PLDI 2016 Tutorial
Automata-Based String Analysis
Model Counting

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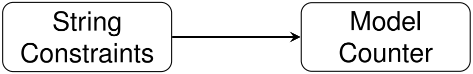
Overview

Overview

String
Constraints

Overview



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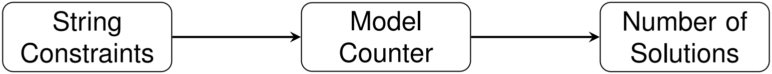

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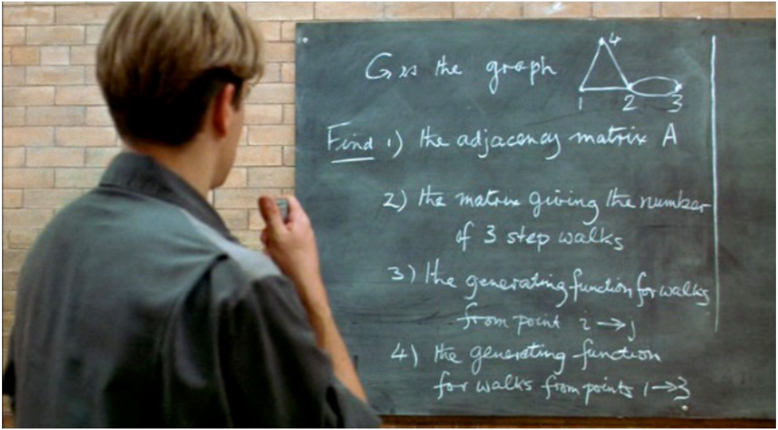

Outline

- Motivation and Background
- Model Counting Boolean Formulas
- String Model Counting
  - Automata-Based Methods
  - Non-Automata-Based Method
- String Model Counting Benchmarks

A Motivating Example

An adversary learns a password. User must select a new password.

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Policy for selecting a new password.

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```
public Boolean NewPWCheck(String new_p, old_p){
    if( old_p.contains(new_p)               || ...
        new_p.contains(old_p)               || ...
        old_p.reverse().contains(new_p))    || ...
        new_p.contains(old_p.reverse())       ){
        System.out.println("Too similar.");
        return false;
    } else
        return true;
}
```

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Constraints on possible values of NEW_P

```
(not (contains (toLower NEW_P) "abc-16"))
(not (contains (toLower NEW_P) "61-cba"))
(not (contains "abc-16" (toLower NEW_P)))
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If password length $= n$, then there are $|\Sigma|^n$ possible passwords.

If adversary knows `old_p` and the policy ...

- how much is the reduction in search space?
- what is the probability of guessing the new password?

Motivation

In general, we want to answer questions regarding

- probability of program behaviors,
- number of inputs that cause an error,
- amount of information flow,
- information leakage,
- other, as yet unforeseen, applications...

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These are **quantitative** questions which require **model counting**.

Motivation

Techniques for model counting for other theories

Boolean Logic Formulas

- DPLL
- Random sampling based
- Approximations

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Linear Integer Arithmetic:

- LattE
- Barvinok

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- security critical functions,
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- databases,
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Software for string constraint model counting

- Automata-Based Model Counter (ABC) [Aydin, et. al. CAV 2015]
- String Model Counter (SMC) [Luu, et. al. PLDI 2014]

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Recall the classic (boolean) SAT problem

Given a formula ϕ from propositional logic, is it possible to assign all variables the values T (true) or F (false) so that the formula is true?

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$$(x, y, z, w, v) = (T, F, T, F, T).$$

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A satisfying assignment is called a **model** for ϕ.

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Given a formula ϕ over some theory (Boolean, LIA, Strings, ...)

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Difficulty of Model Counting

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Model counting is “at least as hard” as satisfiability check.

$$|\phi| > 0 \iff \phi \text{ is satisfiable}$$

Model Counting Boolean SAT

x	y	z	w	v	F
F	F	F	F	F	F
⋮	⋮	⋮	⋮	⋮	⋮
T	F	F	T	T	F
T	F	T	F	F	F
T	F	T	F	T	T
T	F	T	T	F	F
T	F	T	T	T	T
T	T	F	F	F	F
T	T	F	F	T	F
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⋮	⋮	⋮	⋮	⋮	⋮
T	F	F	T	T	F
T	F	T	F	F	F
T	**F**	**T**	**F**	**T**	**T**
T	F	T	T	F	F
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T	T	F	F	F	F
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ϕ has 6 models.

