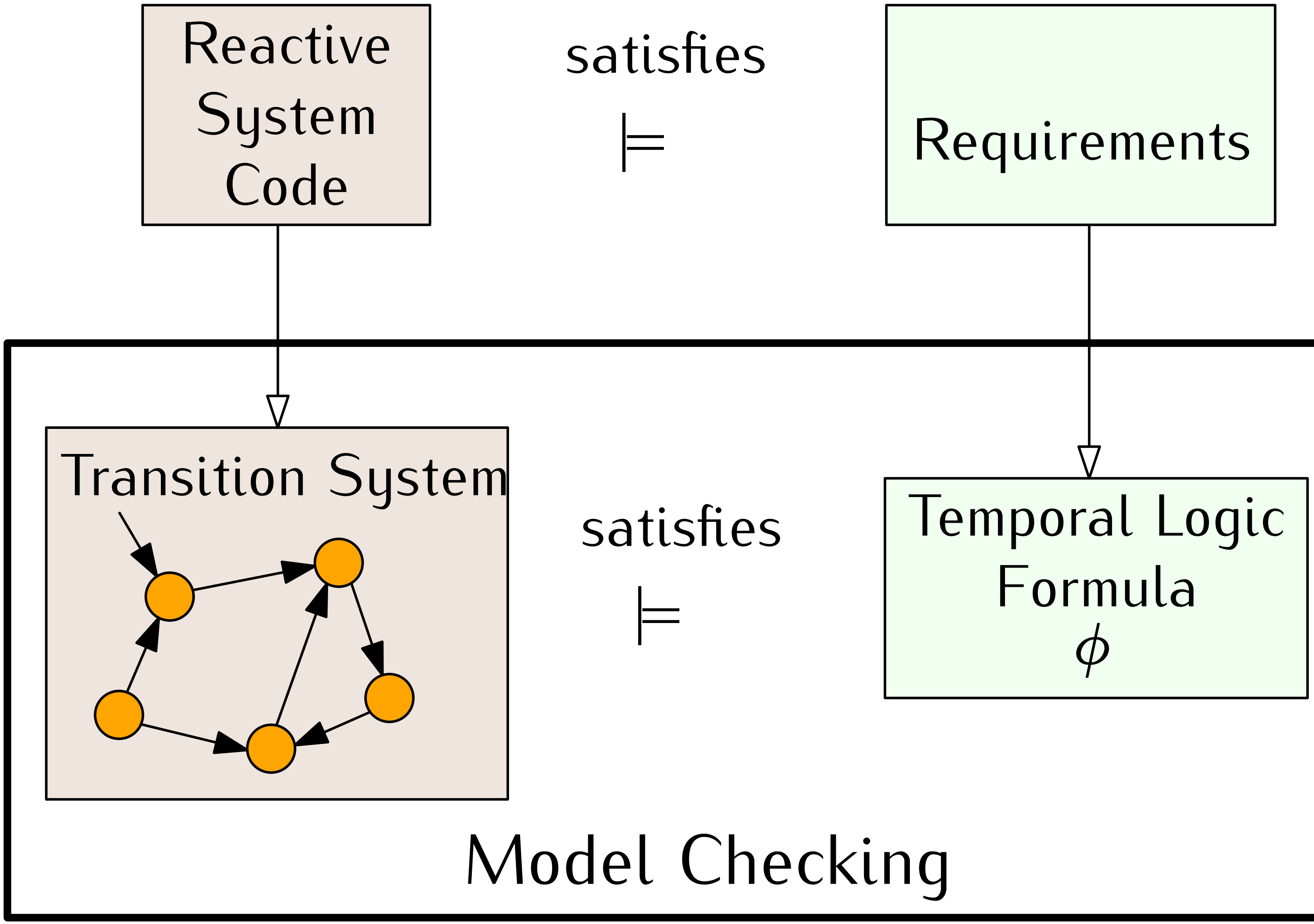
CS 181u Applied Logic

Lecture 13

LTL Model Checking

The Big Picture



LTL Model Checking

$$\mathcal{M} \models \phi \Leftrightarrow \forall \pi \ [\pi \models \phi]$$

LTL Model Checking

The goal of LTL Model Checking: given a transition system $\mathcal{M}$ and an LTL property ϕ,

1. determine if $\mathcal{M} \models \phi$, and
2. if $\mathcal{M} \not\models \phi$, then give a counterexample execution path from $\mathcal{M}$.

LTL Model Checking Algorithm

$$\mathcal{M} \models \phi \Leftrightarrow \forall \pi \ [\pi \models \phi]$$

LTL Model Checking

LTL Model Checking Algorithm Overview

1. Construct a Büchi automaton for $\neg\phi$, $A_{\neg\phi}$.
2. Construct a Büchi automaton for $\mathcal{M}$, $A_{\mathcal{M}}$.
3. Compute the automaton product $A_{\neg\phi} \times A_{\mathcal{M}}$.
4. Check if $A_{\neg\phi} \times A_{\mathcal{M}}$ has an accepting path.
   (a) No accepting path $\quad \Rightarrow \quad \mathcal{M} \models \phi$.
   (b) An accepting path corresponds to a counterexample execution.

Büchi Automata

A **Büchi Automaton**, $\mathcal{A}$, is a tuple
$\mathcal{A} = (\Sigma, S, \rightarrow, I, F)$, where

- Σ is an alphabet of transition symbols,
- S is a set of states,
- $\rightarrow$ is a transition relation,
- I is a set of initial states, and
- F is a set of accepting (a.k.a Final) states.

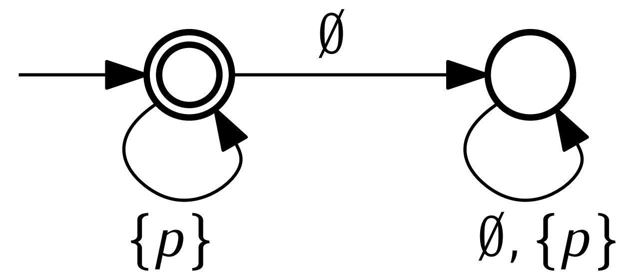
A Büchi Automaton, $\mathcal{A}$, accepts languages of *infinite words*. That is, for a Büchi automaton, $\mathcal{A}$, $\mathcal{L}(\mathcal{A}) \subseteq \Sigma^\omega$.

Acceptance condition: a Büchi automaton, $\mathcal{A}$, accepts an infinite word u if **there exists** an execution path of $\mathcal{A}$ when run on u that visits the set of states of F infinitely often.

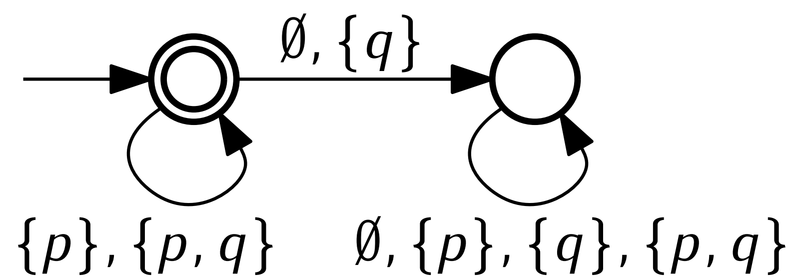
LTL and Büchi Automata

Property: Any LTL formula for atomic propositions AP has a Büchi automaton with alphabet $\Sigma = \mathcal{P}(AP)$.

<u>Example 1</u>: $\phi = G\ p,\ \ AP = \{p\}$



<u>Example 2</u>: $\phi = G\ p,\ \ AP = \{p, q\}$



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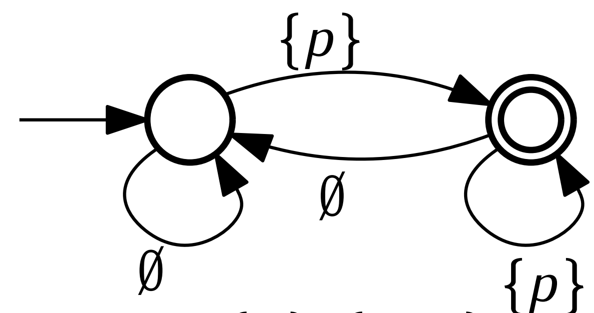

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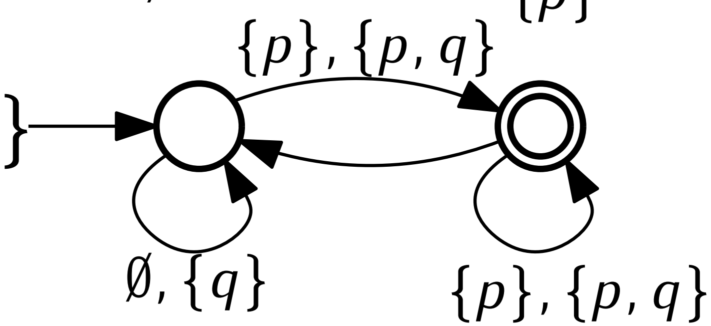
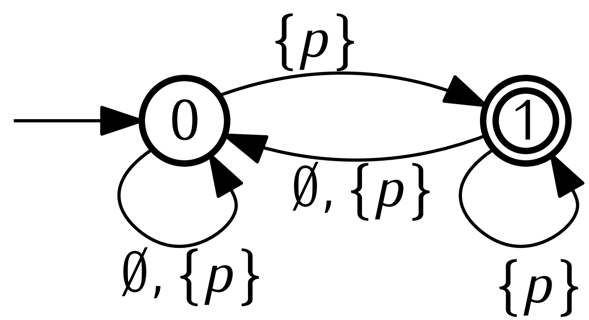


Non-deterministic Büchi Automata

A **non-deterministic Büchi automaton** allows multiple outgoing transitions with the same label. Acceptance condition is the same as before.

Example 3’: $\phi = GF\ p$, $AP = \{p\}$



Non-determinism: for example, from state 0, this automaton can either stay at state 0 or go to state 1 on transition labeled $\{p\}$.

For any sequence of sets of propositions that always has a p in the future, there is a correponding execution path in $\mathcal{A}_{GFp}$ that visits state 1 infinitely often.

1. LTL to BA conversion

There is an algorithm that constructs a Büchi automaton (BA) from any LTL formula. However, we will not give it in this lecture.

There is an online tool for converting LTL to BA:

`http://www.lsv.fr/~gastin/ltl2ba/index.php`

2. Transition System to Büchi Automata

Given a transition system $\mathcal{M} = (S, \rightarrow, I)$ and labelling function $L : S \rightarrow AP$, we can construct a Büchi automaton $\mathcal{A}_{\mathcal{M}} = (\Sigma, S', \Rightarrow, I', F)$ where

- $\Sigma = \mathcal{P}(AP)$
- $S' = S \cup \{i\}$ (new initial state i)
- $\rightarrow$ is a transition relation,
- $I' = \{i\}$ (state i is the only initial state),
- $F = S'$ (all states are accepting states), and
- the new transition relation, $\Rightarrow$ is as defined in the following slides.

2. Transition System to Büchi Automata

Convert the transition relation, $\rightarrow$, in system $\mathcal{M}$ to the transition relation, $\Rightarrow$, in the Büchi automaton, $\mathcal{A}_{\mathcal{M}}$.

