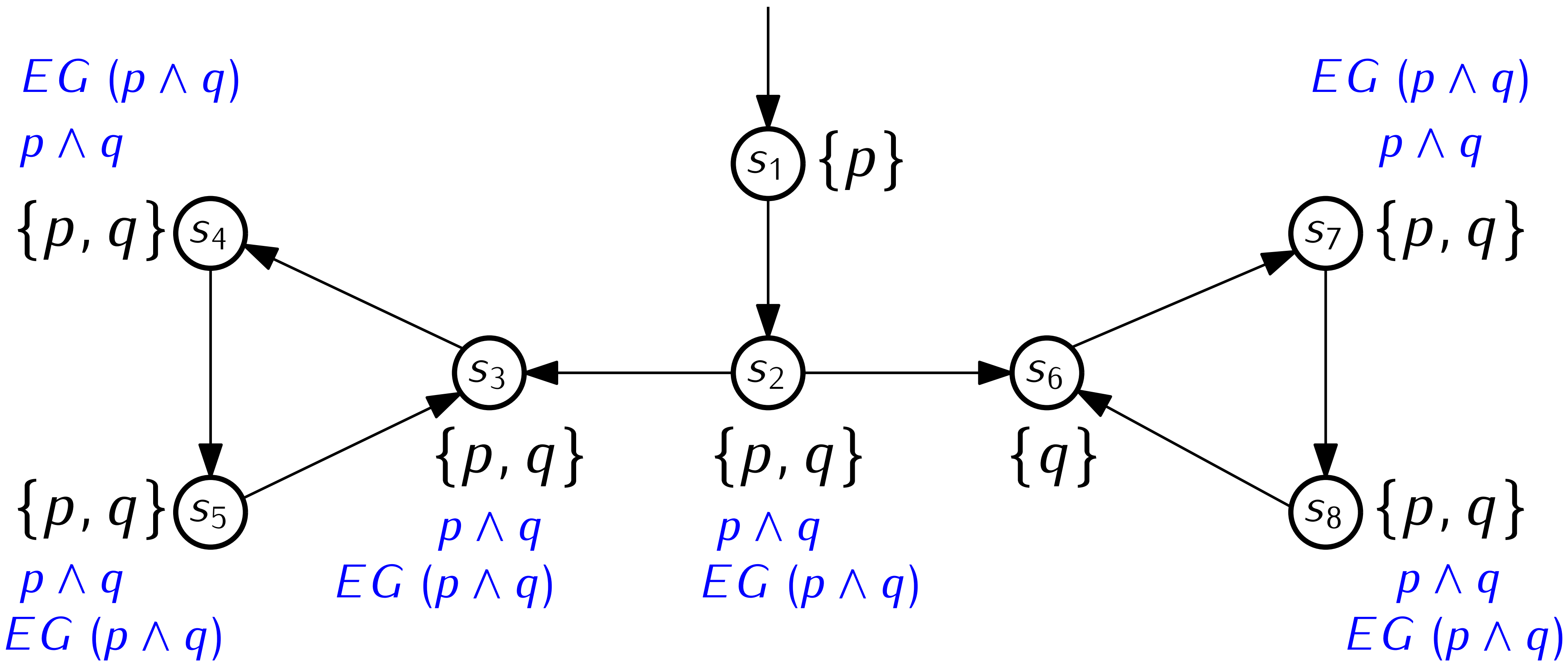
The General CTL Model Checking Algorithm

Example $EX\ \underline{EG\ (p \wedge q)}$



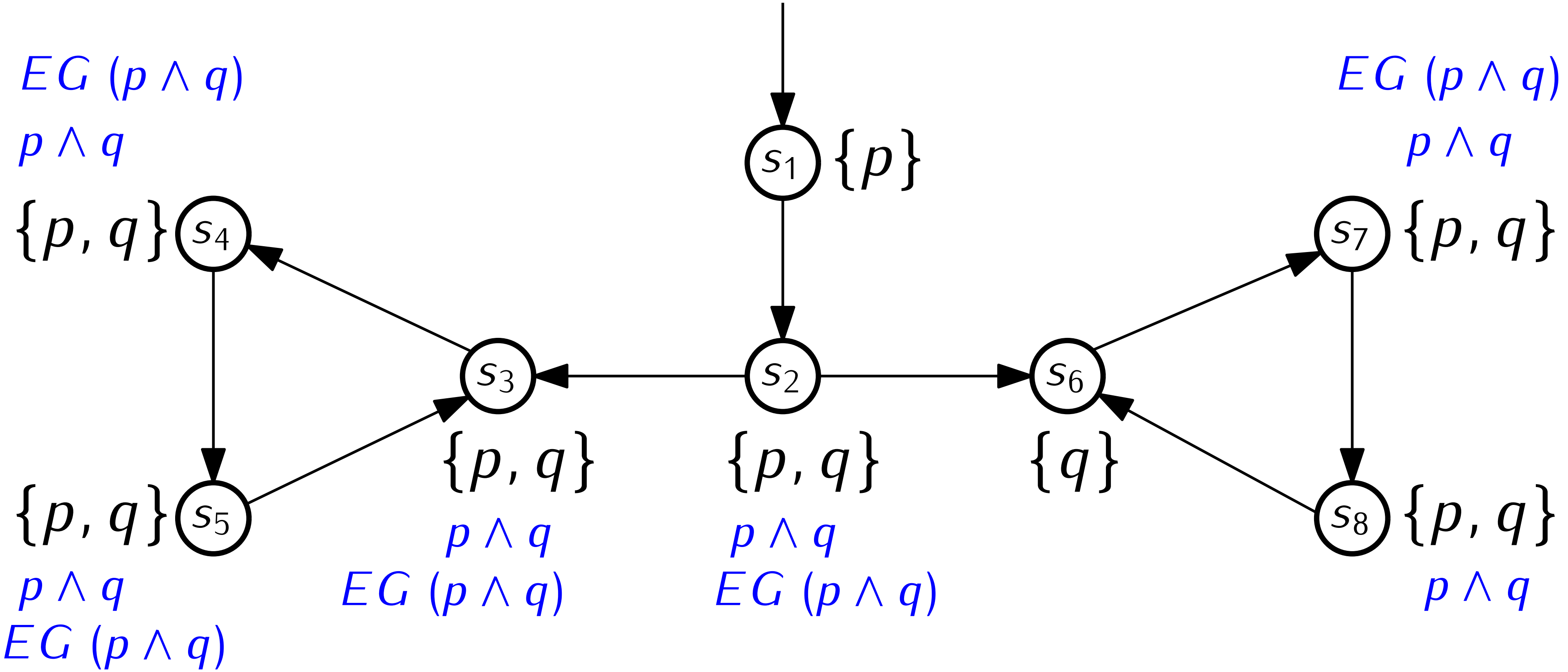
The General CTL Model Checking Algorithm

