Automated Quantification of Software Side-Channel Vulnerabilities

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Overview

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Program

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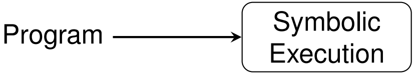

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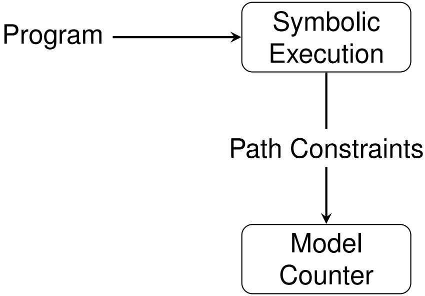
Program
Symbolic Execution
Path Constraints
Model Counter

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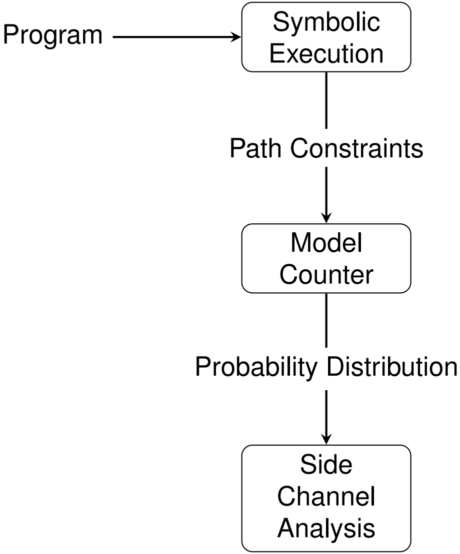
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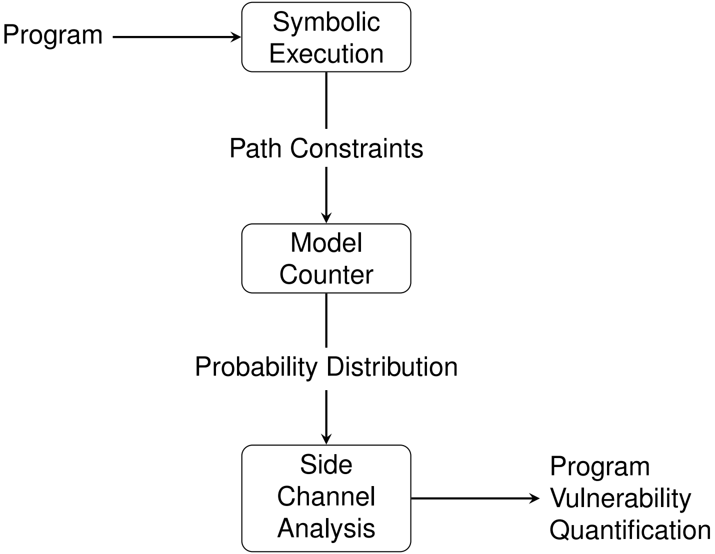
Program
Symbolic Execution
Path Constraints
Model Counter
Probability Distribution
Side Channel Analysis
Program Vulnerability Quantification

Outline

Symbolic Execution
- Software Verification
- Symbolic Execution
- Probabilistic Symbolic Execution
- SMT Solvers

Side Channel Analysis
- Background and Information Theory
- Via Probabalistic Symbolic Execution

Model Counting
- Boolean Logic
- Strings
- Linear Ineger Arithmetic

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Software Verification

Goal: Given a program, determine if executions satisfy some property.

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Software verification problem is undecidable!

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Feasible Software verification techniques must deal with state space explosion.

Work on Software Verification

- Geldenhuys. Probabilistic symbolic execution. ISSTA 2012
- Bultan. Symbolic Model Checking of Infinite State Systems Using Presburger Arithmetic. CAV 1997
- Yu. Patching Vulnerabilities with Sanitization Synthesis. ICSE 2011
- Ball. Automatically Validating Temporal Safety Properties of Interfaces. SPIN 2001
- Biere. Symbolic Model Checking without BDDs. TACAS 1999
- Visser. Model Checking Programs. ASE 2003.
- Burch. Symbolic Model Checking: 10^{20} States and Beyond, LICS 1990
- Bryant, Graph-Based Algorithms for Boolean Function Manipulation, IEEE Trans. Computers. 1986
- Cadar. Symbolic execution for software testing in practice: preliminary assessment. ICSE 2011
- Cadar. Symbolic Execution for Software Testing: Three Decades Later. CACM 2013
- Cousot. Abstract Interpretation: A Unified Lattice Model for Static Analysis of Programs by Construction or Approximation of Fixpoints. POPL 1977.
- Cousot. Systematic Design of Program Analysis Frameworks. POPL 1979

Software Verification Tools

A small sample:

- Edmund Clarke. A Tool for Checking ANSI-C Programs. TACAS 2005.
- Holzmann. The Model Checker SPIN. IEEE Trans. Software Eng 1997.
- Musuvathi. CMC: A pragmatic approach to model checking real code. OSDI 2002.
- Yang. Using Model Checking to Find Serious File System Errors. OSDI 2004
- Ball. A decade of software model checking with SLAM. CACM 2011.
- Godefroid, et al. DART: Directed Automated Random Testing. PLDI 2005.
- Sen. CUTE: A Concolic Unit Testing Engine for C. ESEC/FSE 2005.
- SAGE: Whitebox Fuzzing for Security Testing. CACM 2012.