Informally: add a new init state, move the labels from any state to the incoming transitions for that state. We will illustrate two simple rules to do this:

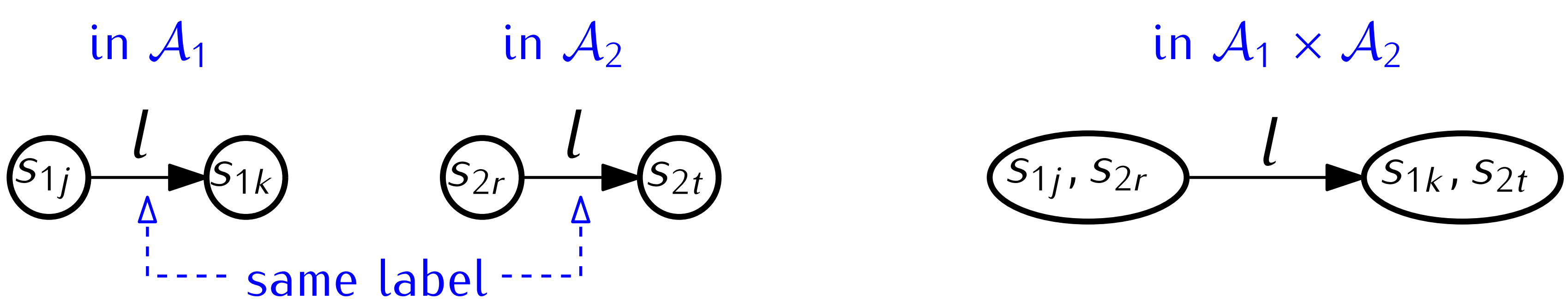
	In $\mathcal{M}$	In $\mathcal{A}_{\mathcal{M}}$
non-initial states	$s_j \longrightarrow \overset{l}{s_k}$	$s_j \xRightarrow{\;l\;} s_k$
initial states	$\longrightarrow \overset{l}{s_k}$	$i \xRightarrow{\;l\;} s_k$

3. Büchi Automata Product

Given Büchi automata
$\mathcal{A}_1 = (\Sigma, S_1, \rightarrow_1, I_1, F_1)$
$\mathcal{A}_2 = (\Sigma, S_2, \rightarrow_2, I_2, F_2)$

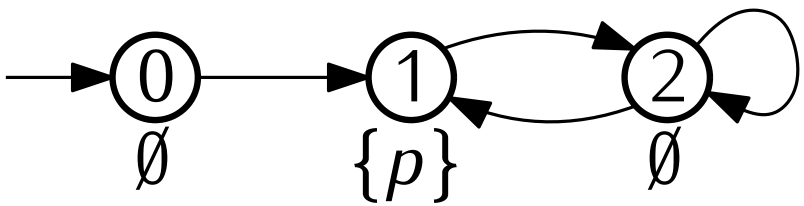
$\mathcal{A}_1 \times \mathcal{A}_2 = (\Sigma, S, \rightarrow, I, F)$ where

- $S = S_1 \times S_2$
- $I = I_1 \times I_2$
- $F = \{(f_1, f_2) : f_1 \in F_1 \wedge f_2 \in F_2\}$,
- $\rightarrow$ is defined for two states and a label l when $\mathcal{A}_1$ and $\mathcal{A}_2$ agree on l. To illustrate:



A Complete Example

Consider this transition system, $\mathcal{M}$, with $AP = \{p\}$



We want to check the property $\phi = F\ G\ \neg p$

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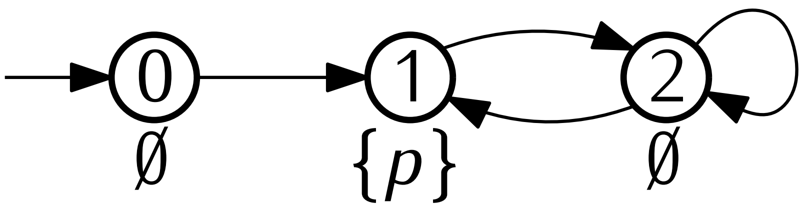
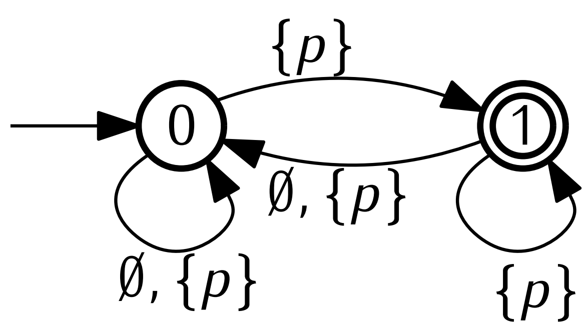


We want to check the property $\phi = F\ G\ \neg p$

Step 1: Construct $A_{\neg\phi}$. $\neg\phi = \neg F\ G\ \neg p = G\ F\ p$
This is the same NBA we saw earlier in the slides (Example 3'):



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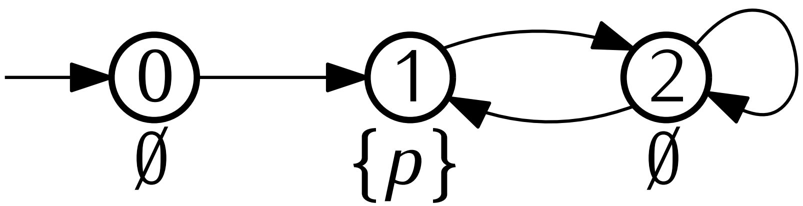

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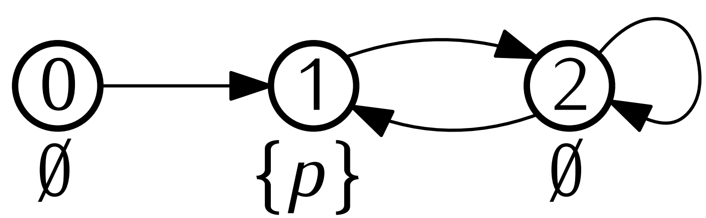
Step 2: Construct $A_{\mathcal{M}}$.



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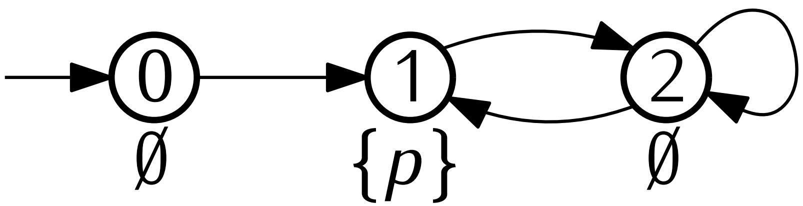

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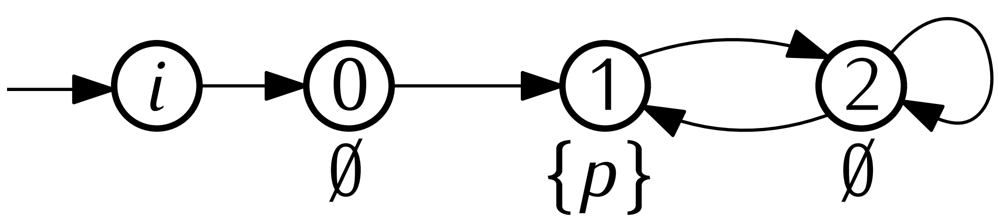

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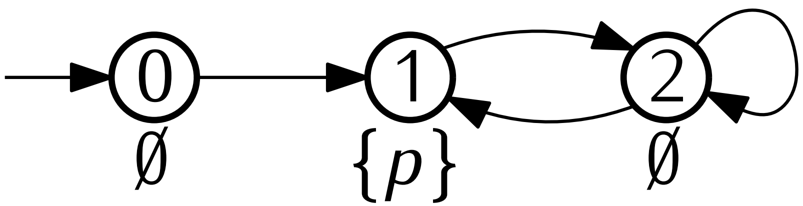

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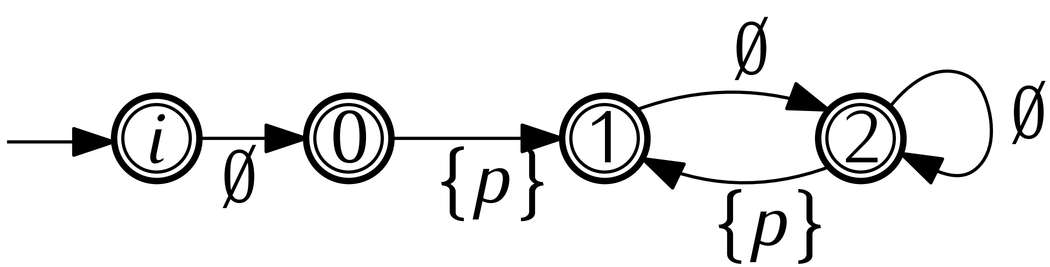
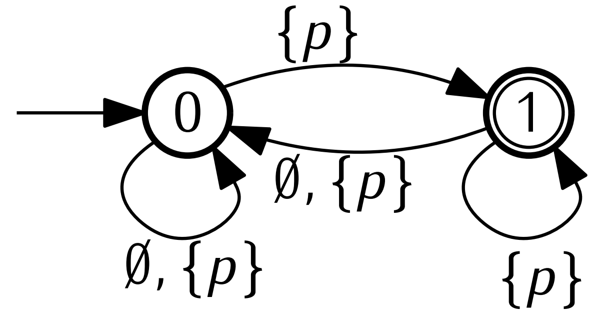
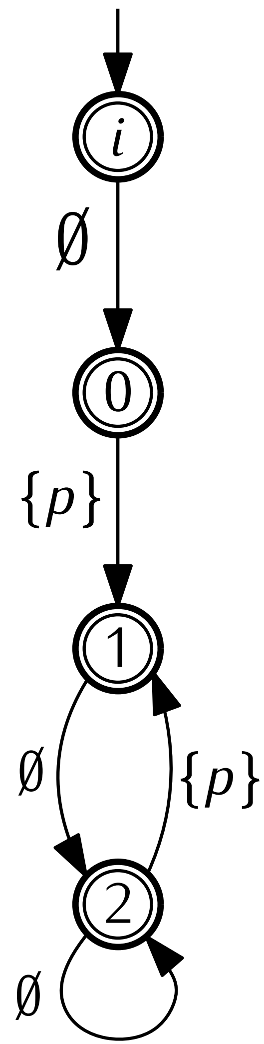


A Complete Example

Step 3: Construct $A_{\mathcal{M}} \times A_{\neg\phi}$.





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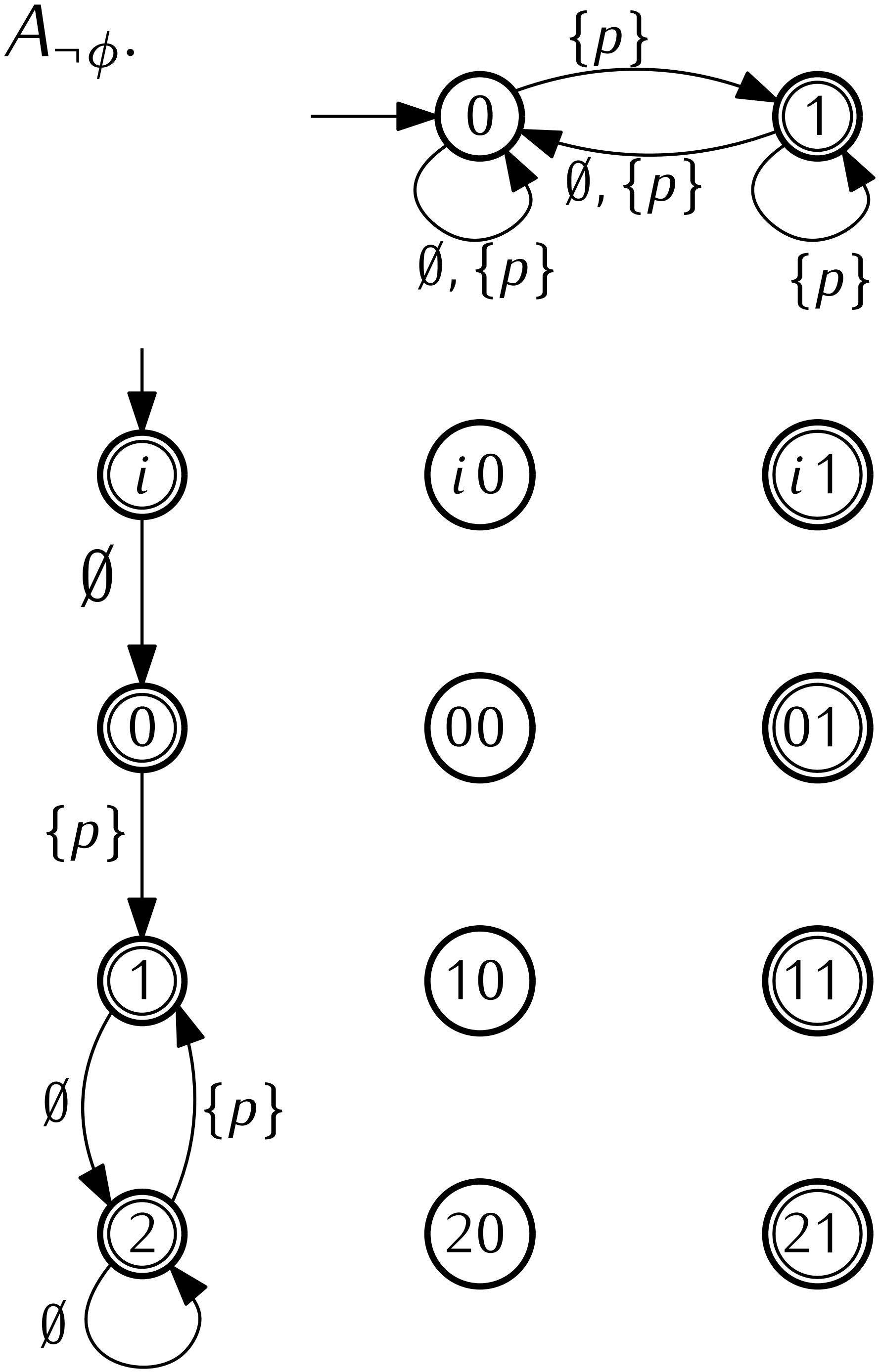