Example $EX\ \underline{EG\ (p \land q)}$



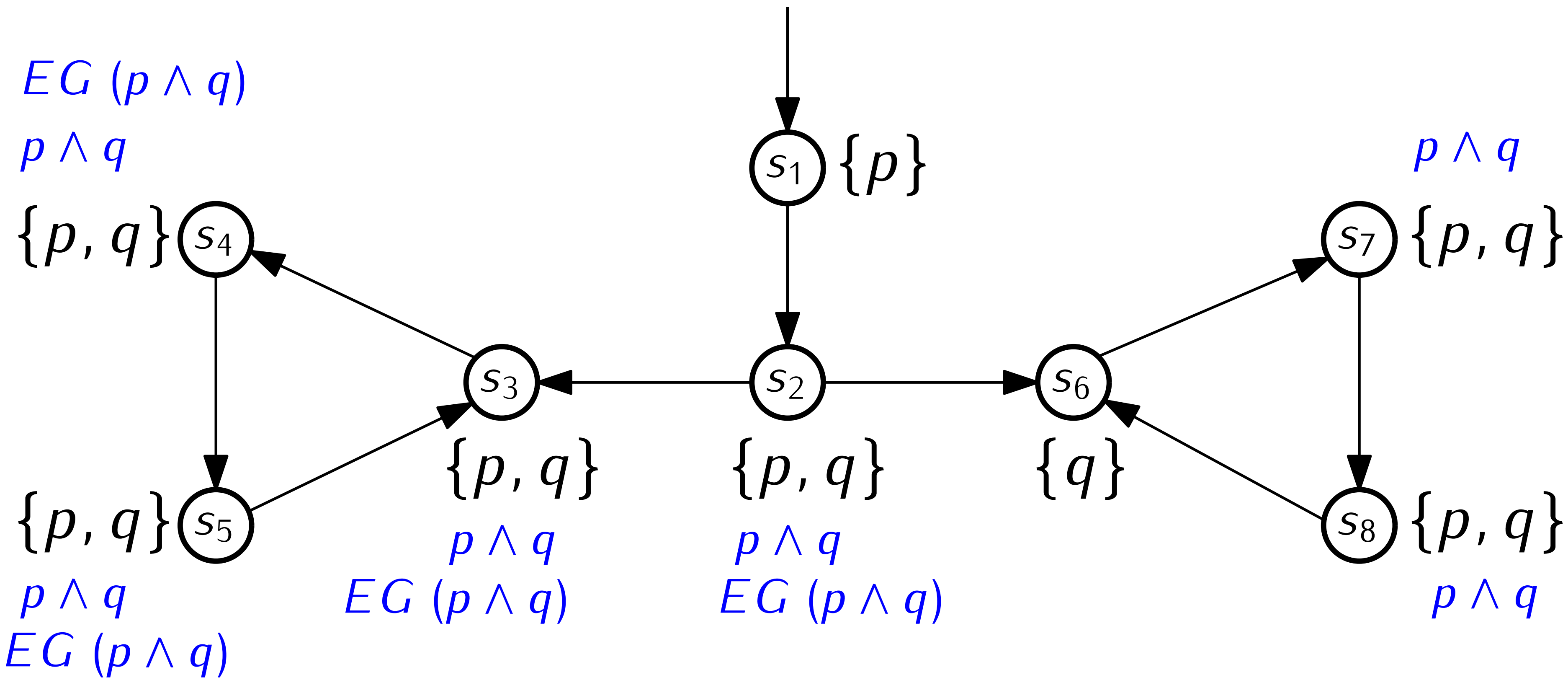
The General CTL Model Checking Algorithm

Example $EX\ \underline{EG\ (p \wedge q)}$



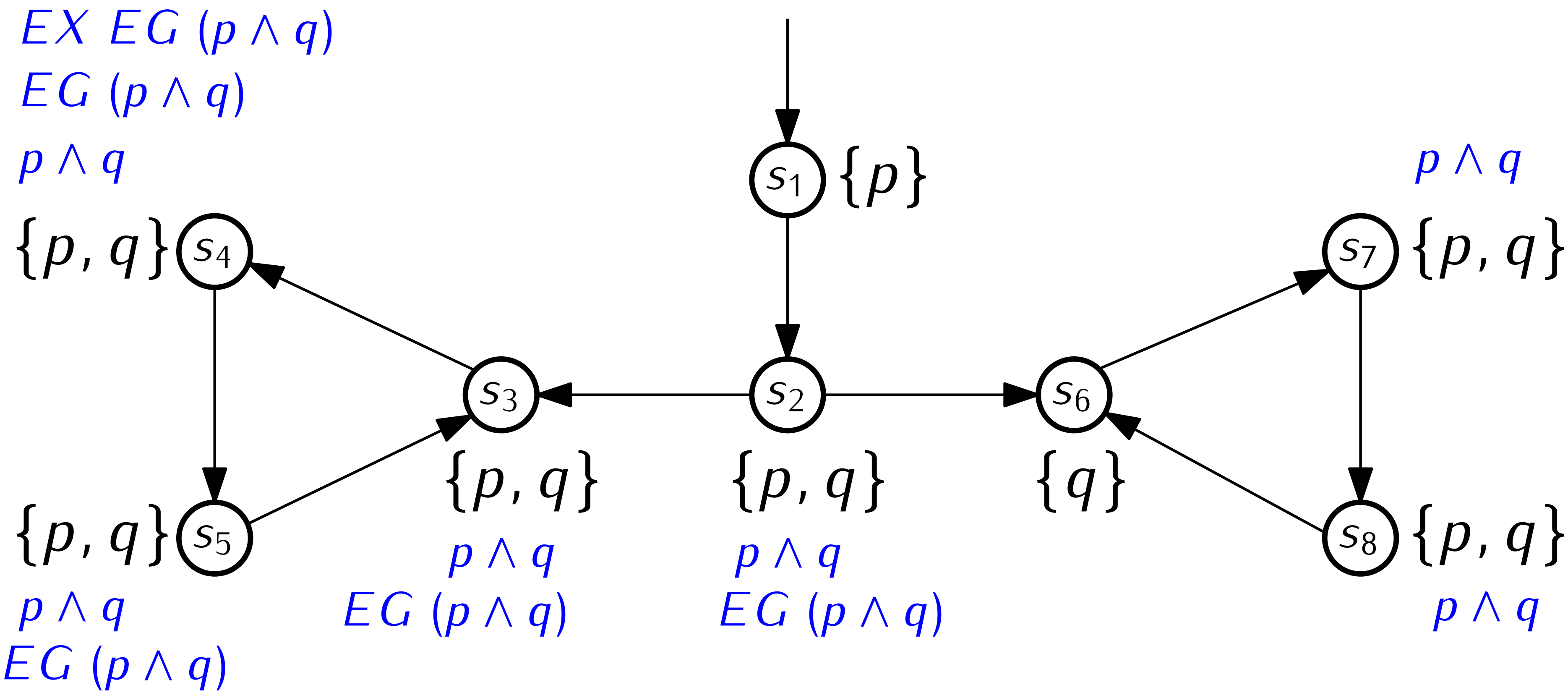
The General CTL Model Checking Algorithm