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Truth table method is $\theta(2^n)$.

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F	F	F	F	F	F
⋮	⋮	⋮	⋮	⋮	⋮
T	F	F	T	T	F
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Truth table method is $\theta(2^n)$.

DPLL method is $O(2^n)$, but is faster in practice.[1]

[1] Birnbaum, et. al. The good old Davis-Putnam procedure helps counting models. JAIR 1999.

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k	X	a_k
0	ε	1

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1	0	1
2	11	1

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1	0	1
2	11	1
3	110	1

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0	ε	1
1	0	1
2	11	1
3	110	1
4	1001, 1100, 1111	3

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k	X	a_k
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2	11	1
3	110	1
4	1001, 1100, 1111	3
5	10010, 10101, 11000, 11011, 11110	5

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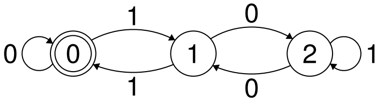

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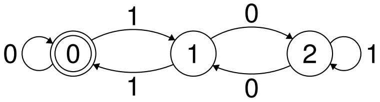

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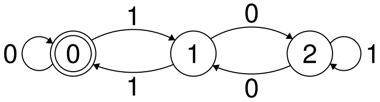

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String Counting $\equiv$ Path Counting

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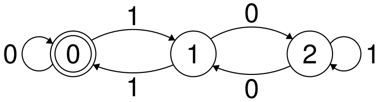
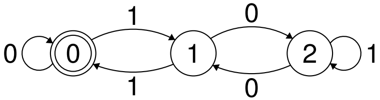


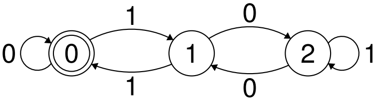
How to count paths of length k?

Deterministic Finite Automata



How to count paths of length k?
Dynamic Programming

Deterministic Finite Automata



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a_k

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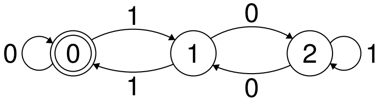

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Dynamic Programming

$a_k(s) =$

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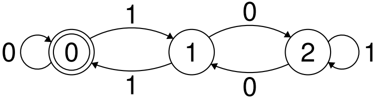

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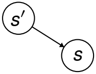

$$a_k(s) = \qquad a_{k-1}(s')$$

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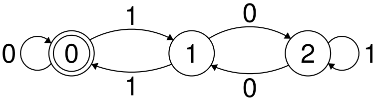

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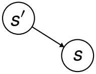

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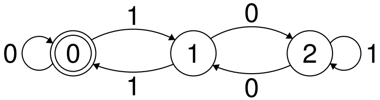

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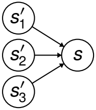

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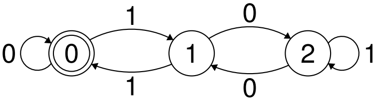

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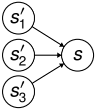

$$a_k(s) = \sum_{s' \to s} a_{k-1}(s')$$

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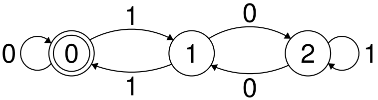

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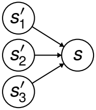

Initial Conditions

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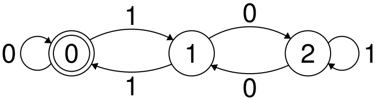

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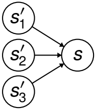

Initial Conditions

$$a_0(0) = 1$$

$$a_k(s) = \sum_{s' \to s} a_{k-1}(s')$$

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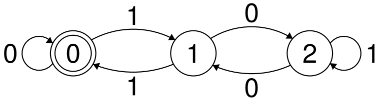
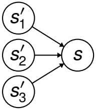


How to count paths of length k?