Outline

Symbolic Execution and Path Constraints

Basic Idea

- Represent program variables as symbolic variables:
  - $x_1 \mapsto X_1, x_2 \mapsto X_2, \ldots, x_n \mapsto X_n$
- Program executions are described by formulas over symbolic variables.
  - $f(X_1, X_2, \ldots, X_n)$
  - Path Constraints

Software Verification With Symbolic Execution

```
function f(x,y)
u = x - y
if(x > y)
  u = u + x
if(u < 0)
  assert false
exit
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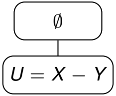


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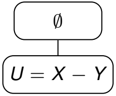


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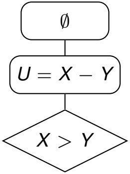


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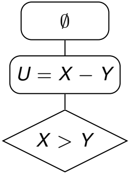


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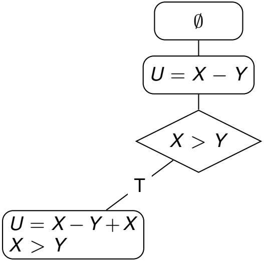


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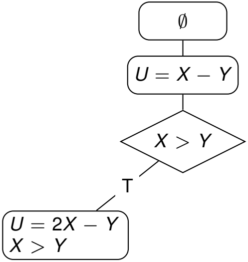


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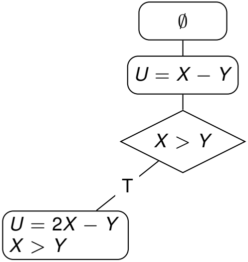


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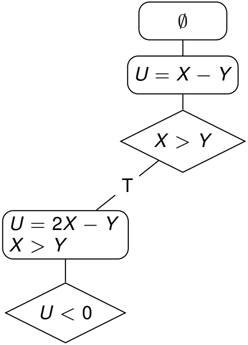

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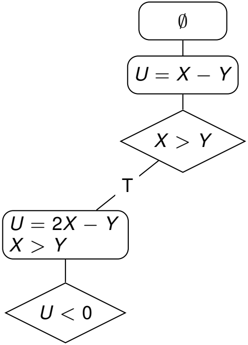

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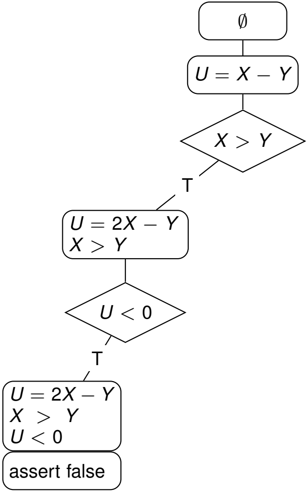

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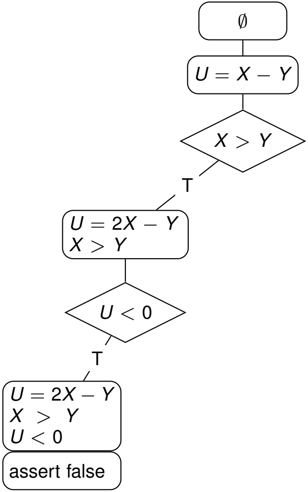

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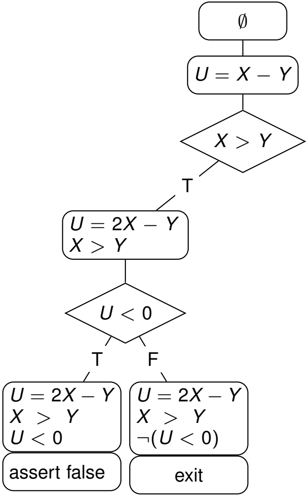

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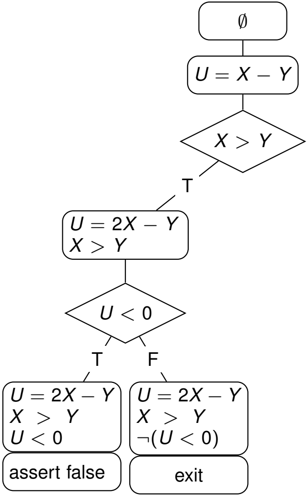

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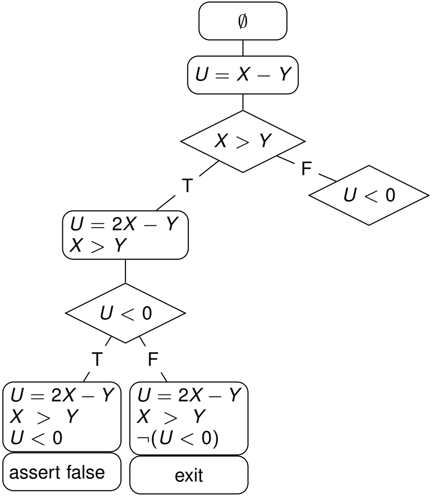