Example $EX\ \underline{EG\ (p \wedge q)}$



The General CTL Model Checking Algorithm

Example $EX\ EG\ (p \wedge q)$



The General CTL Model Checking Algorithm

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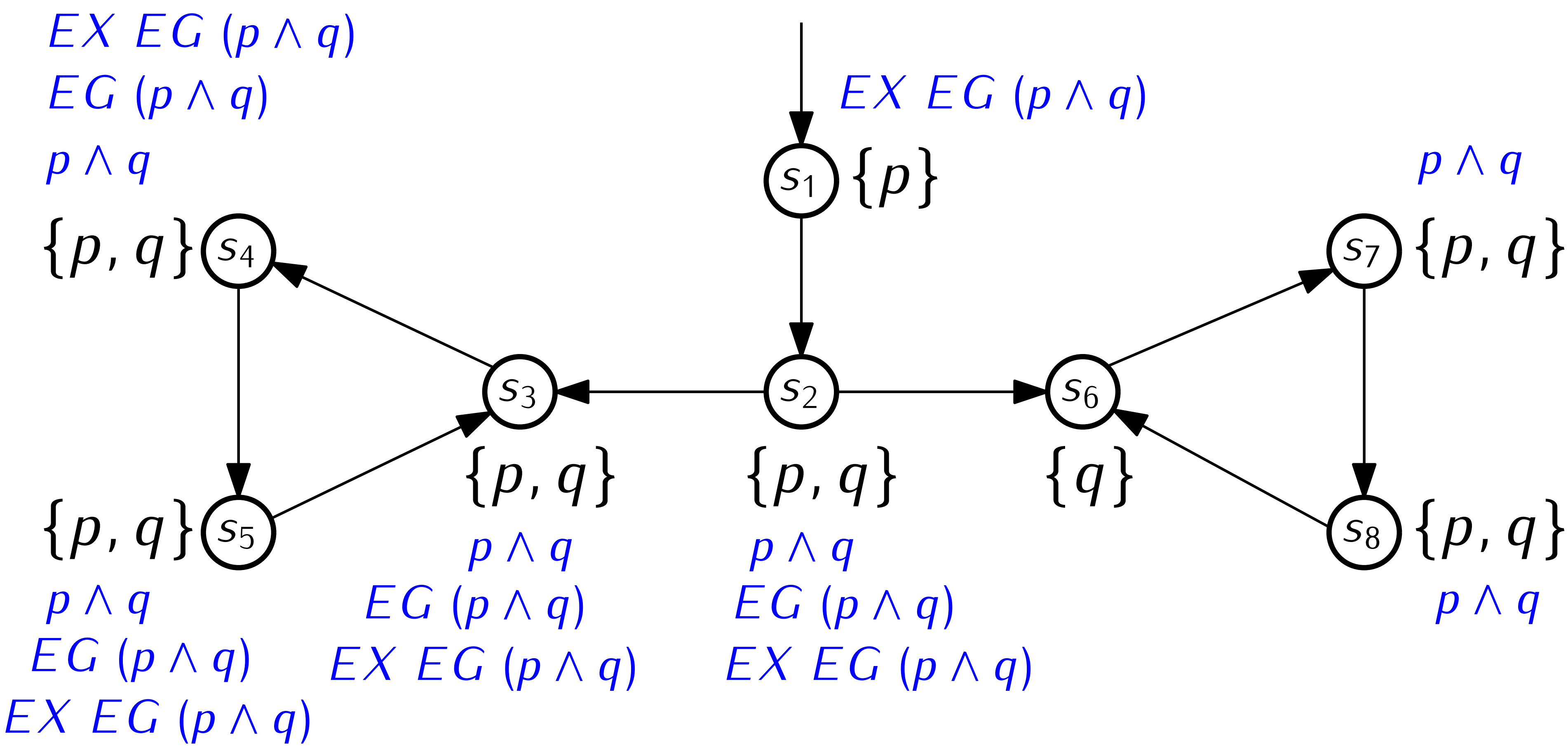
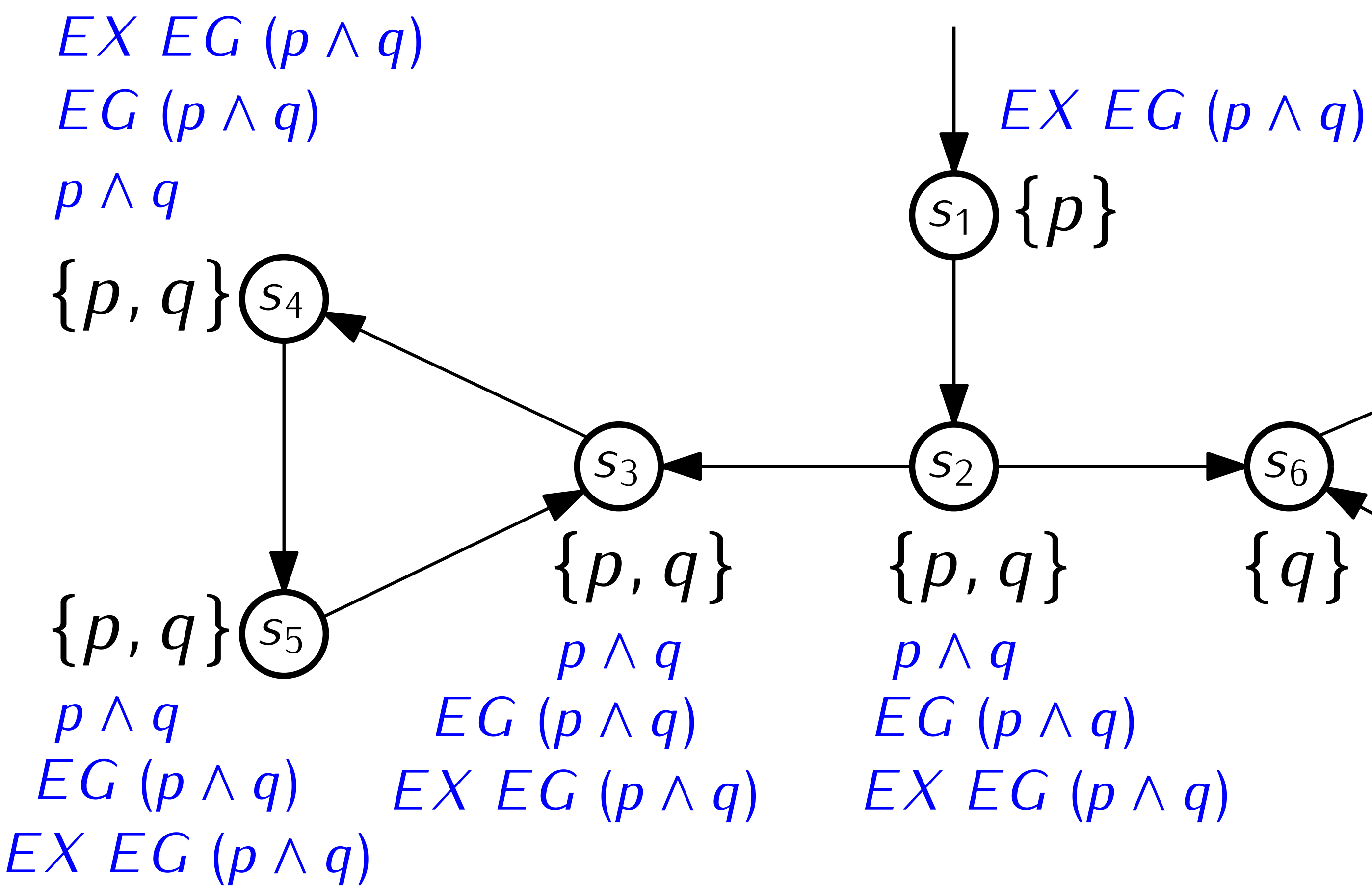