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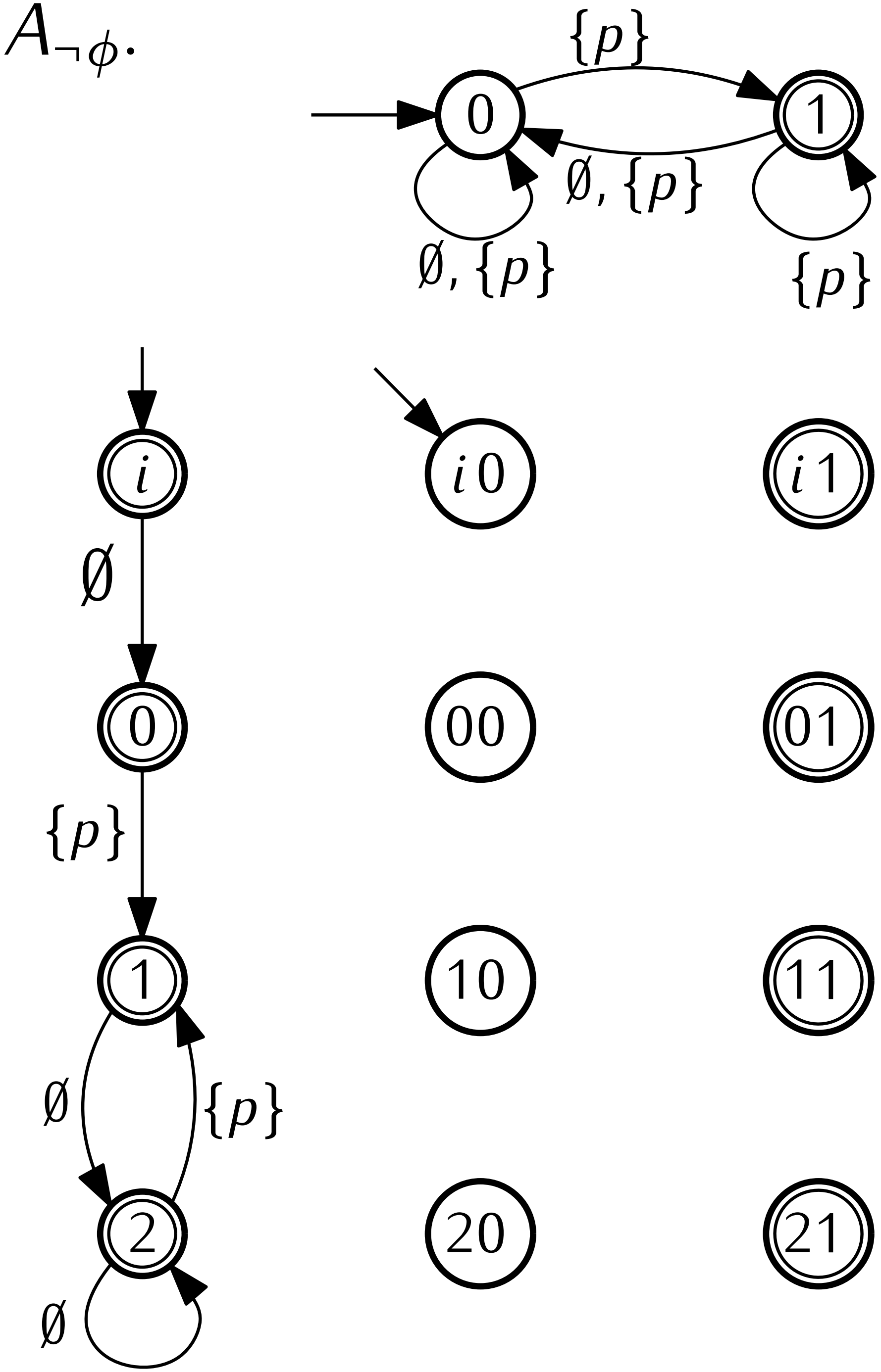

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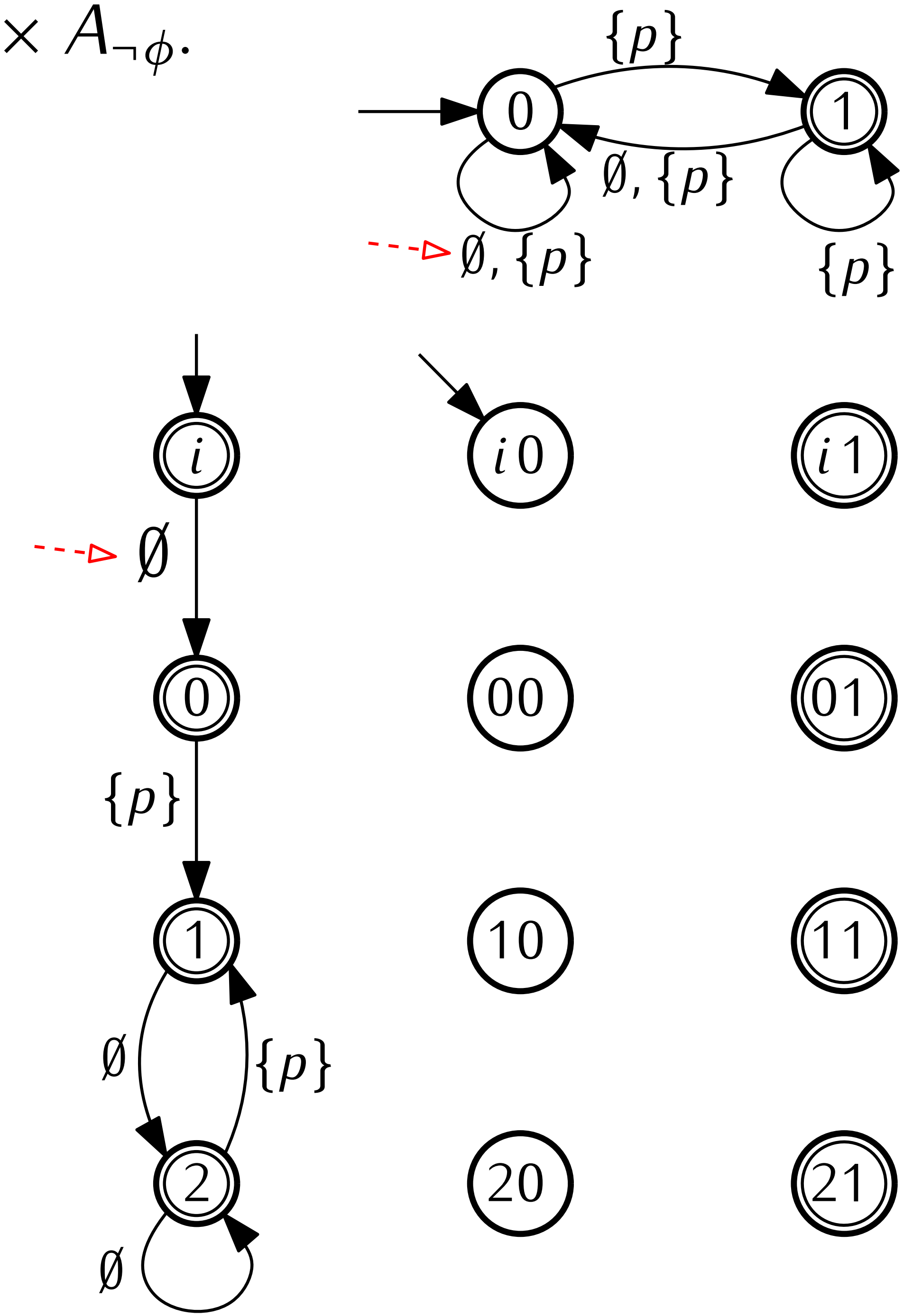

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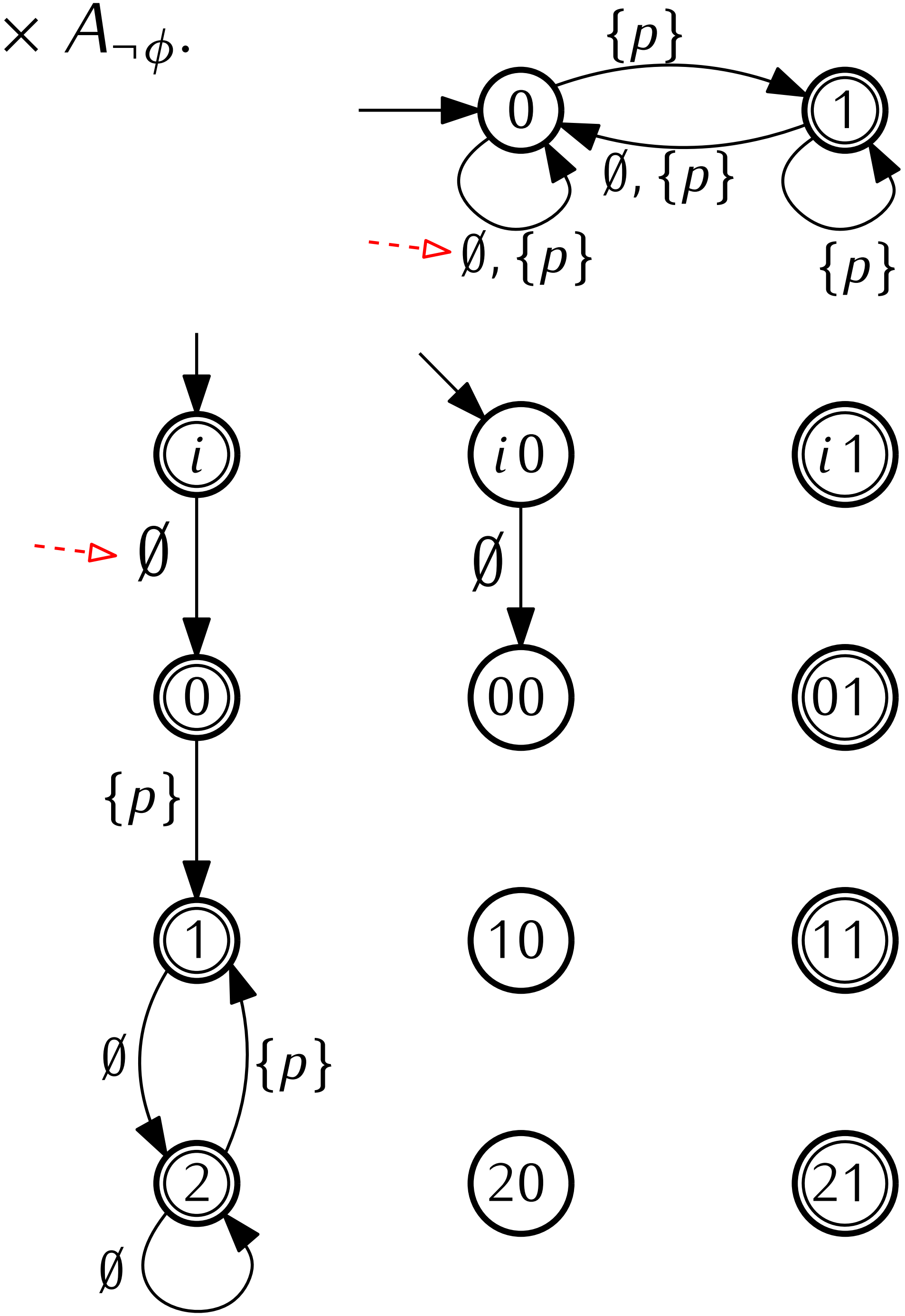