Dynamic Programming

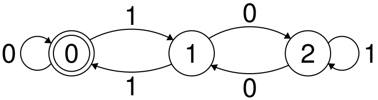


Initial Conditions

$$a_0(0) = 1, a_0(1) = 0, a_0(2) = 0$$

$$a_k(s) = \sum_{s' \to s} a_{k-1}(s')$$

Deterministic Finite Automata



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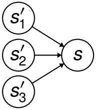

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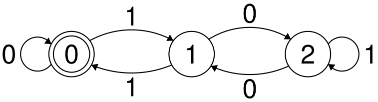
System of Recurrences

$$a_0(k) = a_0(k-1) + a_1(k-1)$$

Deterministic Finite Automata



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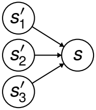

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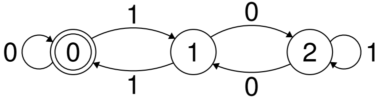
Deterministic Finite Automata



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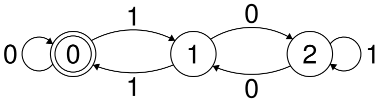

How to count paths of length k?

System of Recurrences

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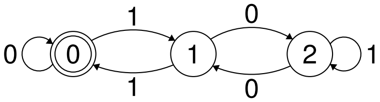
\end{aligned}
$$

Deterministic Finite Automata



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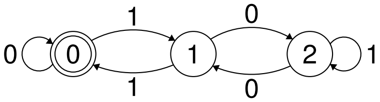
Deterministic Finite Automata



How to count paths of length k?
Matrix Exponentiation

System of Recurrences

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\end{aligned}
$$

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\begin{pmatrix} a_0(k) \\ a_1(k) \\ a_2(k) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}^k \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}
$$

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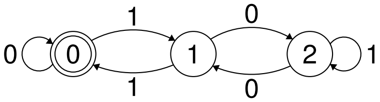
Deterministic Finite Automata



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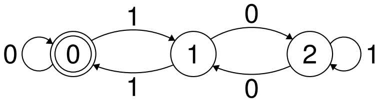
$$a_k = (A^k)_{0,F}$$

Deterministic Finite Automata



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$$a_4 = (A^4)_{0,0} = 3$$

Generating functions are a way to compactly represent (possibly infinite) sequences.

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Sequence element a_k is the k^{th} Taylor series coefficient of $g(z)$.

$$X \in (0|(1(01^*0)^*1))^*$$

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A generating function for language $\mathcal{L}$ encodes

$$a_k = |\{s : s \in \mathcal{L}, \text{len}(s) = k\}|$$

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$$g(z) = 1z^0$$

k	X	a_k
0	ε	1

$$X \in (0|(1(01^*0)^*1))^*$$

A generating function for language $\mathcal{L}$ encodes

$$a_k = |\{s : s \in \mathcal{L}, \mathrm{len}(s) = k\}|$$

$$g(z) = 1z^0 + 1z^1$$

k	X	a_k
0	ε	1
1	0	1

$$X \in (0|(1(01^*0)^*1))^*$$

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$$a_k = |\{s : s \in \mathcal{L}, \text{len}(s) = k\}|$$

$$g(z) = 1z^0 + 1z^1 + 1z^2$$

k	X	a_k
0	ε	1
1	0	1
2	11	1

$$X \in (0|(1(01^*0)^*1))^*$$

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k	X	a_k
0	ε	1
1	0	1
2	11	1
3	110	1

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$$g(z) = 1z^0 + 1z^1 + 1z^2 + 1z^3 + 3z^4$$

k	X	a_k
0	ε	1
1	0	1
2	11	1
3	110	1
4	1001, 1100, 1111	3

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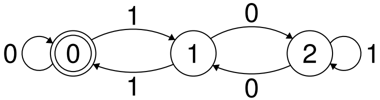
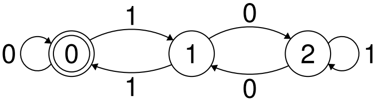
k	X	a_k
0	ε	1
1	0	1
2	11	1
3	110	1
4	1001, 1100, 1111	3
5	10010, 10101, 11000, 11011, 11110	5

Deterministic Finite Automata



How to count paths of length k?