The General CTL Model Checking Algorithm

Example $EX\ EG\ (p \wedge q)$

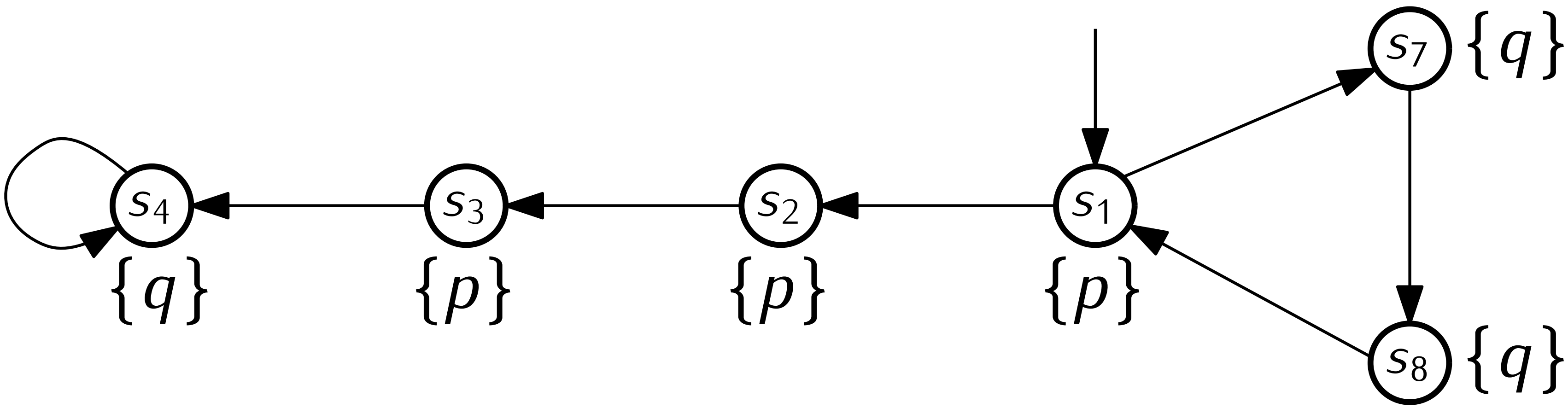


$$s_1, s_2, s_3, s_4, s_5, s_6 \models EX\ EG\ (p \wedge q) \ \wedge s_1 \in I$$

$$\Rightarrow \mathcal{M} \models EX\ EG\ (p \wedge q)$$

The General CTL Model Checking Algorithm

Example $p\ EU(EG\ q)$



The General CTL Model Checking Algorithm

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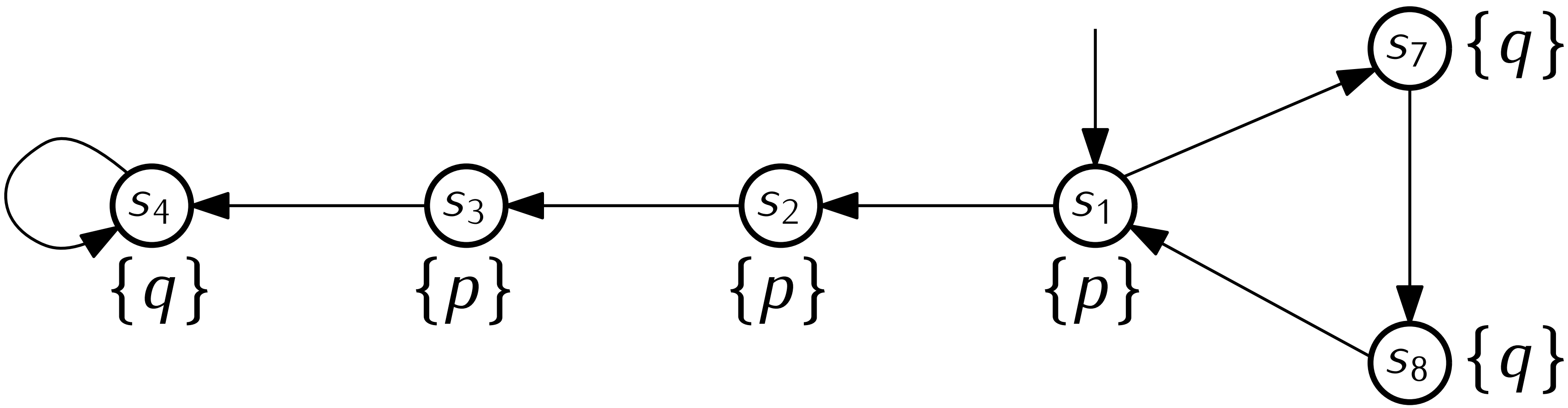
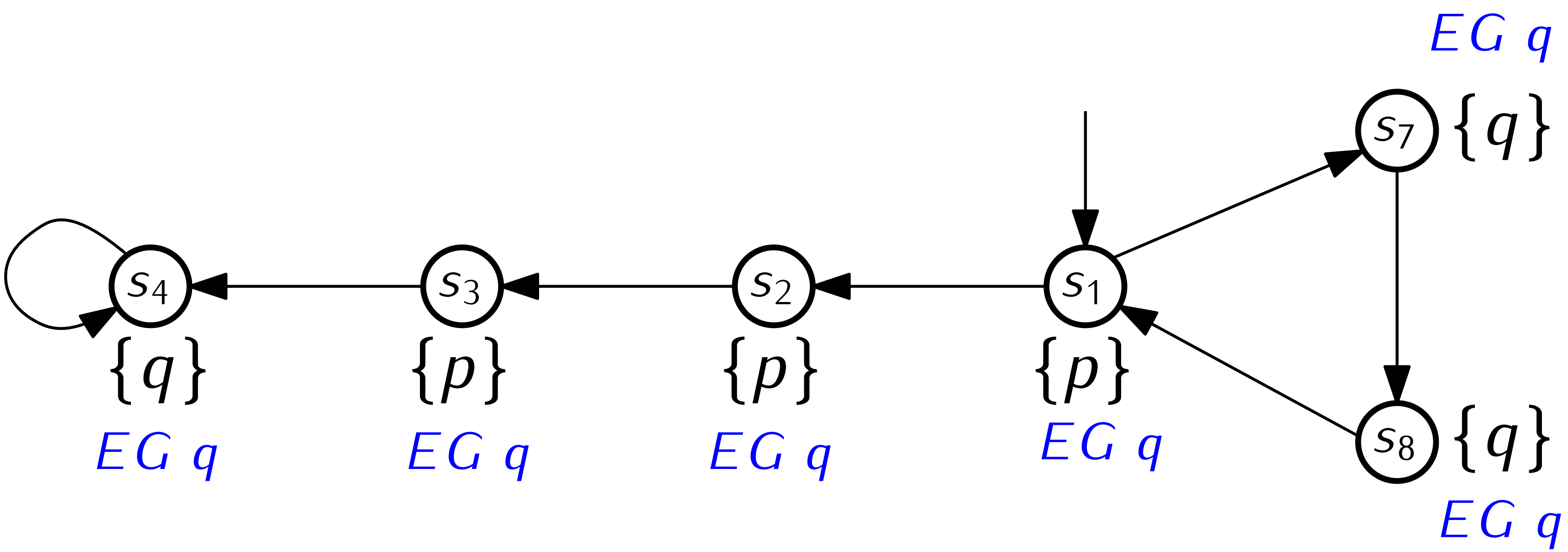