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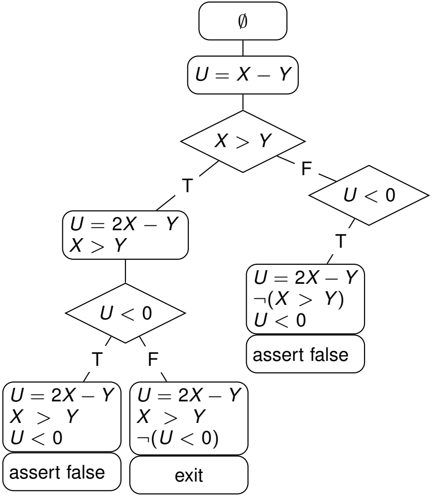

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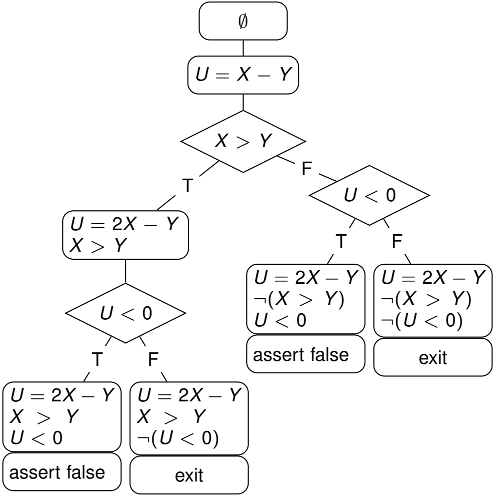

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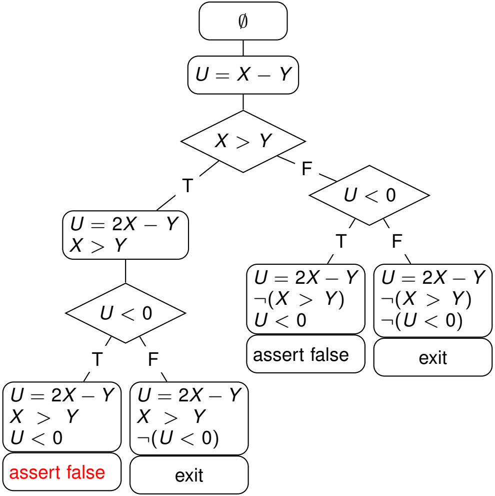

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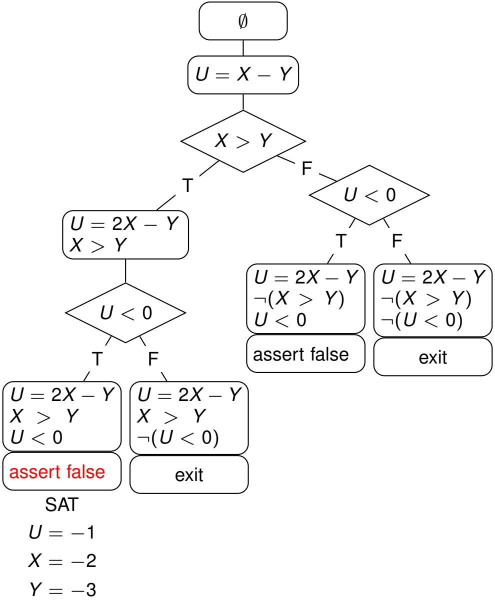

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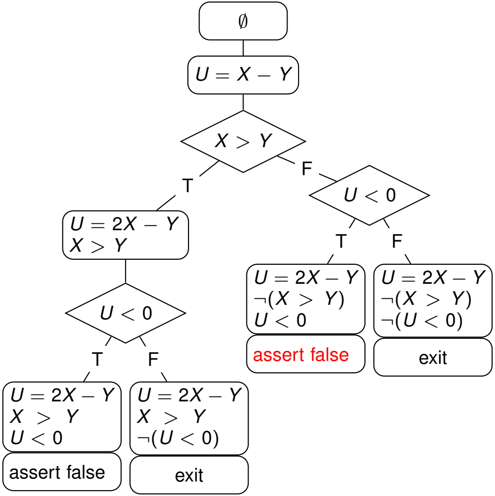

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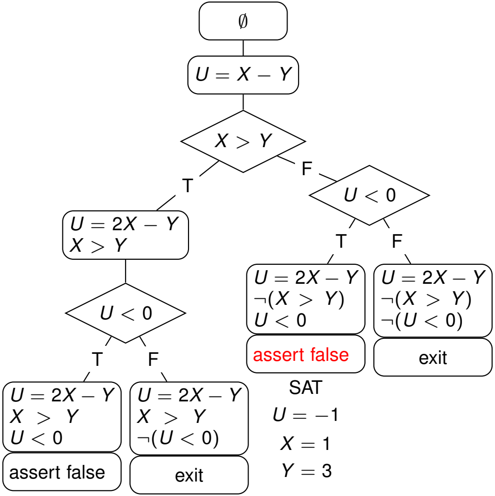

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Let $|D|$ be the size of the input domain D.

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How likely is a certain program behavior?

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Path Constraint Probability

Let $|PC_i|$ be the number of solutions to PC_i.

Let $|D|$ be the size of the input domain D.

Assuming D is uniformly distributed:

$$p(PC_i) = \frac{|PC_i|}{|D|}$$

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bool checkPIN(guess[])
for(i = 0; i < 4; i++)
 if(guess[i] != PIN[i])
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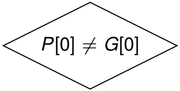
P: PIN, *G*: guess

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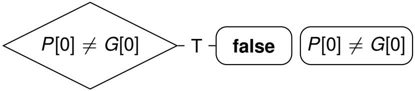


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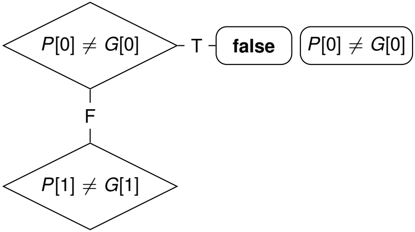


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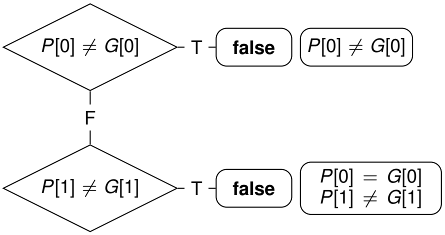

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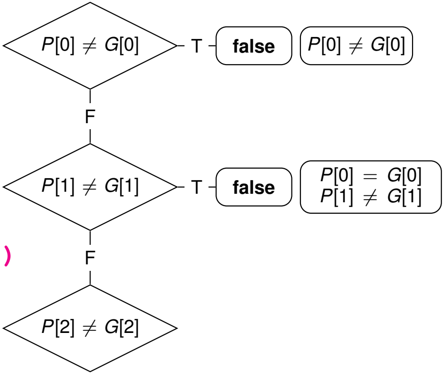

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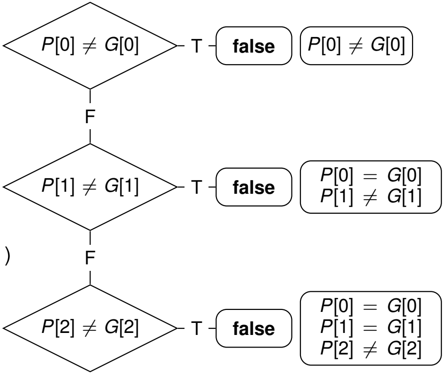