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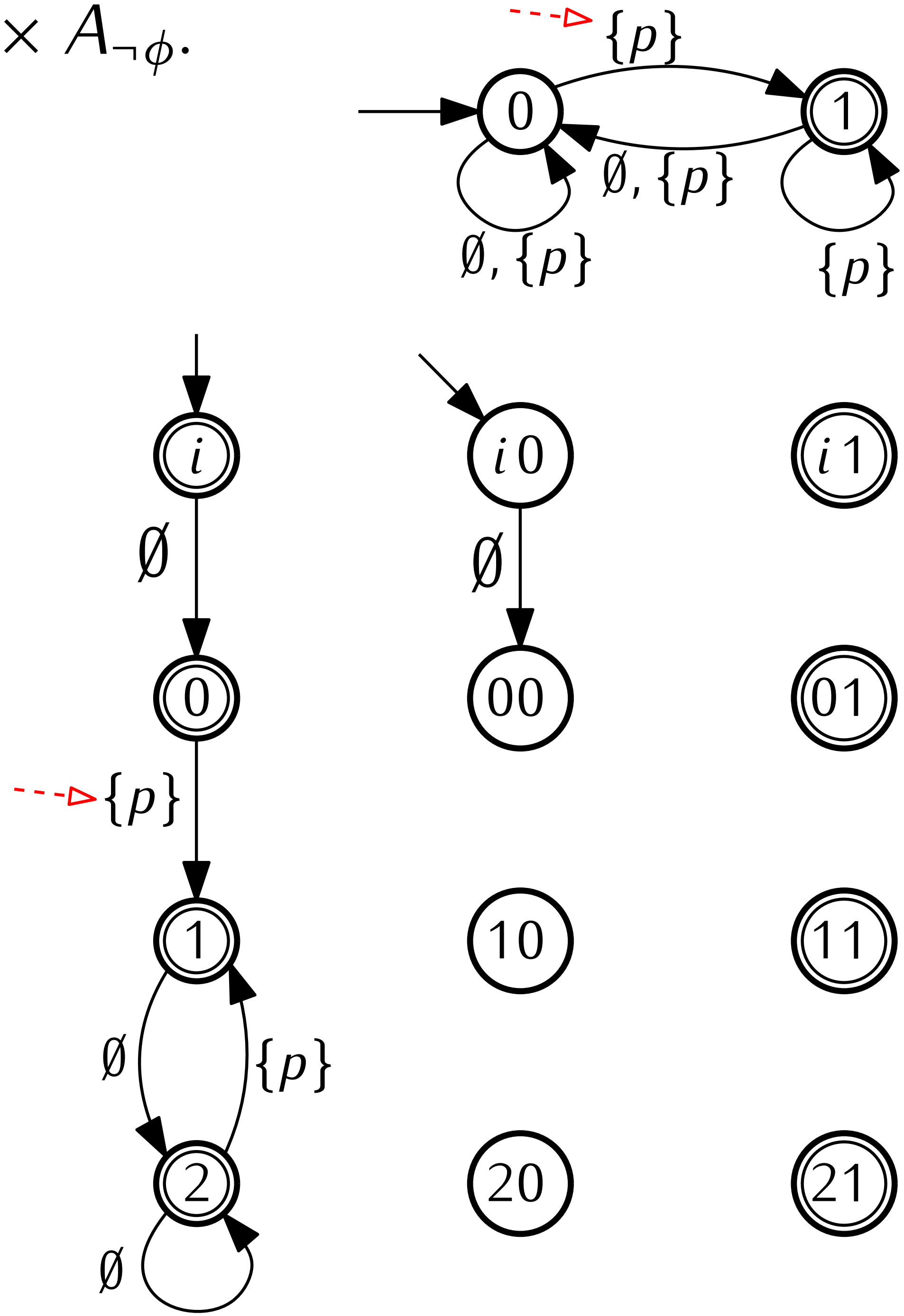

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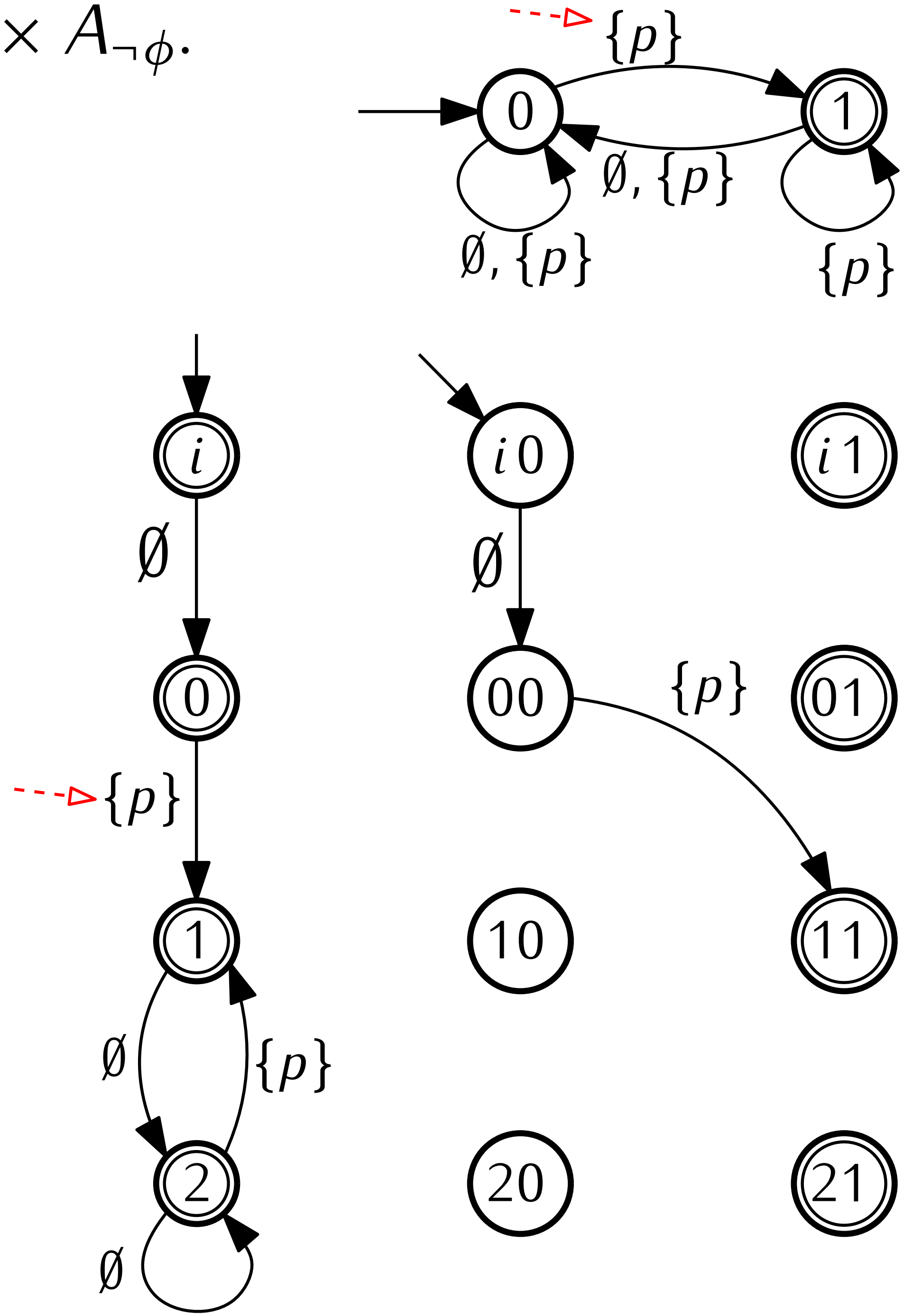

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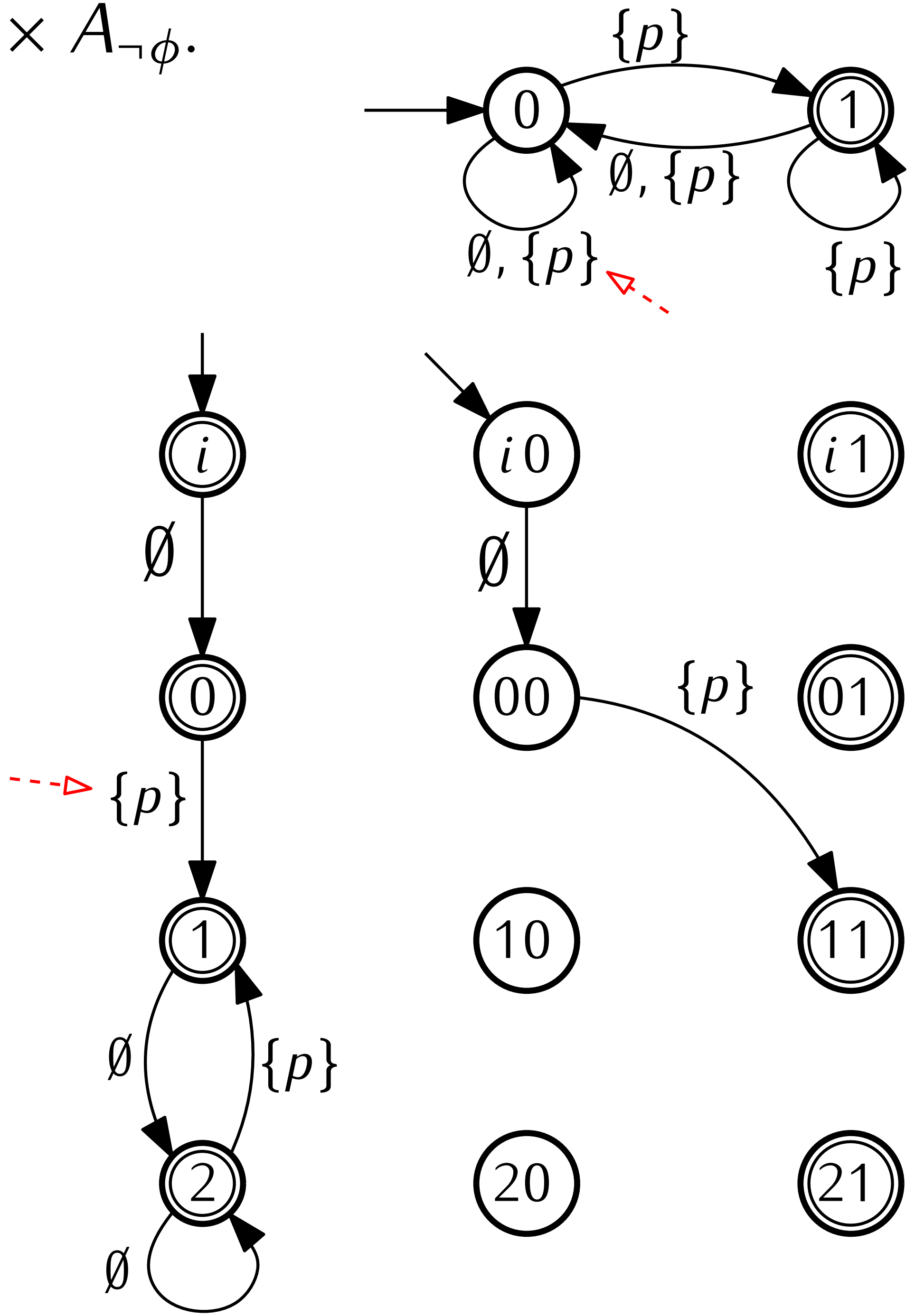

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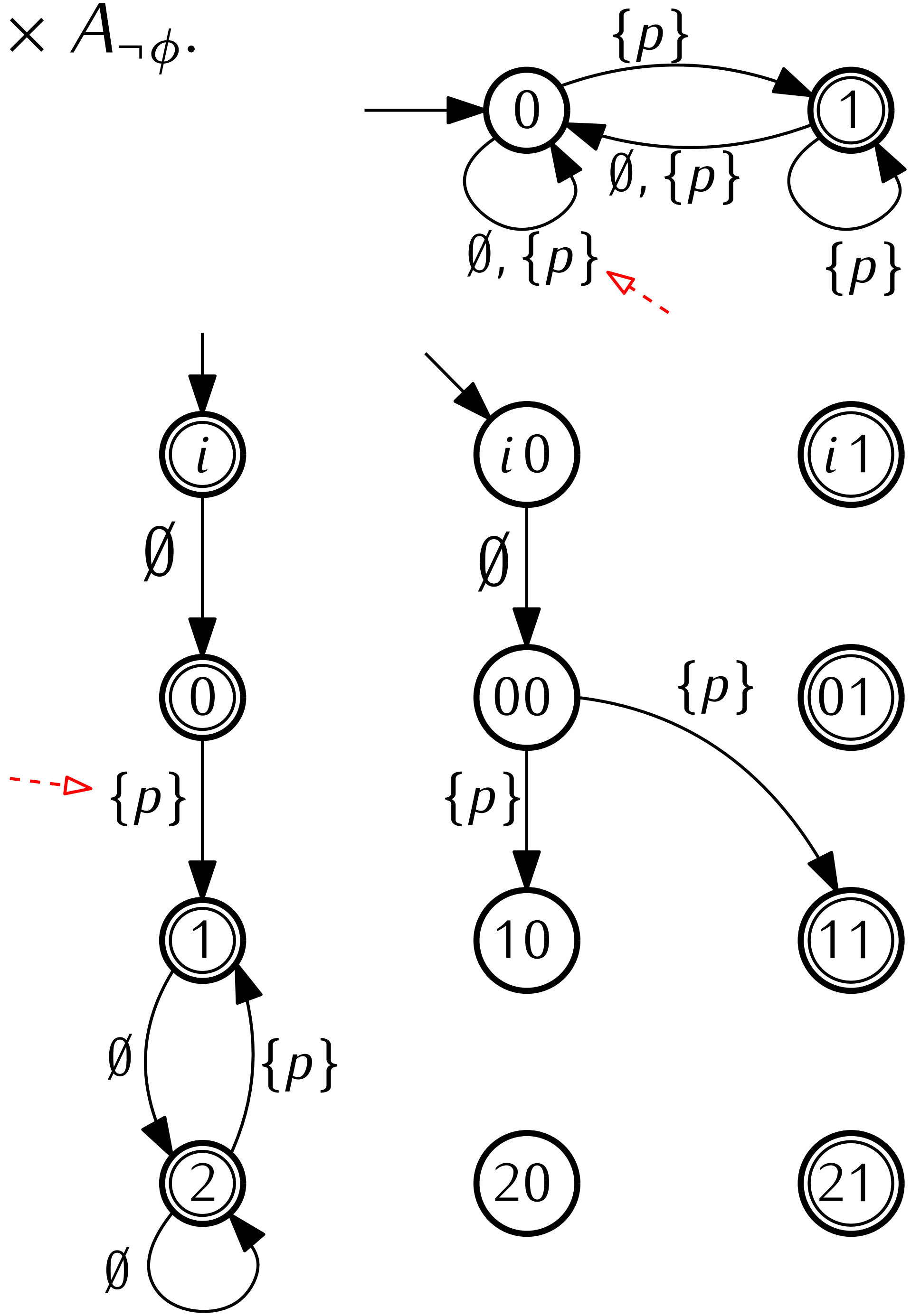