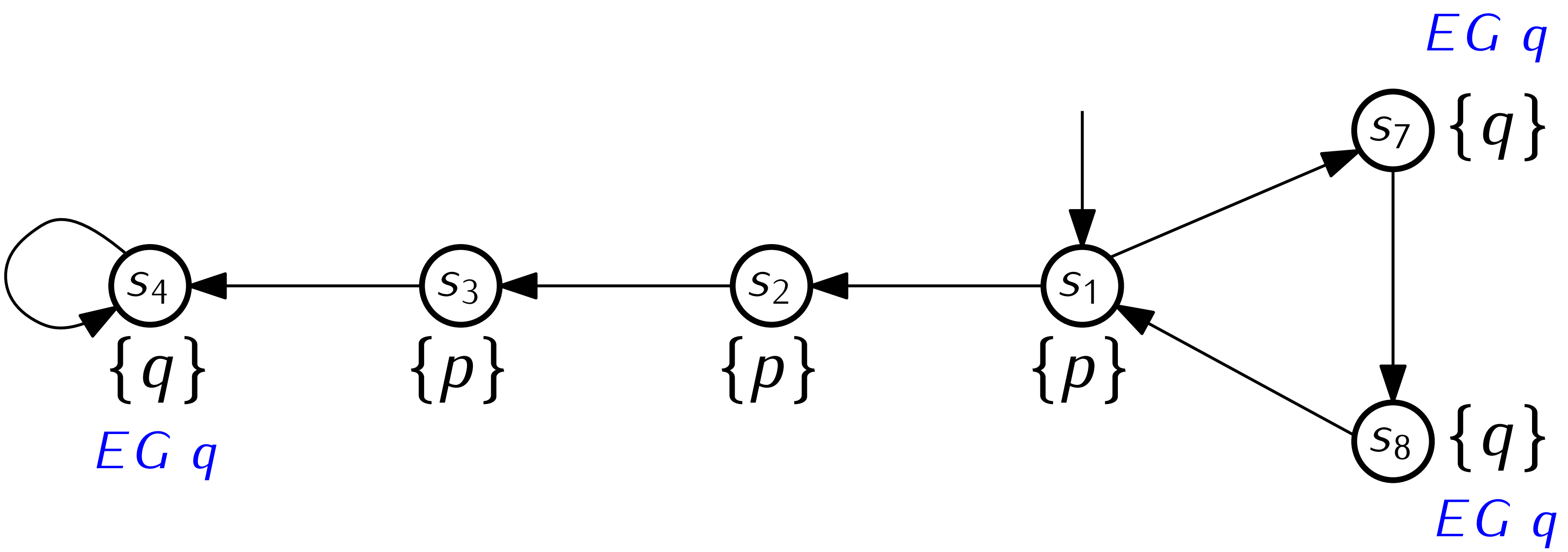
The General CTL Model Checking Algorithm

Example $p \; EU(EG \; q)$



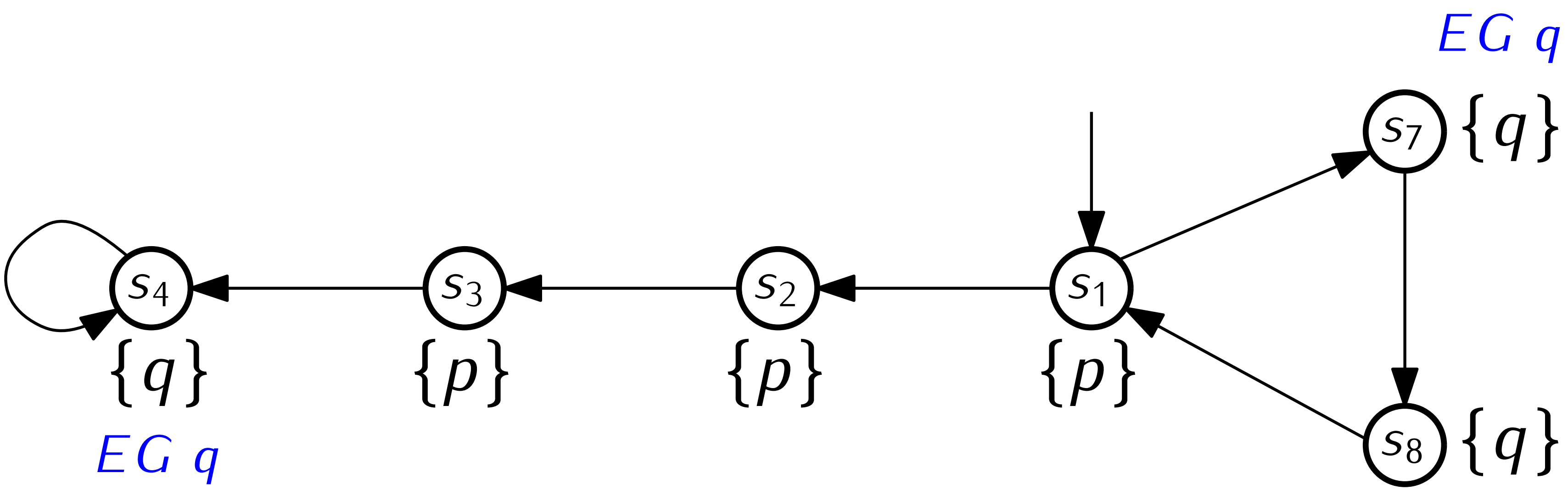
The General CTL Model Checking Algorithm