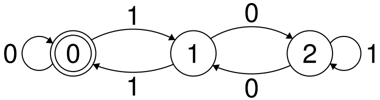
Deterministic Finite Automata



How to count paths of length k?
Generating Functions

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad g(z) = \frac{\det(I - zA : i, j)}{(-1)^n \det(I - zA)}$$

Deterministic Finite Automata



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$$g(z) = \frac{1 - z - z^2}{(z-1)\,(2z^2 + z - 1)}$$

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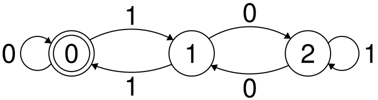

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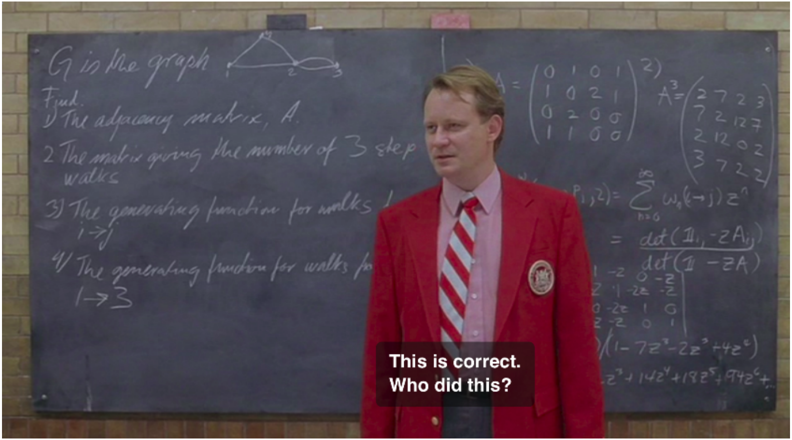
$$g(z) = 1z^0 + 1z^1 + 1z^2 + 1z^3 + 3z^4 + 5z^5 + \ldots$$

Good job, Will Hunting!!!



Automata-Based Model Counter (ABC)

CAV 2015: Automata-Based Model Counting for String Constraints.
Abdulbaki Aydin, Lucas Bang, Tevfik Bultan:

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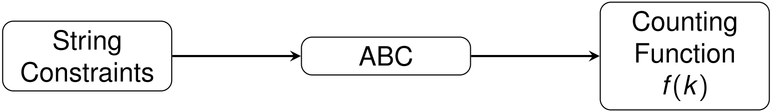

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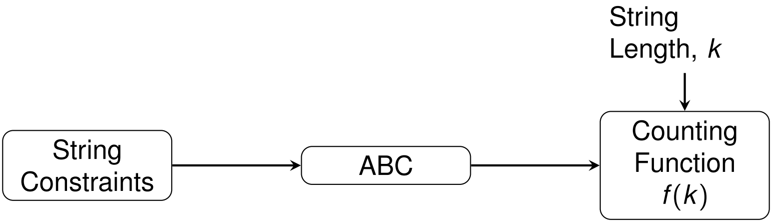

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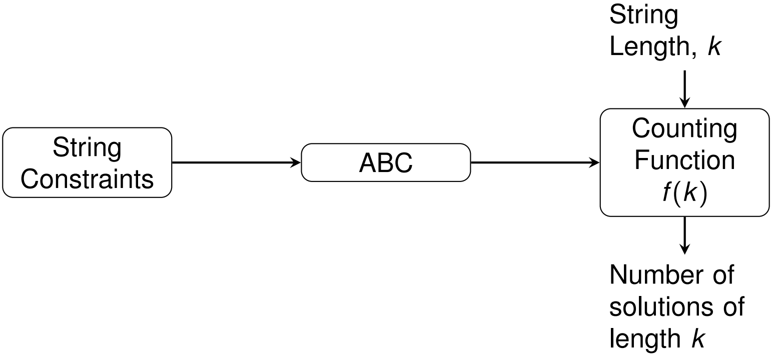

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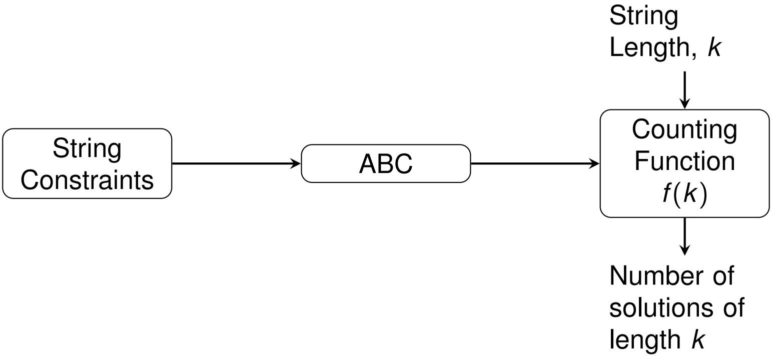

Idea: Convert string constraints to DFA. Count paths in DFA.