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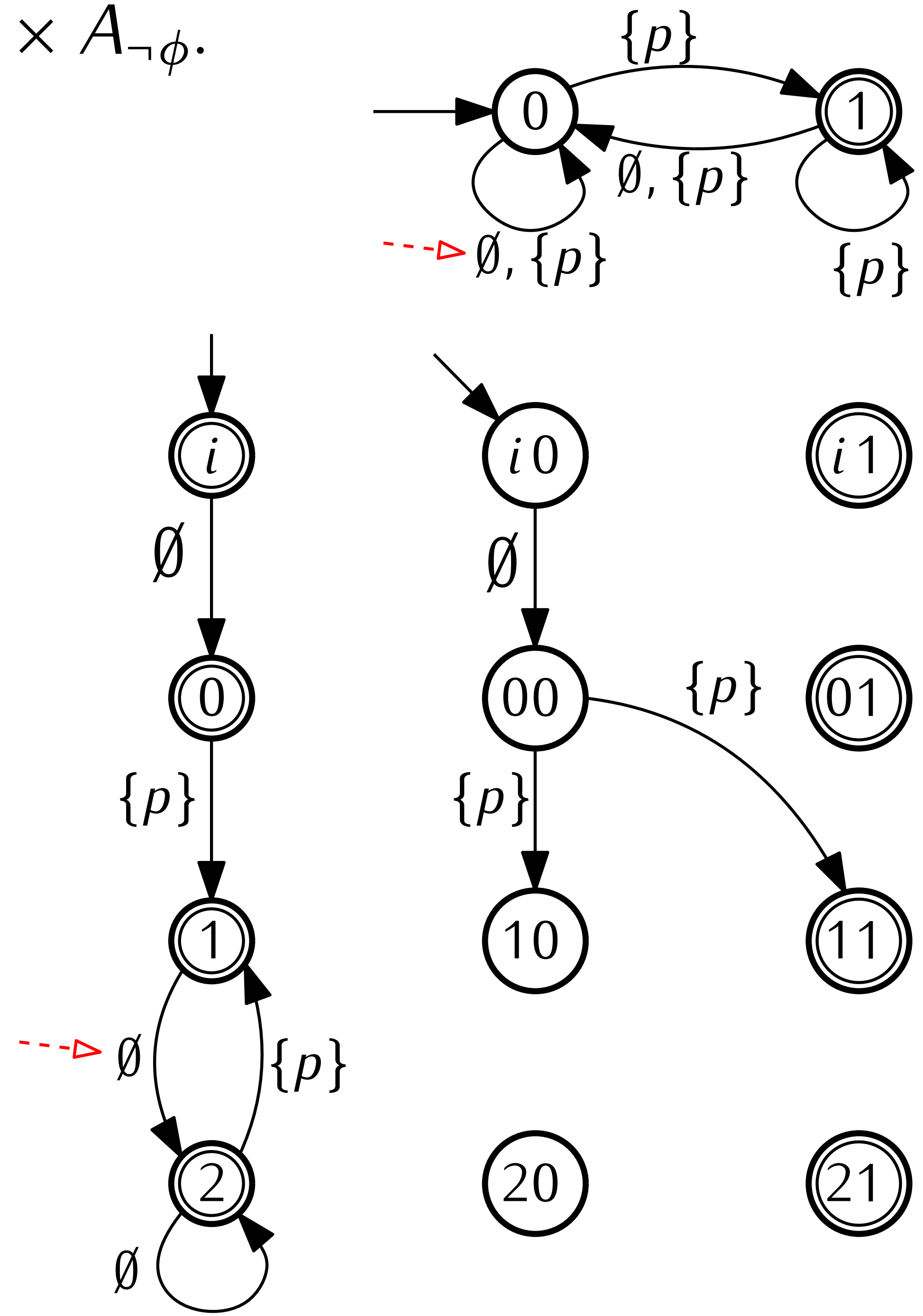

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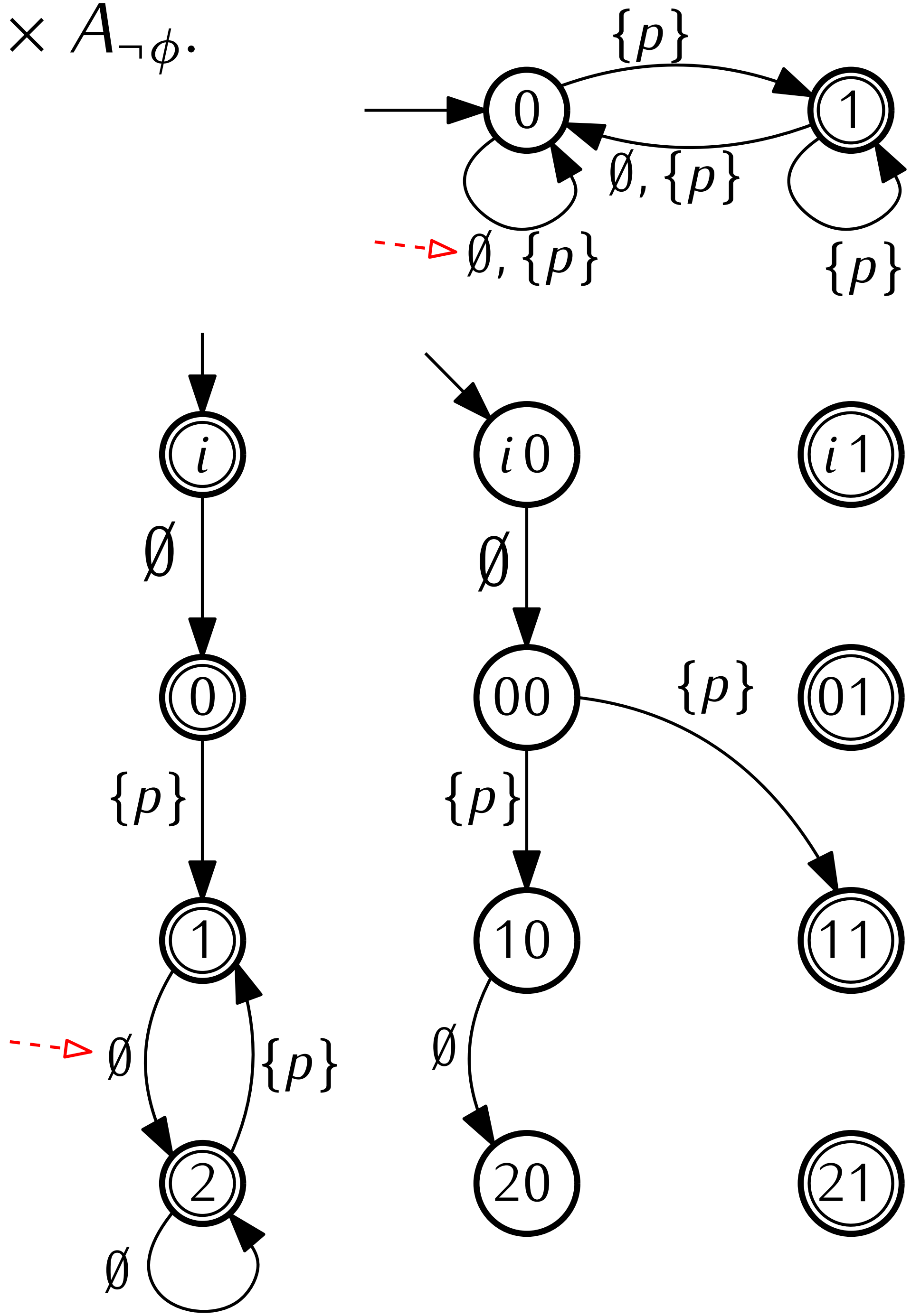

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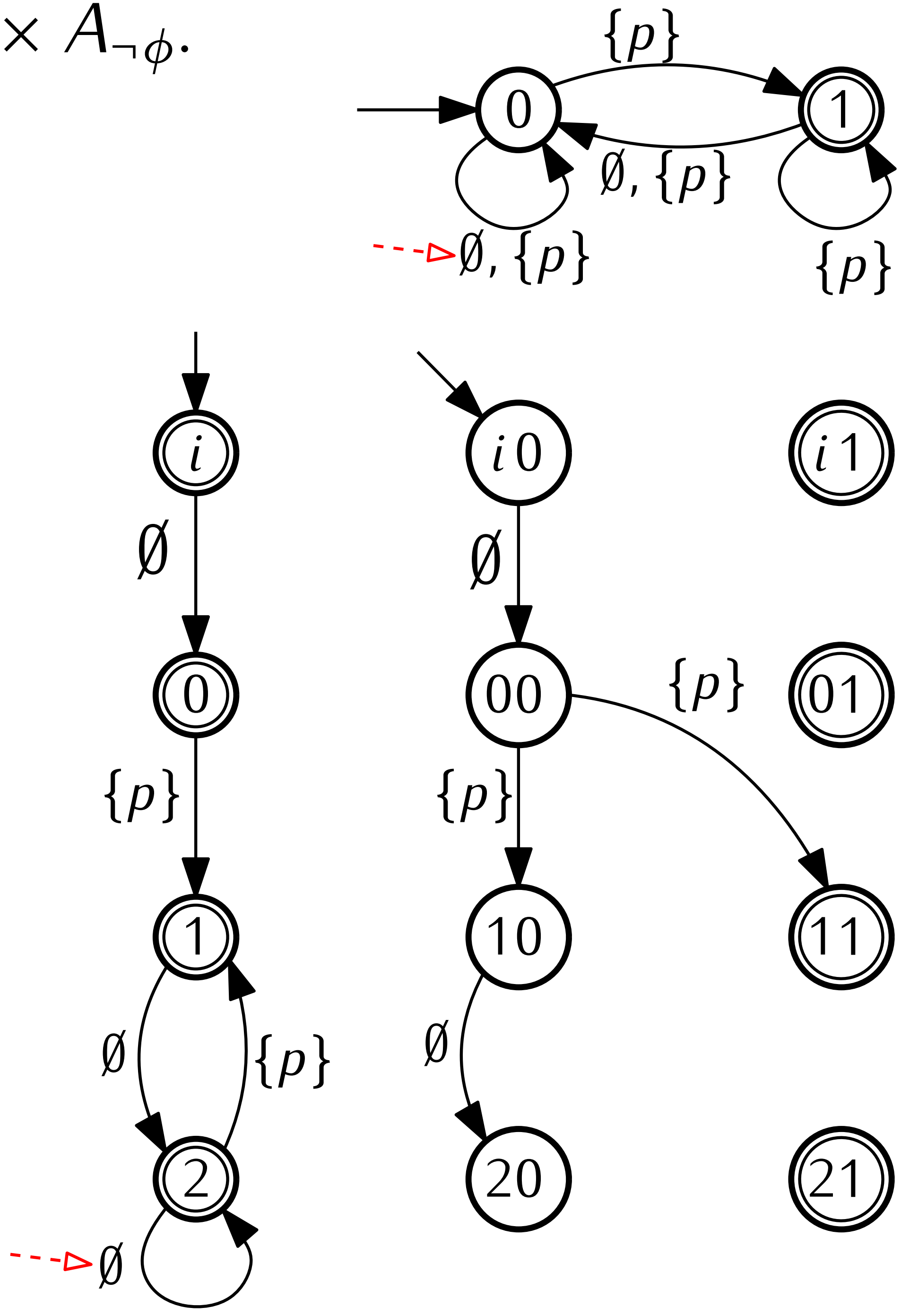