Example $p\ EU(EG\ q)$



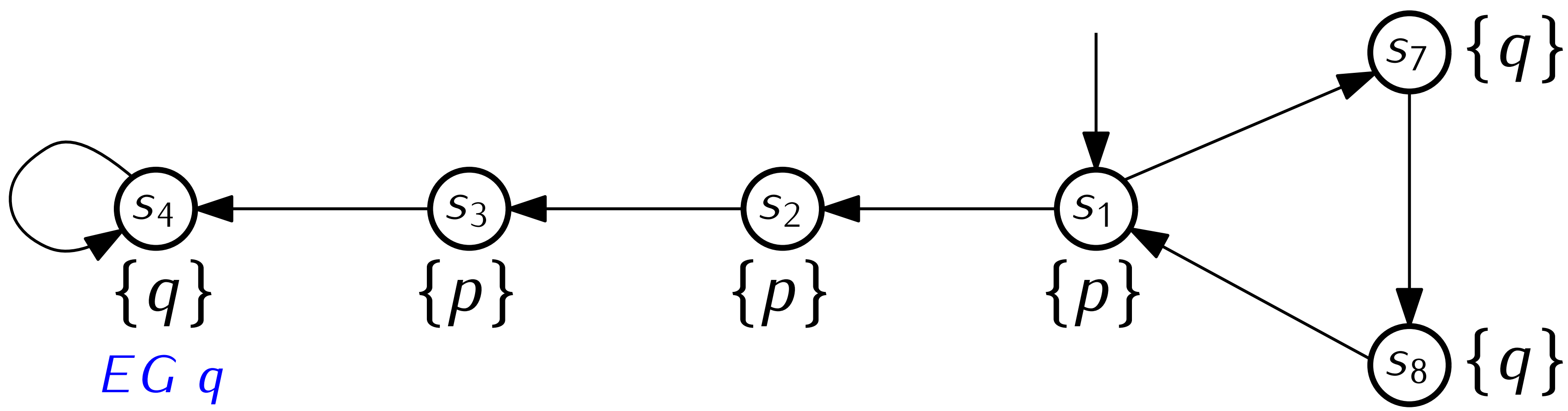
The General CTL Model Checking Algorithm

Example $p \; EU(EG \; q)$



The General CTL Model Checking Algorithm

Example $p \; EU(\underline{EG \; q})$



The General CTL Model Checking Algorithm

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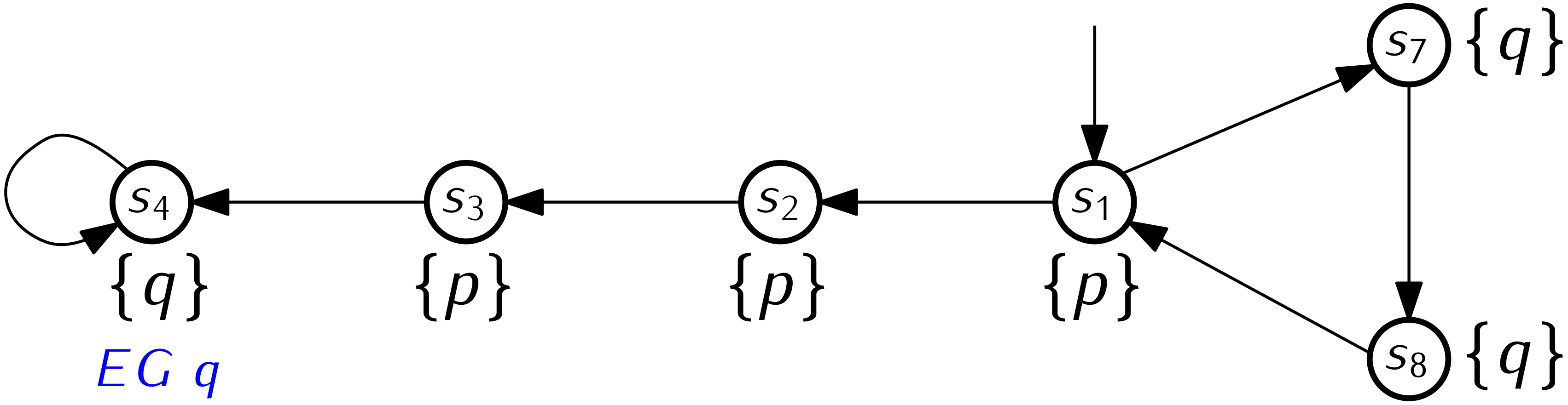