Password Changing Policy

Constraint on NEW_P

```
(declare-fun NEW_P () String)

(not (contains (toLower NEW_P) "abc-16"))
(not (contains "abc-16" (toLower NEW_P)))
(not (contains (toLower NEW_P) "61-cba"))
(not (contains "61-cba" (toLower NEW_P)))

(check-sat)
(model-count)
```

Password Changing Policy

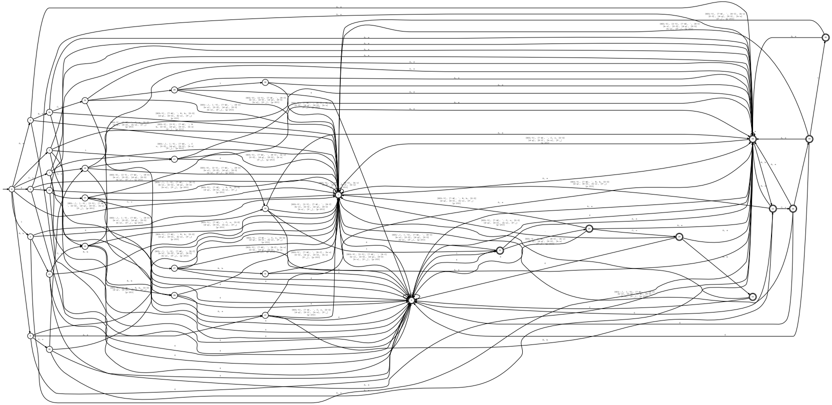

Figure : Solution DFA for all possible values of NEWP.

Password Changing Policy

$$
\begin{pmatrix}
0 & 1 & 1 & 1 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
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\end{pmatrix}
$$

Figure : Transition matrix for DFA for all possible values of NEWP.

Password Changing Policy

$$
\begin{pmatrix}
0 & 1 & 1 & 1 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
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$$

Figure : Transition matrix for DFA for all possible values of NEWP.

Password Changing Policy

Generating function which enumerates NEW_P:

$$g(z) = \frac{8096z^{12} - 8128z^{11} + 32z^{10} + 16z^{7} - 16z^{6} - 256z^{2} + 257z - 1}{194304z^{17} + 225920z^{16} + 241984z^{15} + \ldots + z^{5} - 6114z^{4} - 2280z^{3} - 247z^{2}}$$

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To answer our quantitative question:

- Brute force searching for password length = 6: $256^6 = 2^{48}$ passwords.

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- Brute force searching for password length = 6: $256^6 = 2^{48}$ passwords.
- If adversary knows `old_p` and the policy: $1103823437965 \approx 2^{40.0056}$ passwords.

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To answer our quantitative question:

- Brute force searching for password length = 6: $256^6 = 2^{48}$ passwords.
- If adversary knows `old_p` and the policy: $1103823437965 \approx 2^{40.0056}$ passwords.
- Reduces search space by about factor of $2^{7.9944}$

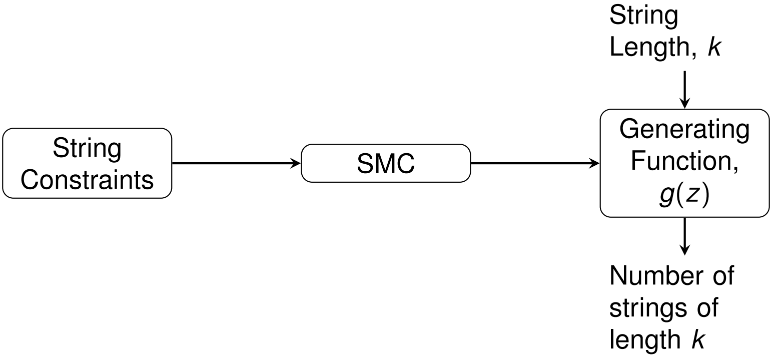
Outline

- Motivation and Background
- Model Counting Boolean Formulas
- String Model Counting
  - Automata-Based Methods
  - Non-Automata-Based Method
- String Model Counting Benchmarks

SMC Model Counting

PLDI 2014: A Model Counter For Constraints Over Unbounded Strings. Luu, Shinde, Saxena, Demsky.