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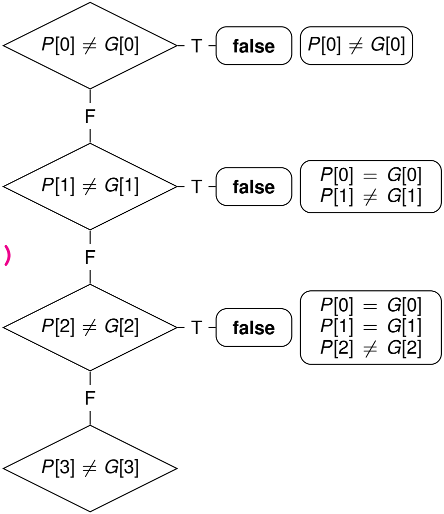

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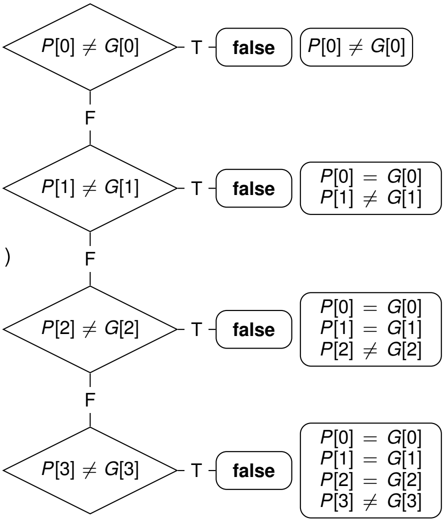

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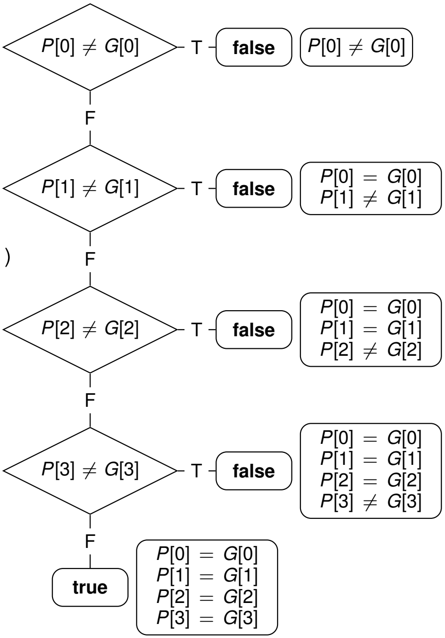

Probabilistic Symbolic Execution

i	0	1	2	3	4
PC_i	$P[0] \neq G[0]$	$P[0] = G[0]$ $P[1] \neq G[1]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] \neq G[2]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] = G[2]$ $P[3] \neq G[3]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] = G[2]$ $P[3] = G[3]$
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Probabilistic Symbolic Execution

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$$p_i = \frac{|PC_i|}{|D|}$$

Probabilistic Symbolic Execution

Assume binary 4 digit PIN. P has 4 bits, G has 4 bits. $|D| = 2^8 = 256$.

i	0	1	2	3	4
PC_i	$P[0] \neq G[0]$	$P[0] = G[0]$ $P[1] \neq G[1]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] \neq G[2]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] = G[2]$ $P[3] \neq G[3]$	$P[0] = G[0]$ $P[1] = G[1]$ $P[2] = G[2]$ $P[3] = G[3]$
$\|PC_i\|$	?????				
p_i					

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$\|PC_i\|$	128				
p_i	?????				

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$\|PC_i\|$	128				
p_i	1/2				

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$\lvert PC_i \rvert$	128	64			
p_i	1/2	?????			

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$\|PC_i\|$	128	64	32		
p_i	1/2	1/4	1/8		

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$\|PC_i\|$	128	64	32	16	
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$\|PC_i\|$	128	64	32	16	16
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$$p_i = \frac{|PC_i|}{|D|}$$