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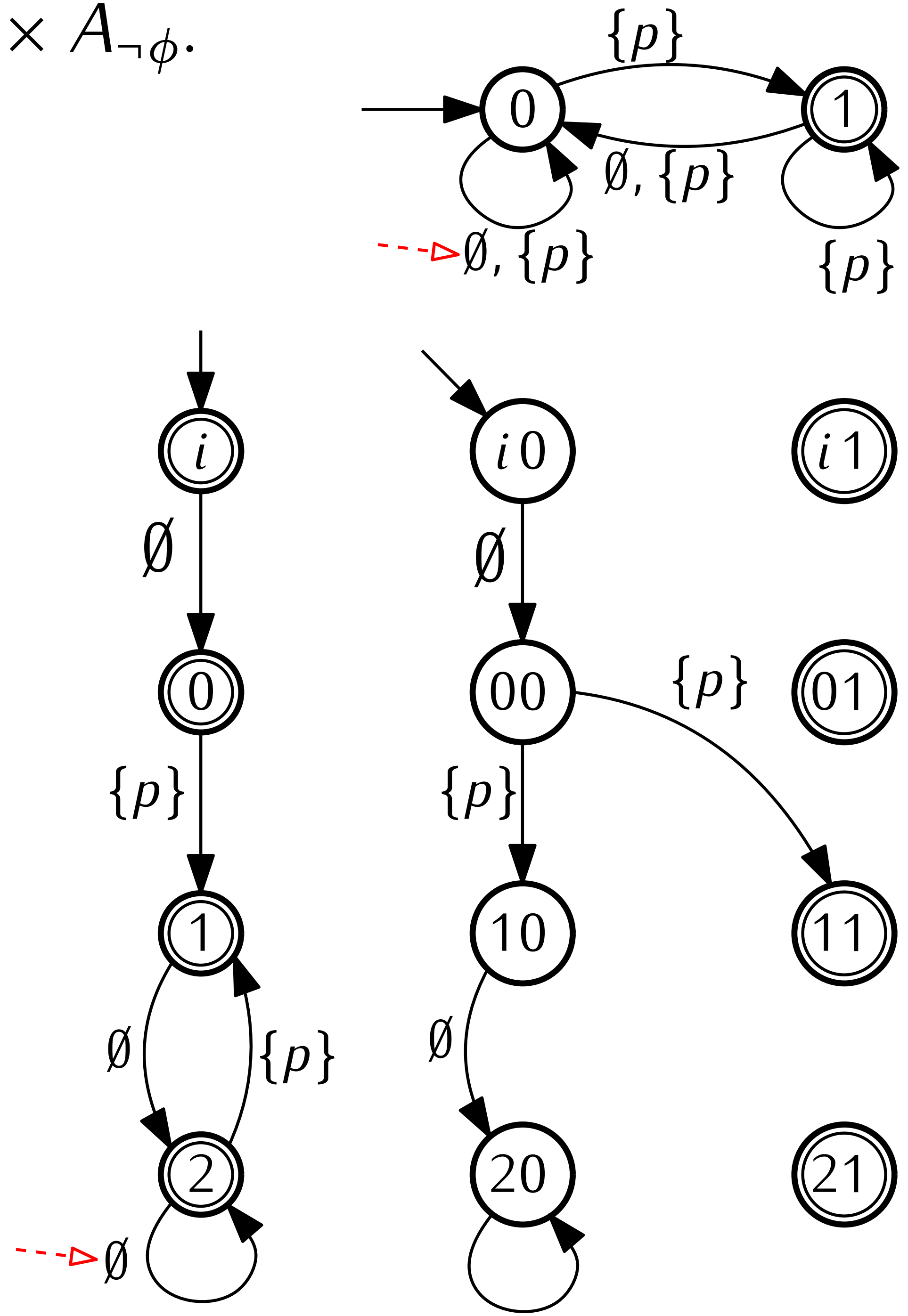

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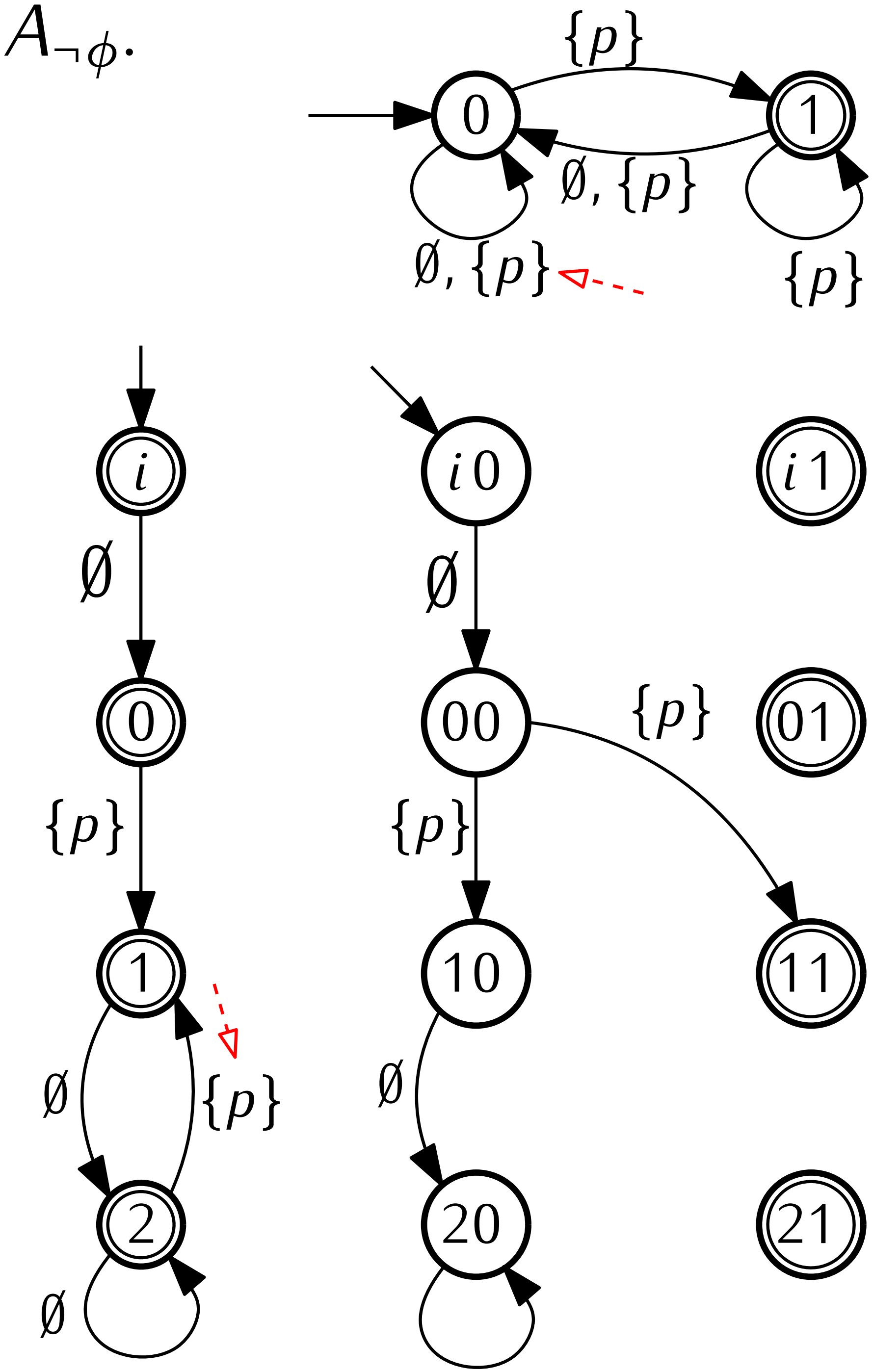

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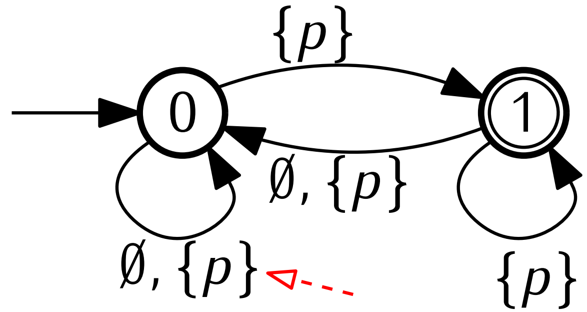

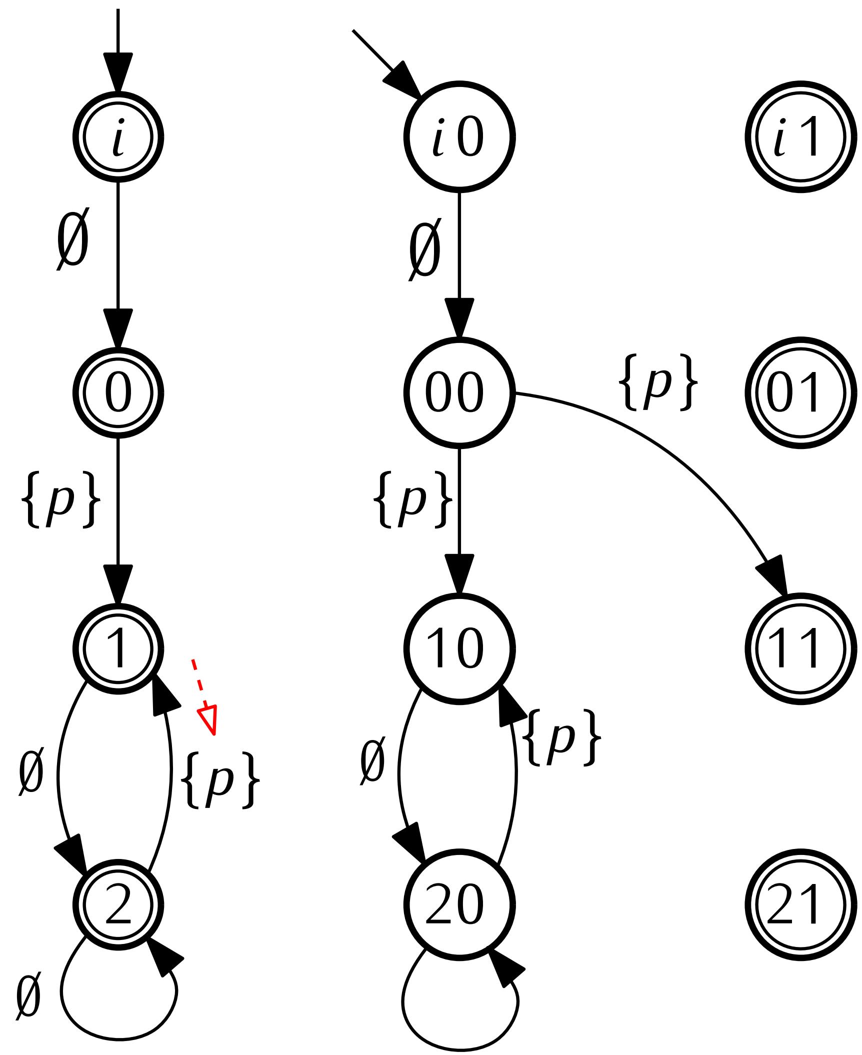

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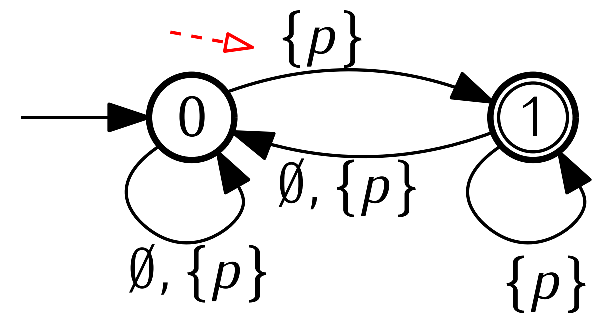