SMC Tool Online: https://github.com/loiluu/smc



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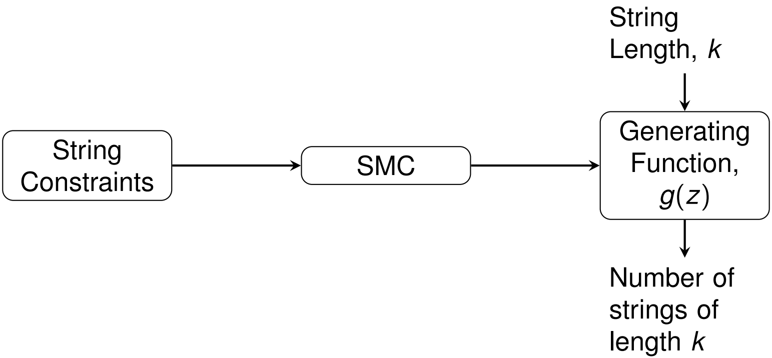

Idea: go directly from constraints to $g(z)$ using transformations.

SMC Model Counting

For a regular expression constraint, generating function can be derived recursively.

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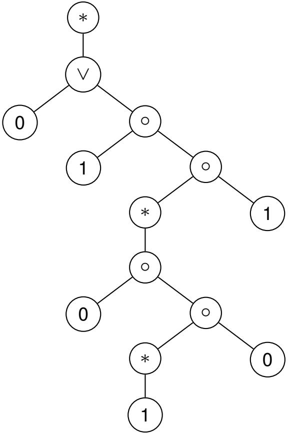
A^* & \mapsto & 1/(1 - A(z))
\end{array}
$$

Regular Expressions

$$X \in (0|(1(01^*0)^*1))^*$$



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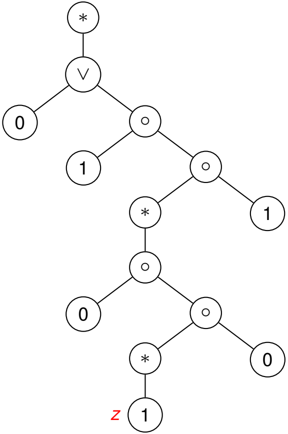

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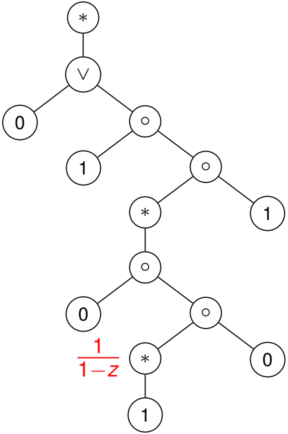

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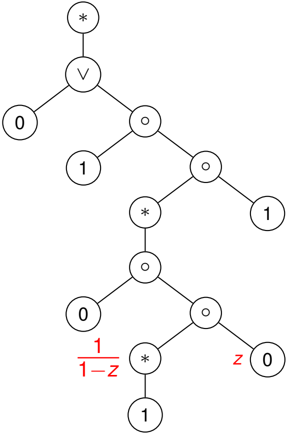

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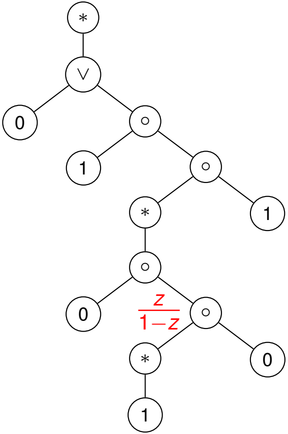

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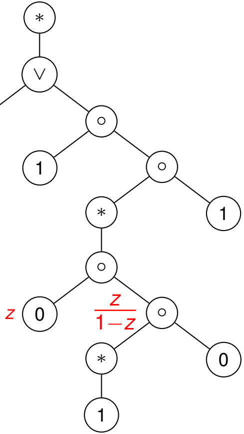