A measure of program vulnerability

Probability that an adversary can guess a prefix of length i in 1 guess is given by p_i.

Outline

Satisfiability Modulo Theories (SMT) Solvers

Problem: how to solve path constraints?

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Satisfiability Modulo Theories (SMT) Solvers

SMT solvers determine the satisfiability of formulas from combinations of theories including:

- Linear Integer Arithmetic (LIA)
- Strings
- Bitvectors
- Arrays
- Uninterpreted Functions

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- Uninterpreted Functions

Existing SMT solvers include: Z3, CVC4, MathSAT, ...

Work in SMT Solvers

- Birnbaum. The good old Davis-Putnam procedure helps counting models. JAIR 1999
- Vijay Ganesh. Decision Procedures for Bit-Vectors, Arrays and Integers(PhD. Thesis) 2007.
- Jha. Engineering an efficient SMT solver for bit-vector arithmetic. CAV 2009.
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- Davis. A Computing Procedure for Quantification Theory. JACM 1960.
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- Kroening. Decision Procedures - an algorithmic point of view. TCS 2008
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Satisfiability Modulo Theories (SMT) Solvers

Davis-Putnam-Logemann-Loveland (DPLL) Algorithm

A decision procedure for satisfiability of Boolean formulas in conjunctive normal form (CNF-SAT).

Satisfiability Modulo Theories (SMT) Solvers

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A decision procedure for satisfiability of Boolean formulas in conjunctive normal form (CNF-SAT).

This is **the core** algorithm used in SMT solvers.

Davis-Putnam-Logemann-Loveland (DPLL) Algorithm