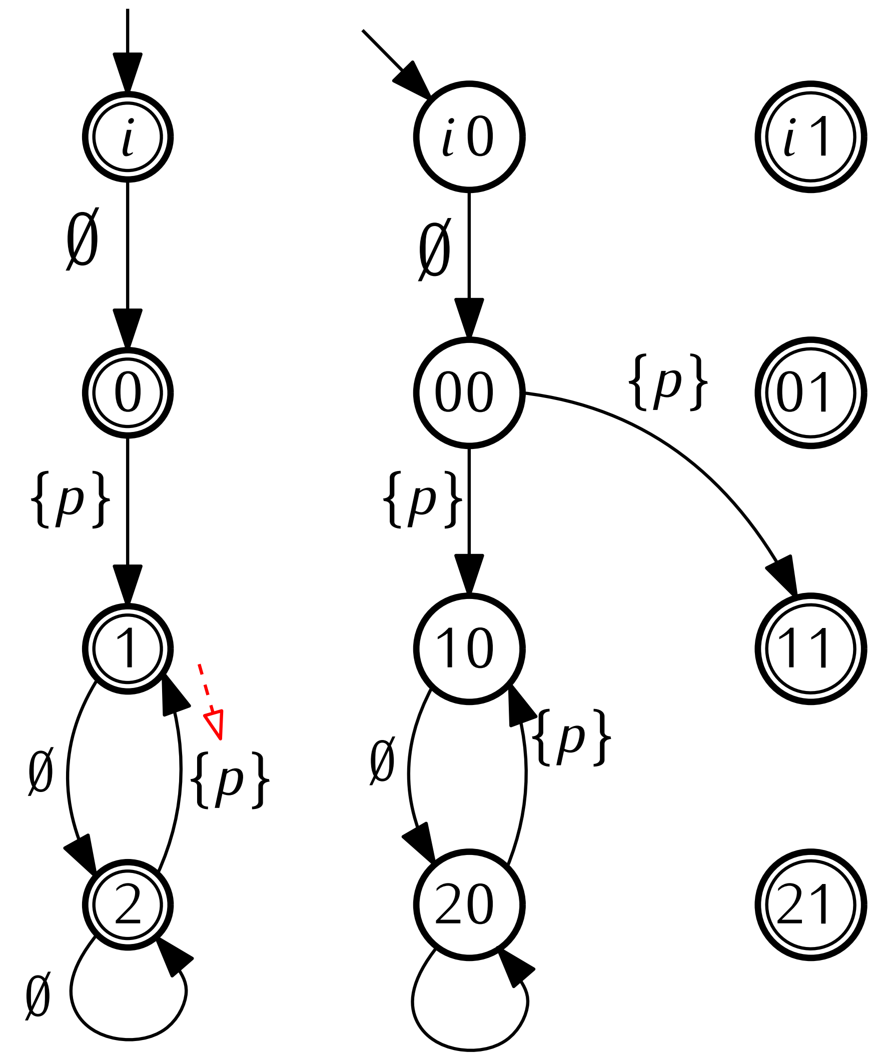

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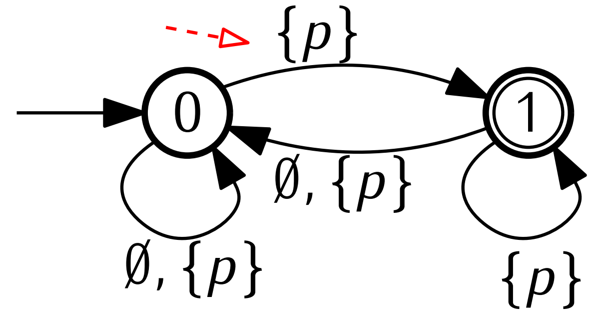

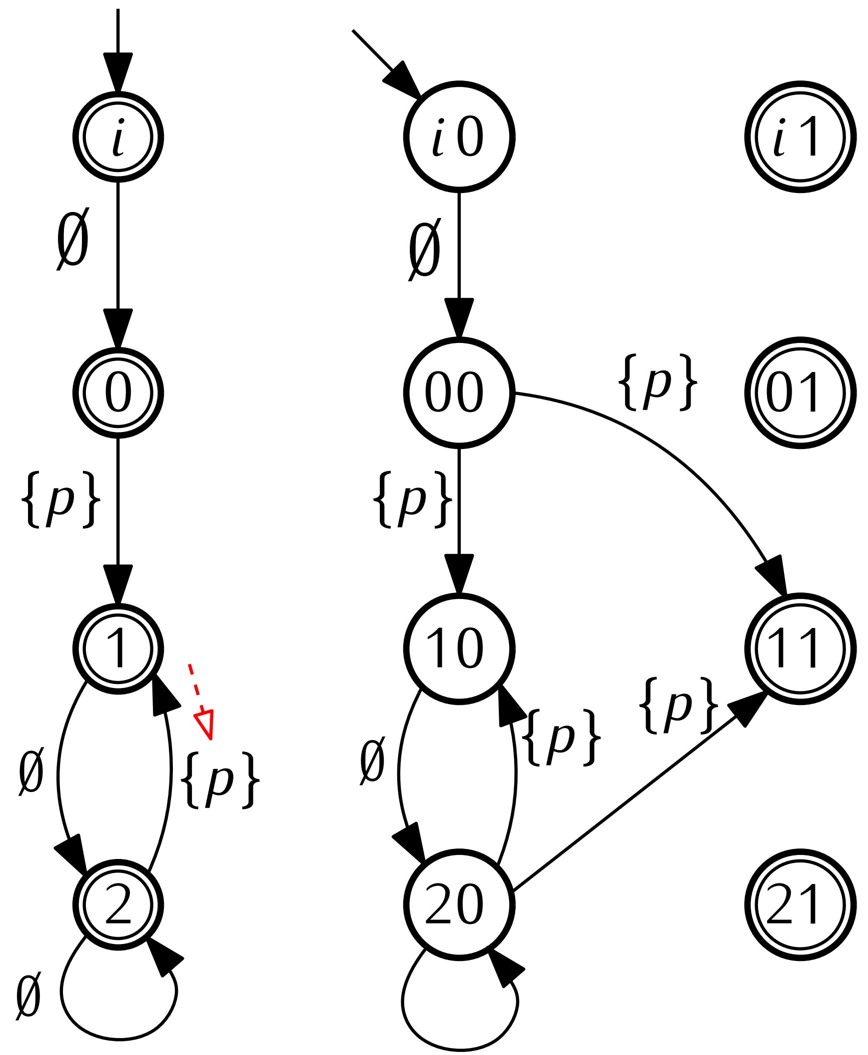

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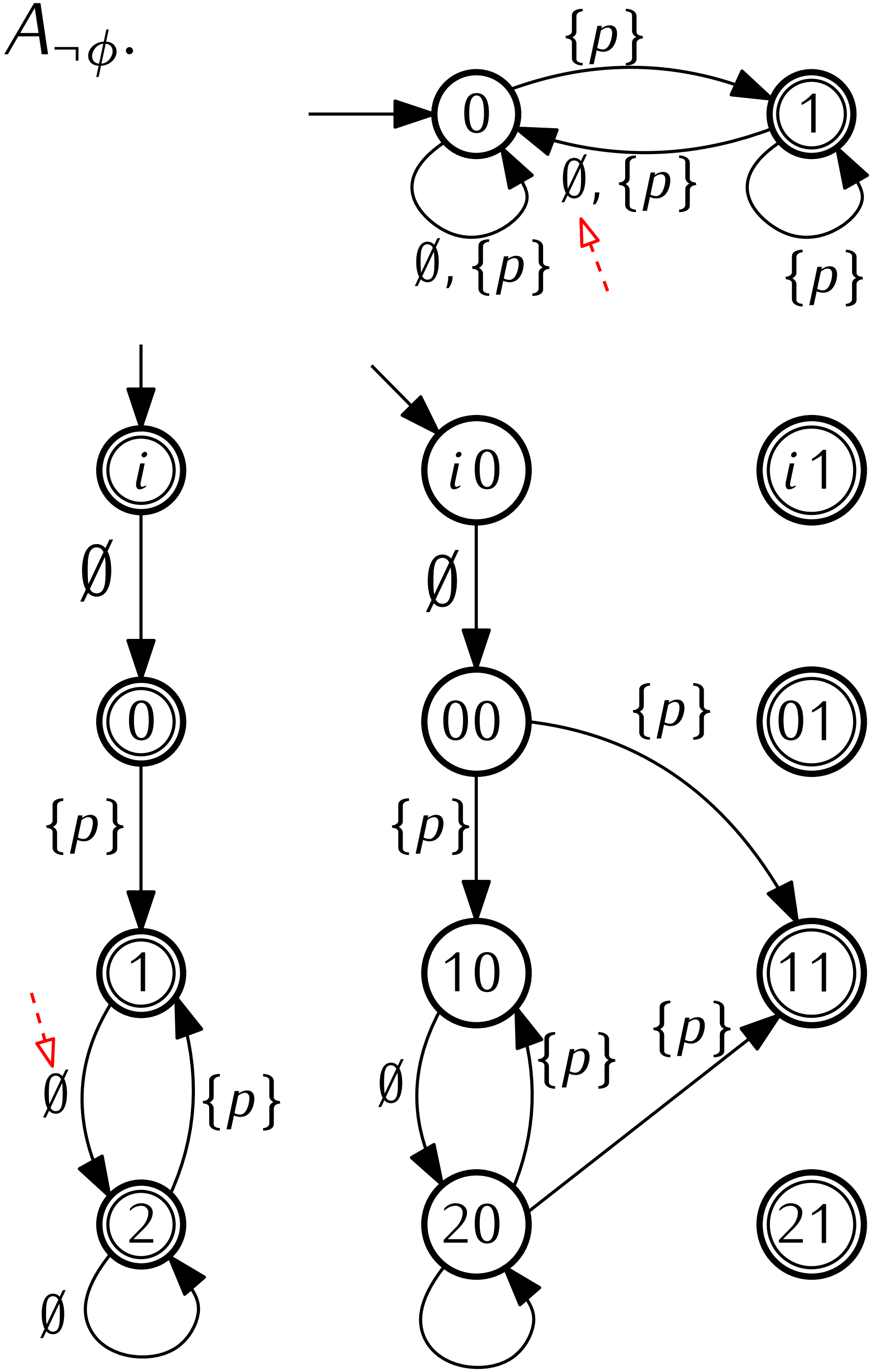

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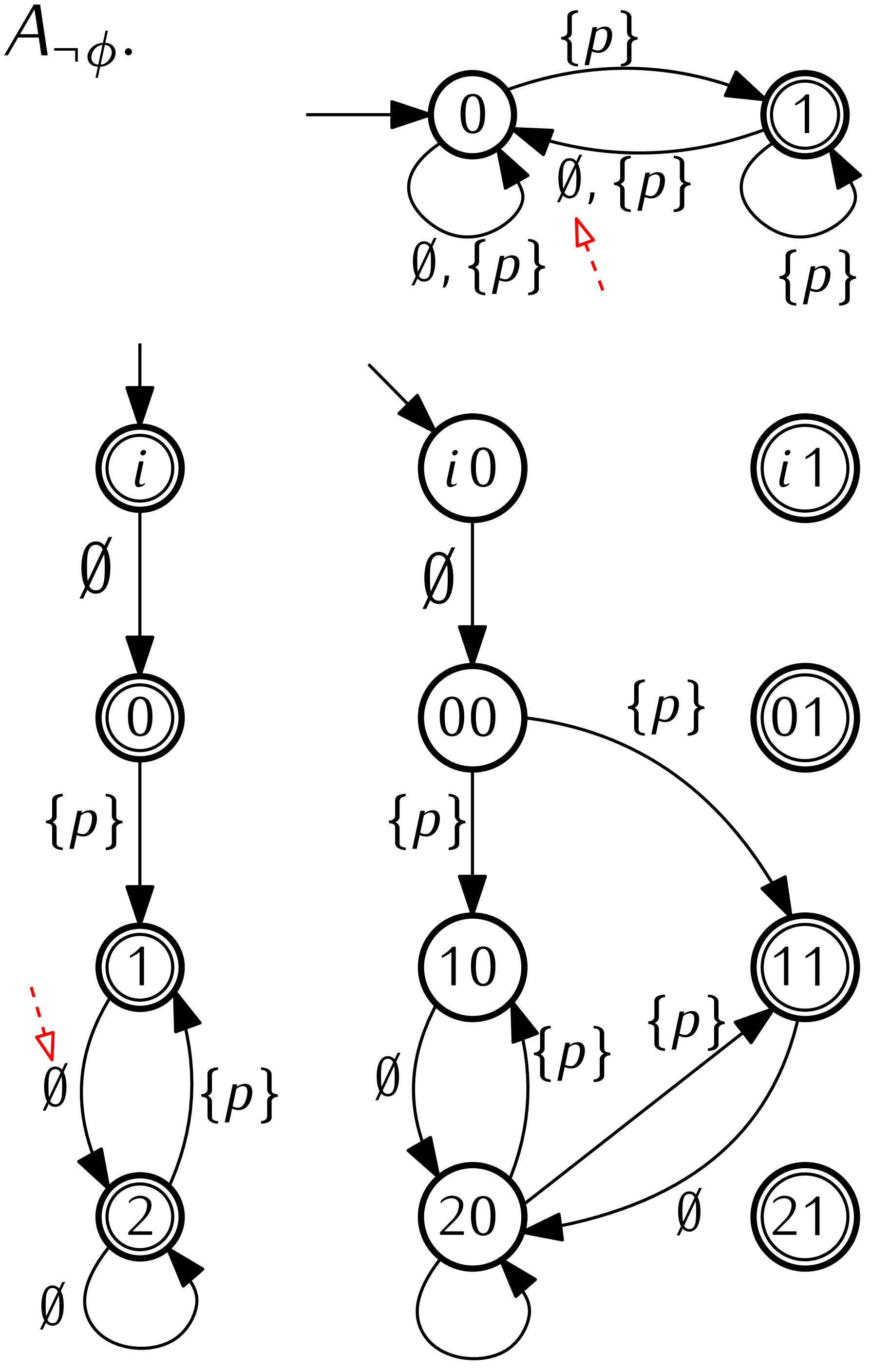