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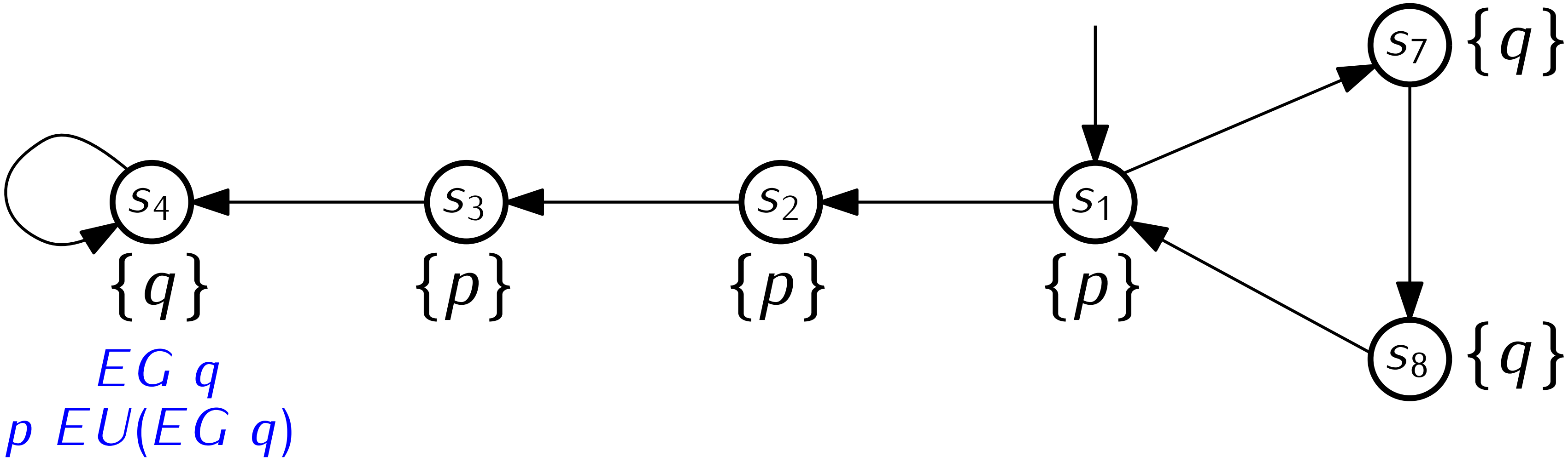
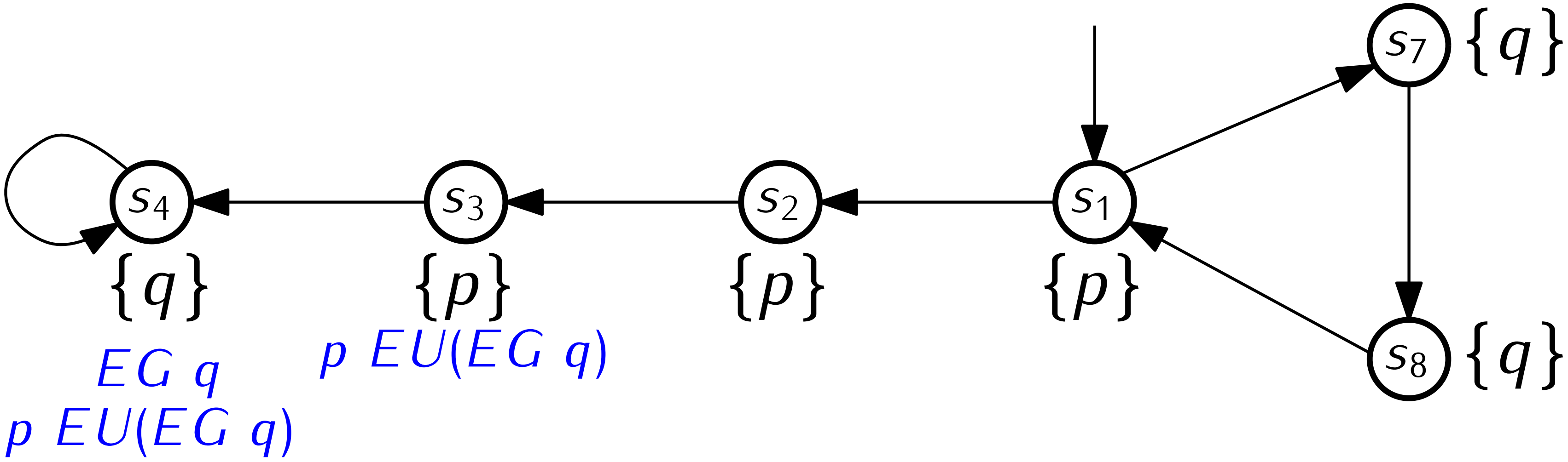


The General CTL Model Checking Algorithm

Example $p\ EU(EG\ q)$



The General CTL Model Checking Algorithm

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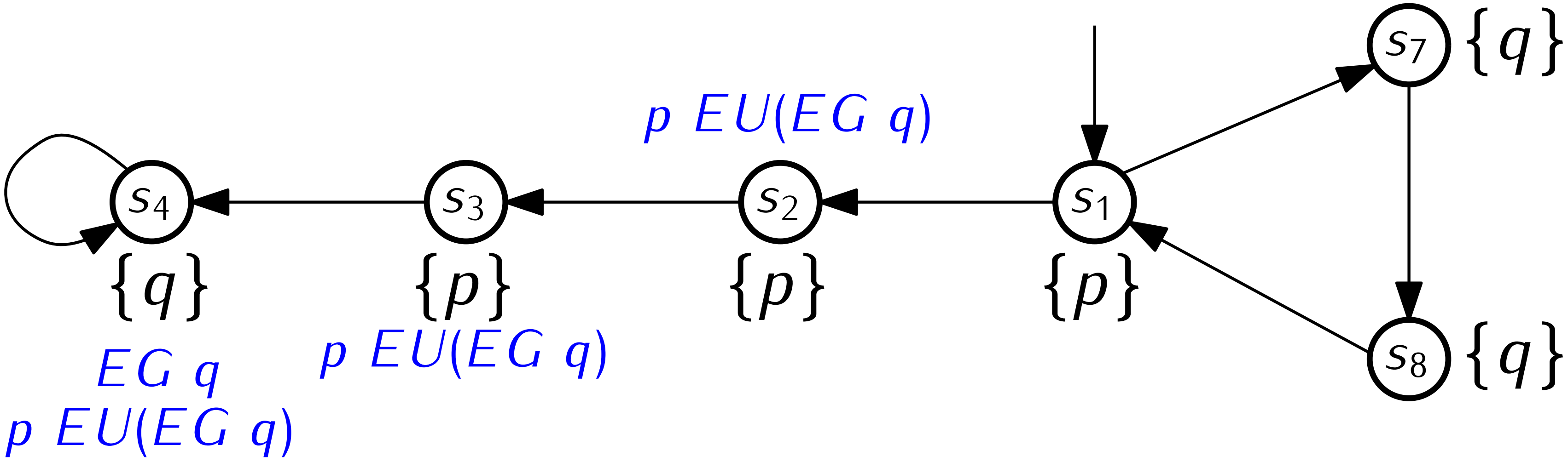