```
Function : DPLL(φ)
Input    : CNF formula φ over n variables
Output   : true or false, the satisfiability of F
begin
  | UnitPropagate(φ)
  | if φ has false clause then return false
  | if all clauses of φ satisfied then return true
  | x ← SelectBranchVariable(φ)
  | return DPLL(φ[x ↦ true]) ∨ DPLL(φ[x ↦ false])
end
```

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Satisfiability Modulo Theories (SMT) Solvers

DPLL uses **Unit Propagation**.

$$\phi = \{x \vee y \neg x \vee z, z \vee w, x, y \vee v\}$$

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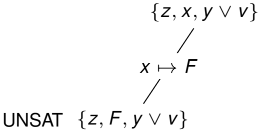
$$\phi' = \{z, x, y \vee v\}$$

DPLL Execution Example

$$\{z, x, y \vee v\}$$

DPLL Execution Example



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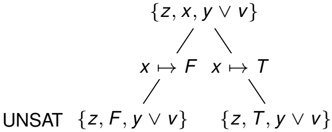

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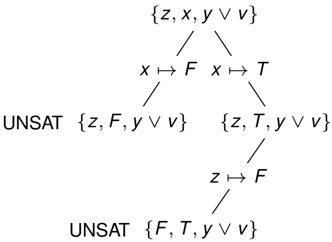

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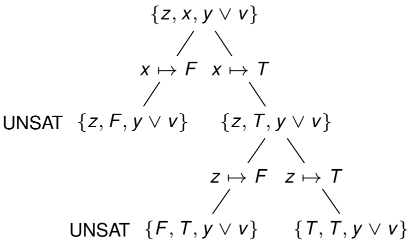

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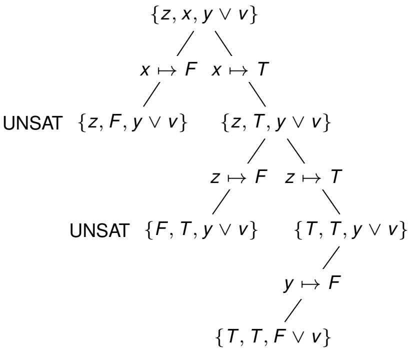

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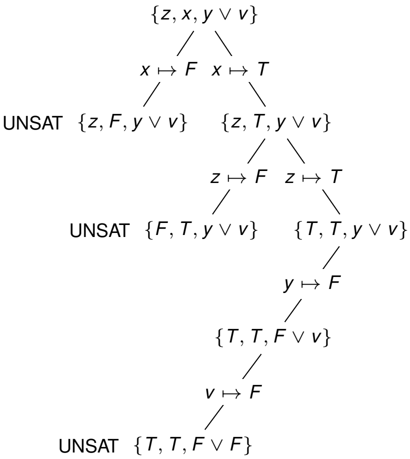

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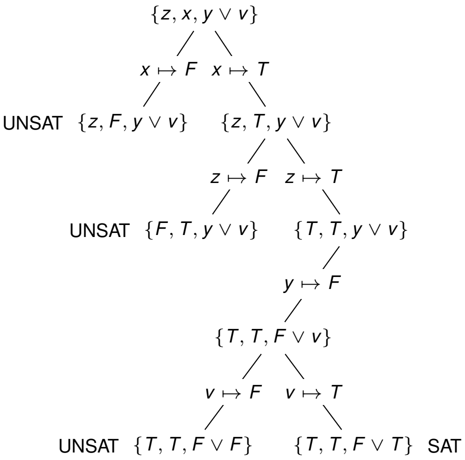

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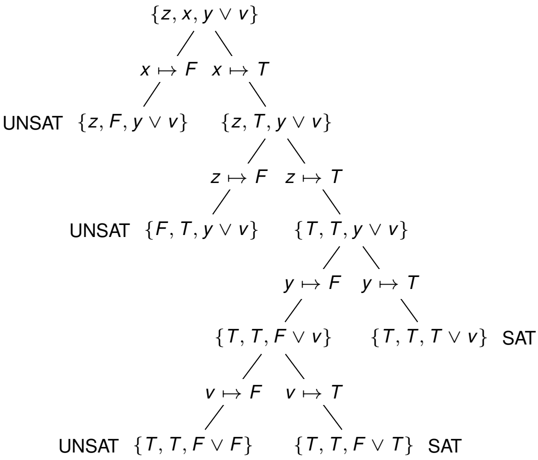

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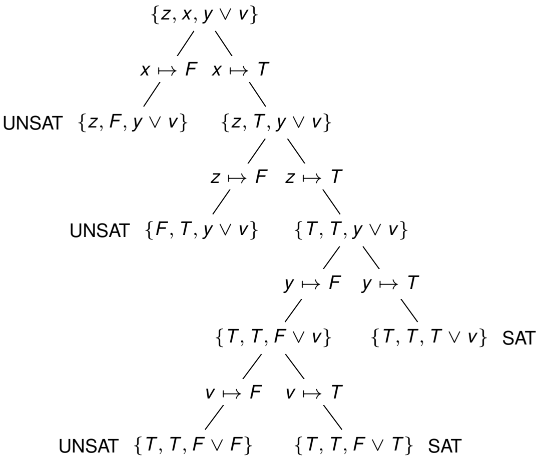

Result: ϕ is satisfiable.

Software Verification With Symbolic Execution

Symbolic Execution

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- Summarizes program executions with path constraints.

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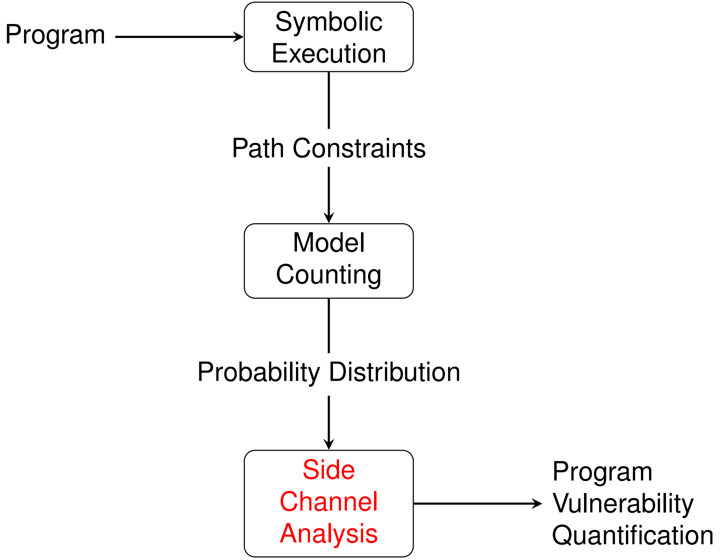
Variants of Symbolic Execution

- Standard
  - Cadar. Symbolic execution for software testing in practice: preliminary assessment. ICSE 2011
  - Cadar. Symbolic Execution for Software Testing: Three Decades Later. CACM 2013
- Probabilistic
  - Geldenhuys. Probabilistic symbolic execution. ISSTA 2012

Overview



Program
Symbolic Execution
Path Constraints
Model Counting
Probability Distribution
Side Channel Analysis
Program Vulnerability Quantification

Outline

What is a side channel?

How's the weather?

What is a side channel?

How's the weather?

Direct Channel: Go outside and look up.

What is a side channel?

How's the weather?

Direct Channel: Go outside and look up.

But, I'm too busy working on my MAE.

What is a side channel?

How's the weather?

Direct Channel: Go outside and look up.

But, I'm too busy working on my MAE.

Side Channel: Did Bo ride his bike today?

What is a side channel?

How's the weather?

Direct Channel: Go outside and look up.

But, I'm too busy working on my MAE.

Side Channel: Did Bo ride his bike today?

Learn some information through an indirect observation.

Observe Bo instead of the weather.

Side Channel Analysis

As a software verification problem

Side Channel Analysis

As a software verification problem

Verify that a program does not leak “too much” confidential information to an adversary who can observe:

- Computation time
- Power usage
- Memory allocations
- Network packet size
- Keystroke time

Side Channel Analysis

First considered at the hardware level.