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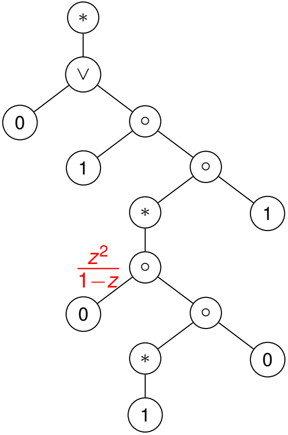

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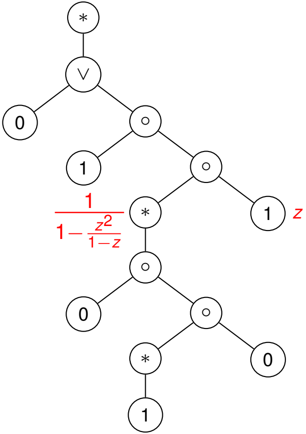

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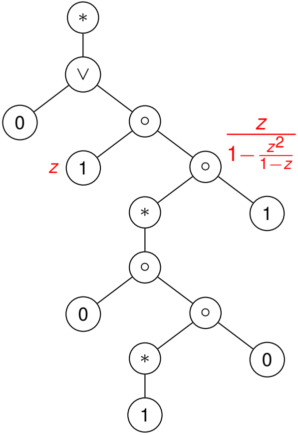

$$\frac{z}{1-\frac{z^2}{1-z}}$$

Regular Expressions

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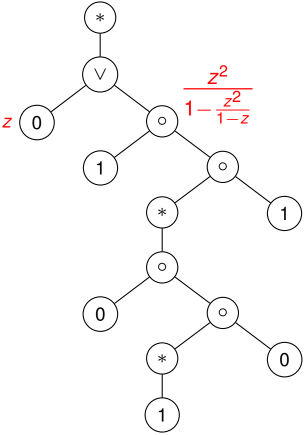

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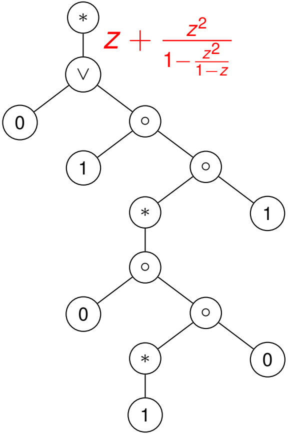

$$z + \frac{z^2}{1-\frac{z^2}{1-z}}$$

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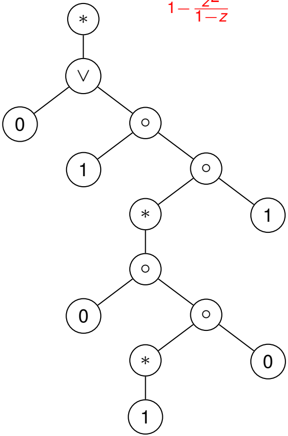
$$\frac{1}{1-z-\frac{z^2}{1-\frac{z^2}{1-z}}}$$



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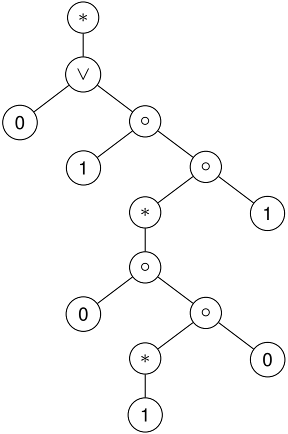

Generating Function:

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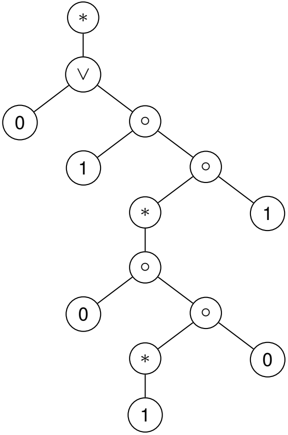