The General CTL Model Checking Algorithm

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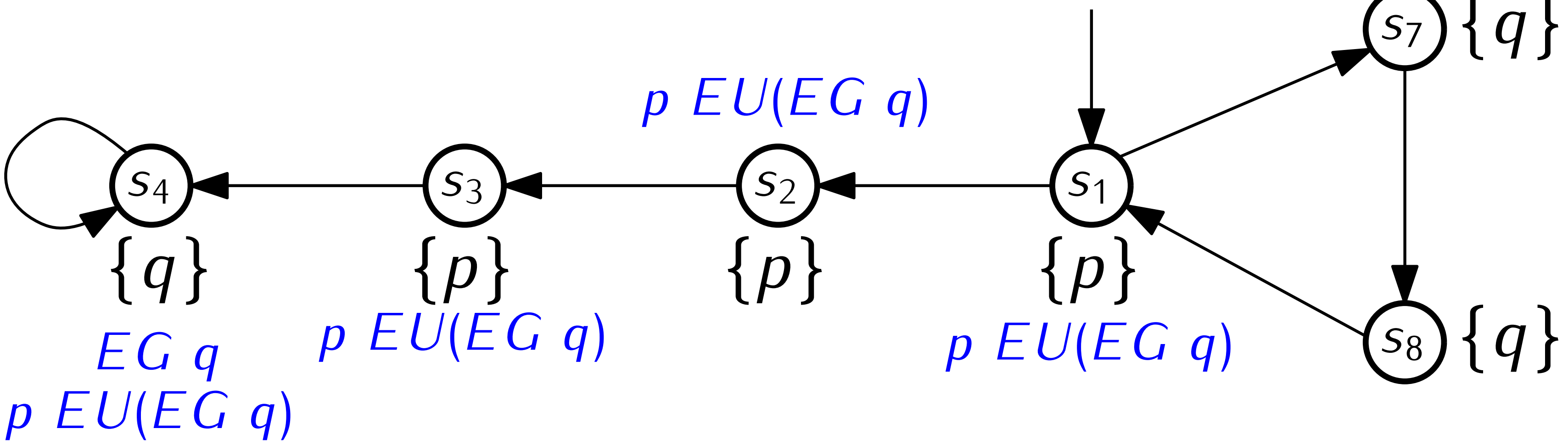
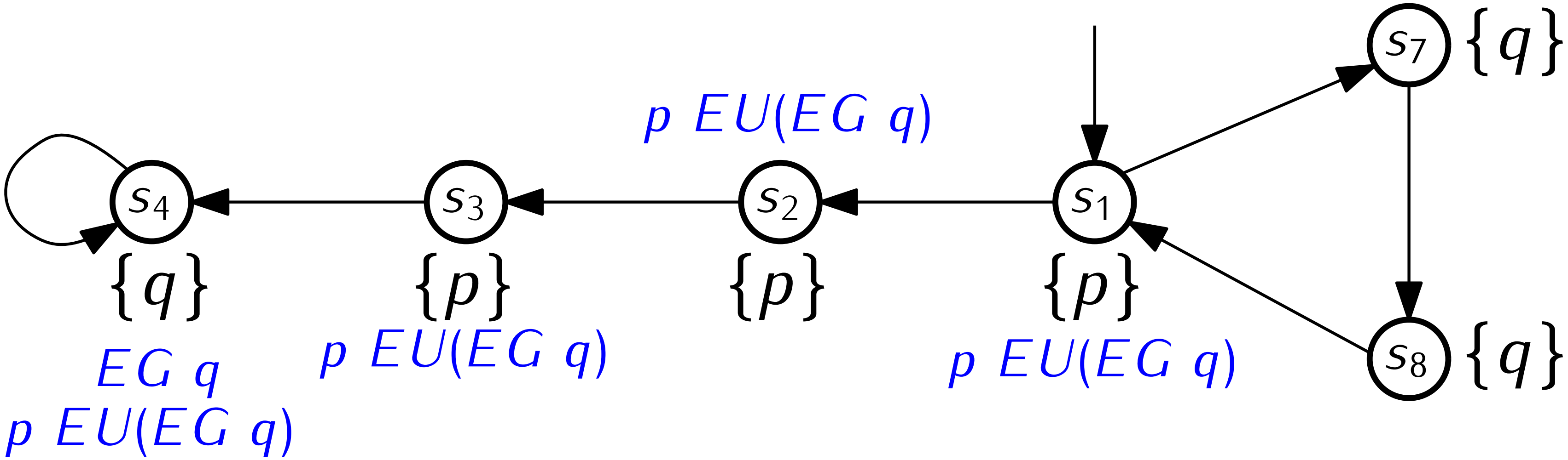


The General CTL Model Checking Algorithm

Example $p\ EU(EG\ q)$



$$s_1, s_2, s_3, s_4 \models p\ EU(EG\ q) \land s_1 \in I \Rightarrow \mathcal{M} \models p\ EU(EG\ q)$$

The General CTL Model Checking Algorithm

$\mathsf{SAT}(\phi) =$

The General CTL Model Checking Algorithm

$\mathsf{SAT}(\phi) =$

case

The General CTL Model Checking Algorithm

SAT(ϕ) =

case

ϕ is $\top$: return S

The General CTL Model Checking Algorithm

$\text{SAT}(\phi) =$

case

ϕ is $\top$: return S

ϕ is p_i : return $\{s : p \in L(S)\}$

The General CTL Model Checking Algorithm

$\mathsf{SAT}(\phi) =$

case

ϕ is $\top$: return S

ϕ is p_i : return $\{s : p \in L(S)\}$

ϕ is $\phi \wedge \psi$: return $\mathsf{SAT}(\phi) \cap \mathsf{SAT}(\psi)$

The General CTL Model Checking Algorithm

$\mathsf{SAT}(\phi) =$

case

ϕ is $\top$: return S

ϕ is p_i : return $\{s : p \in L(S)\}$

ϕ is $\phi \wedge \psi$: return $\mathsf{SAT}(\phi) \cap \mathsf{SAT}(\psi)$

ϕ is $\phi \wedge \psi$: return $\mathsf{SAT}(\phi) \cup \mathsf{SAT}(\psi)$

The General CTL Model Checking Algorithm

SAT(ϕ) $=$

case

ϕ is $\top$: return S

ϕ is p_i : return $\{s : p \in L(S)\}$

ϕ is $\phi \wedge \psi$: return SAT(ϕ)$\cap$ SAT(ψ)

ϕ is $\phi \wedge \psi$: return SAT(ϕ)$\cup$ SAT(ψ)

ϕ is $\neg\phi$: return S – SAT(ϕ)

The General CTL Model Checking Algorithm

SAT(ϕ) $=$

case

ϕ is $\top$: return S

ϕ is p_i : return $\{s : p \in L(S)\}$

ϕ is $\phi \wedge \psi$: return SAT(ϕ)$\cap$ SAT(ψ)

ϕ is $\phi \wedge \psi$: return SAT(ϕ)$\cup$ SAT(ψ)

ϕ is $\neg\phi$: return S – SAT(ϕ)

ϕ is $EX\phi$: return SAT$_{EX}$(ϕ)

The General CTL Model Checking Algorithm

$\mathsf{SAT}(\phi) =$

case

ϕ is $\top$: return S

ϕ is p_i : return $\{s : p \in L(S)\}$

ϕ is $\phi \wedge \psi$: return $\mathsf{SAT}(\phi) \cap \mathsf{SAT}(\psi)$

ϕ is $\phi \wedge \psi$: return $\mathsf{SAT}(\phi) \cup \mathsf{SAT}(\psi)$

ϕ is $\neg\phi$: return $S - \mathsf{SAT}(\phi)$

ϕ is $EX\phi$: return $\mathsf{SAT}_{EX}(\phi)$

ϕ is $EU\phi$: return $\mathsf{SAT}_{EU}(\phi)$

The General CTL Model Checking Algorithm

$\text{SAT}(\phi) =$

case

ϕ is $\top$: return S

ϕ is p_i : return $\{s : p \in L(S)\}$

ϕ is $\phi \wedge \psi$: return $\text{SAT}(\phi) \cap \text{SAT}(\psi)$

ϕ is $\phi \wedge \psi$: return $\text{SAT}(\phi) \cup \text{SAT}(\psi)$

ϕ is $\neg\phi$: return $S - \text{SAT}(\phi)$

ϕ is $EX\phi$: return $\text{SAT}_{EX}(\phi)$

ϕ is $EU\phi$: return $\text{SAT}_{EU}(\phi)$

ϕ is $EG\phi$: return $\text{SAT}_{EG}(\phi)$

esac