```
int modPow(int num, int privatekey, int publickey)
  int s = 1, y = num, result = 0;
  while (privatekey > 0)
    if (privatekey % 2 == 1)
      result = (s * y) % publickey;
    else
      result = s;
    s = (result * result) % publickey;
    privatekey /= 2;
  return result;
```

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Side Channel Analysis

A lot of research interest

- Geoffrey Smith. On the Foundations of Quantitative Information Flow. FOSSACS 2009
- Pasquale Malacaria. Assessing security threats of looping constructs. POPL 2007
- David Clark. A static analysis for quantifying information flow in a simple imperative language. JCS (2007)
- Jonathan Heusser. Quantifying information leaks in software. ACSAC 2010: 261-269
- Quoc-Sang Phan. Symbolic quantitative information flow. ACM SIGSOFT SEN 2012
- Quoc-Sang Phan. Quantifying information leaks using reliability analysis. SPIN 2014
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Information Gain

Adversarial Model

A malicious adversary can see the observables, O.

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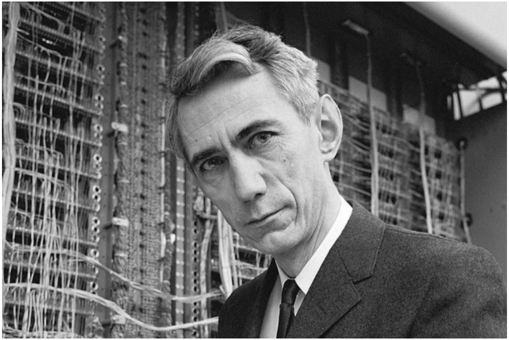
How much can the adversary learn?

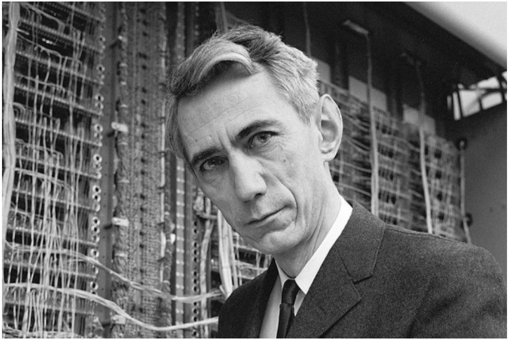
Quantify using information theory.

Information Theory



Claude Shannon

Information Theory



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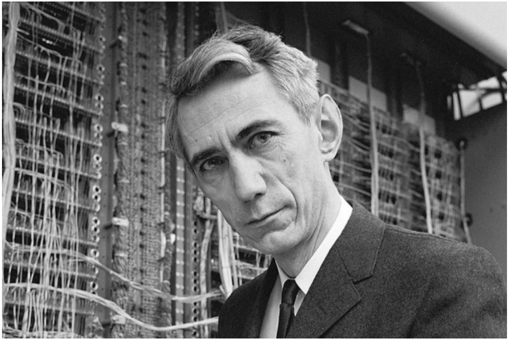

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Logarithm gives the necessary number of bits

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Outline

Software Side Channel Analysis

High Level Idea:

- Define symbolic execution observation model (o_i):

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- Quantify information gain: $H = \sum p_i \log \frac{1}{p_i}$

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bool checkPIN(guess[])
for(i = 0; i < 4; i++)
 if(guess[i] != PIN[i])
  return false
return true
```
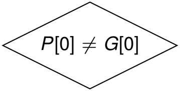