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$$g(z) = \frac{1}{1-z-\frac{z^2}{1-\frac{z^2}{1-z}}}$$

$$= \frac{1-z-z^2}{(z-1)(2z^2+z-1)}$$

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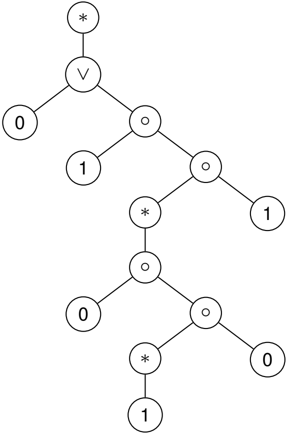

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$$g(z) = 1z^0+1z^1+1z^2+1z^3+3z^4+5z^5+\dots$$

Other operations in SMC

Specialized transformations for other operations

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$$contains(s_1, s_2) \quad \mapsto \quad \frac{z^n}{(1-Mz)(z^n+(1-Mz)c(z))}$$

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\begin{aligned}
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F_1 \vee F_2 &\mapsto [\max(L_1(z), L_2(z)), \min(U_1(z) + U_2(z), G(z))]
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Also handle substring, length, negation, conjunction, . . . ,
with upper and lower bounds.

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Experimental Comparison

Table : Log scaled comparison between SMC and ABC

	bound	SMC lower bound	SMC upper bound	ABC count
nullhttpd	500	3752	3760	3760
ghttpd	620	4880	4896	4896
csplit	629	4852	4921	4921
grep	629	4676	4763	4763
wc	629	4281	4284	4281
obscure	6	0	3	2

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JavaScript Benchmarks

- Kaluza benchmarks, extracted from JavaScript code via DSE, [Saxena, SSP 2010]

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- Small Constraints (19,731):
  - ABC: 19,731 constraints, average 0.32 seconds per constraint
  - SMC: 17,559 constraints, average 0.26 seconds per constraint.

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JavaScript Benchmarks

- Kaluza benchmarks, extracted from JavaScript code via DSE, [Saxena, SSP 2010]
- Small Constraints (19,731):
  - ABC: 19,731 constraints, average 0.32 seconds per constraint
  - SMC: 17,559 constraints, average 0.26 seconds per constraint.
- Big Constraints (1,587):
  - ABC: 1,587 constraints, average 0.34 seconds per constraint
  - SMC: 1,342 constraints, average 5.29 seconds per constraint

ABC Bonus: Model Counting Linear Integer Arithmetic

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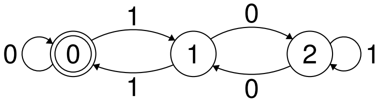

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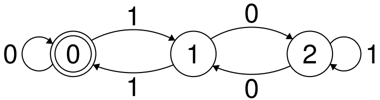

Idea: DFA can represent (some) relations on sets of binary integers. We can use similar techniques that we used for #String to solve #LIA.

Model Counting Linear Integer Arithmetic

Quantifier-Free Linear Integer Arithmetic $(\mathbb{Z}, +, <)$.

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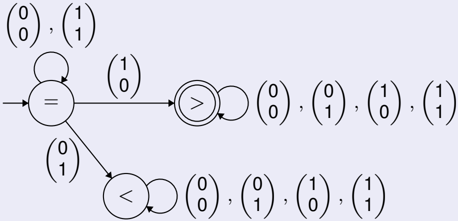
It is possible to represent the solutions to a set of LIA constraints as a binary multi-track DFA.

Binary Multi-track DFA

Solution DFA for LIA constraints.

- Read bits of x and y from most to least significant.
- Alphabet is a tuple of bits: $\begin{pmatrix} b_x \\ b_y \end{pmatrix}$

Solution DFA for the constraint $x > y$.



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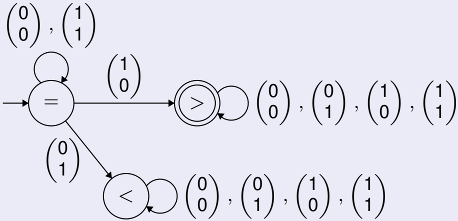

Solutions of length $n \equiv$ solutions within bound 2^n

Model Counting Summary

Counting Techniques for Different Theories

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Counting Techniques for Different Theories

- Boolean
  - Truth Table (Brute Force)
  - DPLL
- Strings
  - DFA with Dynamic Programming, Matrix Multiplication, GFs
  - Regular Expression with GFs
- Linear Integer Arithmetic
  - Binary Multi-track DFA

Related work on model counting

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Thank you.