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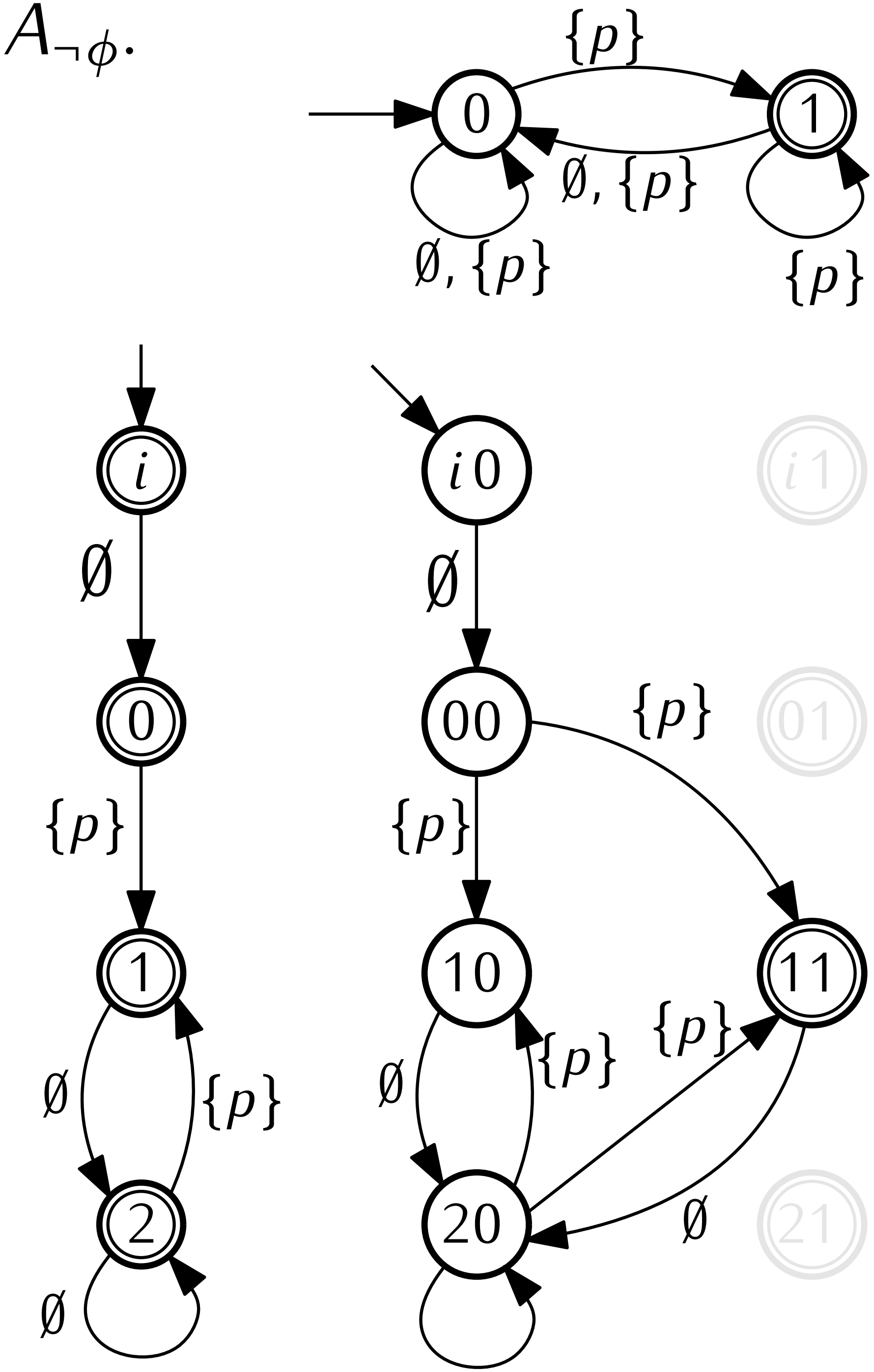

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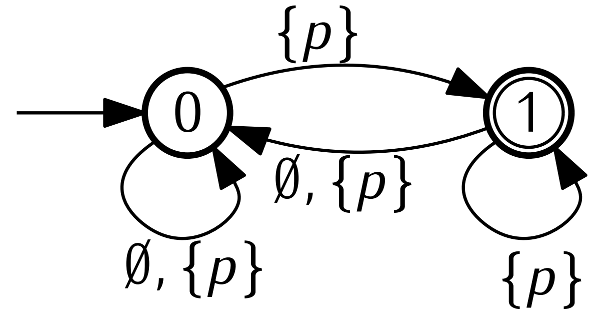
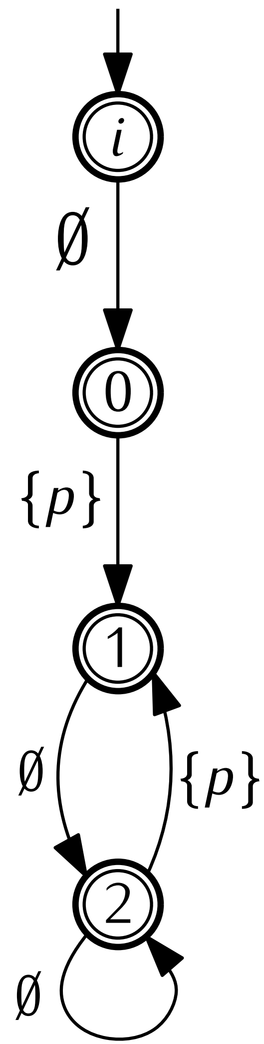
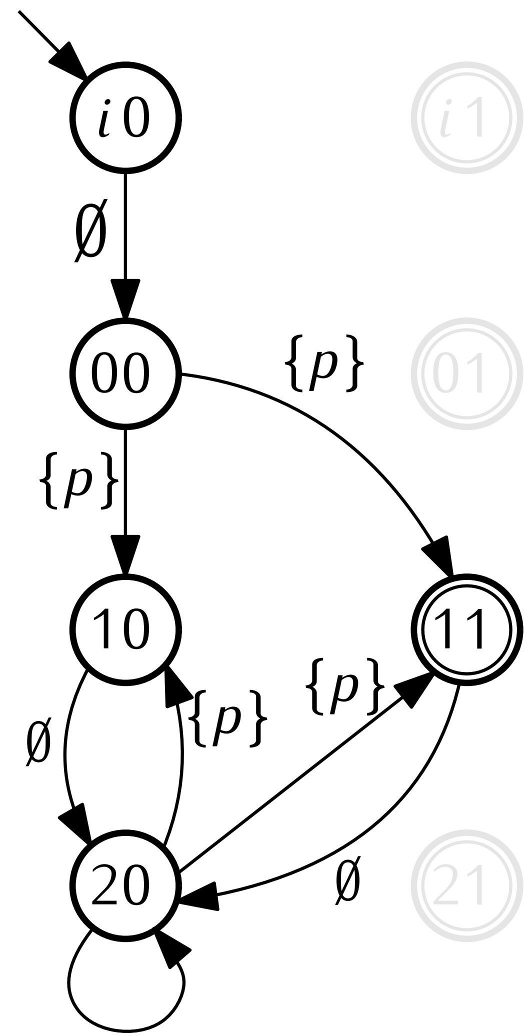


Step 4: Is there an accepting path (a path that visits an accept state infinitely often)?





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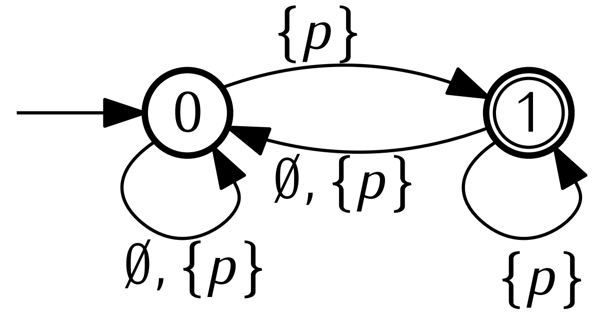

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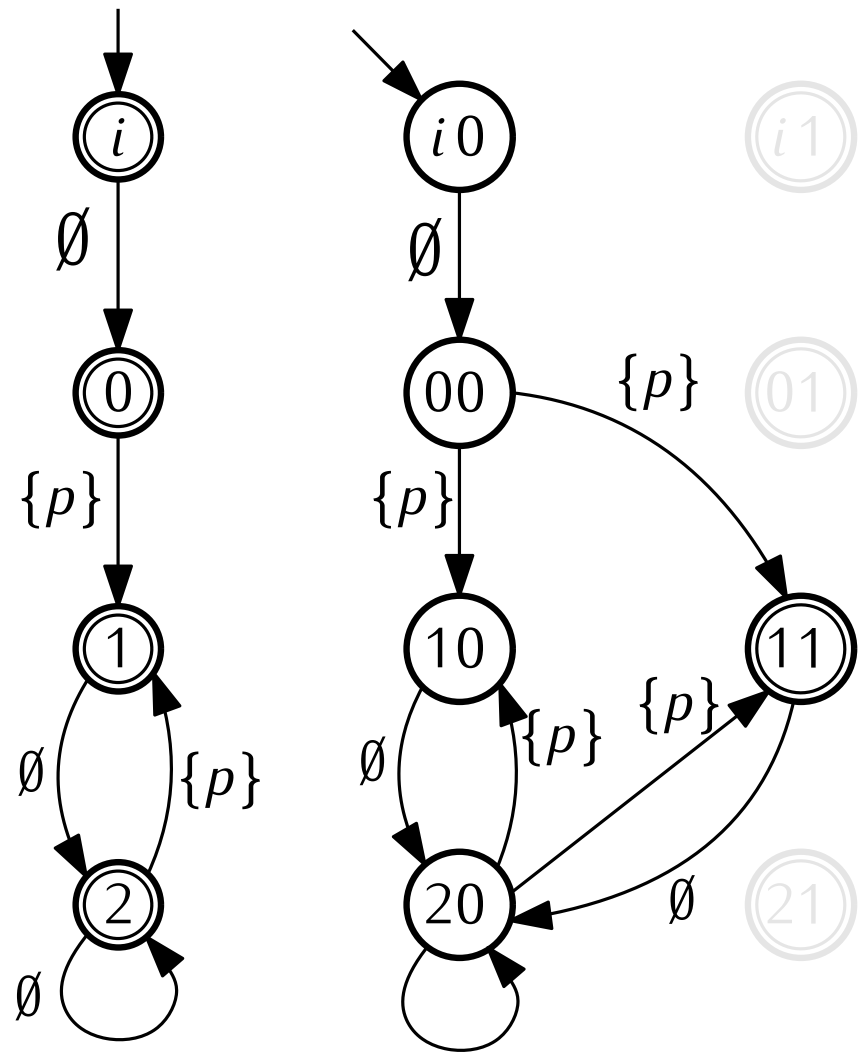
Yes. $i0, 00, 10, (20, 11)^{\omega}$



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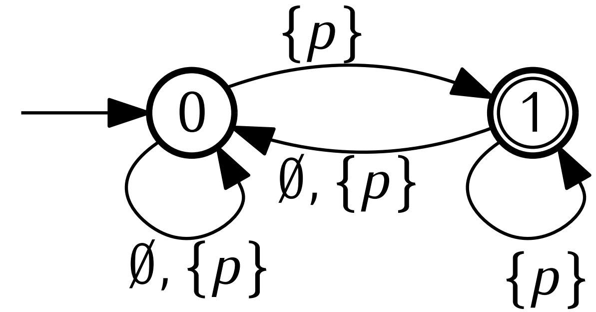

Step 4: Is there an accepting path (a path that visits an accept state infinitely often)?

Yes. $i0, 00, 10, (20, 11)^{\omega}$

This corresponds to $0, (1, 2)^{\omega}$ in the original transition system $\mathcal{M}$. Since we have found a counter example path, $\mathcal{M} \not\models \phi$.