P: PIN, G: guess

$o_i =$ lines of code

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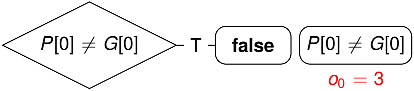


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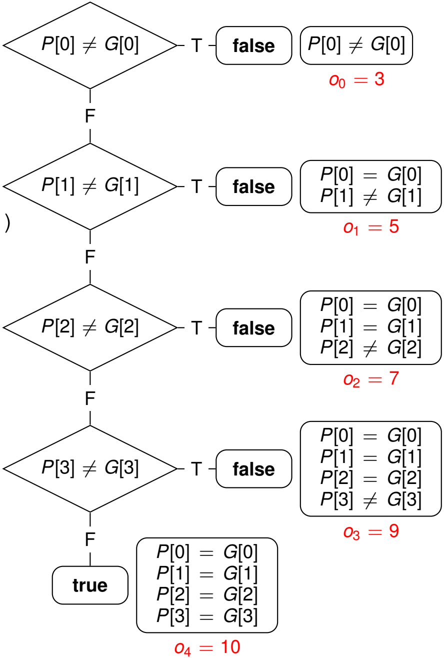

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A measure of program vulnerability

H = expected amount of information that an adversary can gain in 1 guess.

Side Channel Analysis

A more secure 4 digit PIN verification function:

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public verifyPassword (guess[])
  matched = true
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Summary

- Observe non-functional aspects of computatation to learn information.
- Probabalistic symbolic execution provides p_i, o_i
- Quantify information gain: $H = \sum p_i \log \frac{1}{p_i}$

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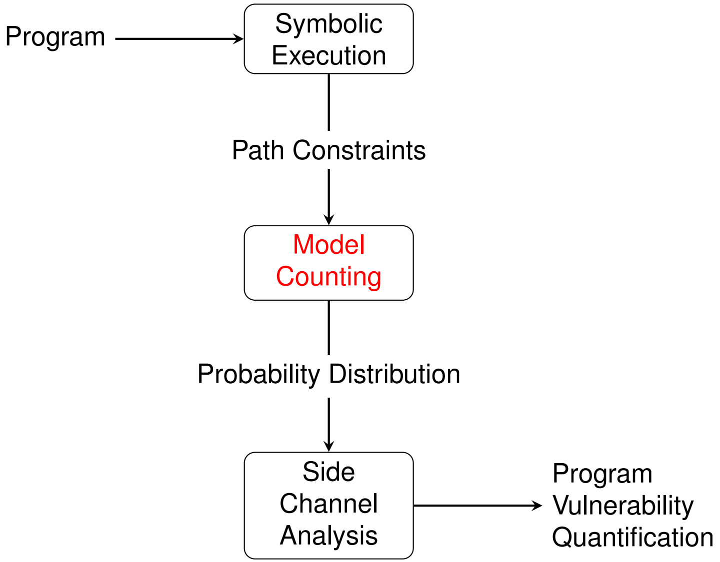
Remaining issues

- How to determine the number of solutions to path constraints?
- Path constraints for real programs could involve boolean formulas, strings, numeric constraints.

Overview



Program
Symbolic Execution
Path Constraints
Model Counting
Probability Distribution
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Program Vulnerability Quantification

Model Counting

Recall the classic (boolean) SAT problem

Given a formula ϕ from propositional logic, is it possible to assign all variables the values T (true) or F (false) so that the formula is true?

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Example:

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A satisfying assignment is called a **model** for ϕ.

Model Counting

The **model counting problem**

Given a formula ϕ over some theory (Boolean, LIA, Strings, ...)

how many models are there for ϕ?

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$$|\phi| > 0 \iff \phi \text{ is satisfiable}$$

Work on Model Counting

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Outline

Model Counting Boolean SAT

x	y	z	w	v	F
F	F	F	F	F	F
⋮	⋮	⋮	⋮	⋮	⋮
T	F	F	T	T	F
T	F	T	F	F	F
T	F	T	F	T	T
T	F	T	T	F	F
T	F	T	T	T	T
T	T	F	F	F	F
T	T	F	F	T	F
T	T	F	T	F	F
T	T	F	T	T	F
T	T	T	F	F	T
T	T	T	F	T	T
T	T	T	T	F	T
T	T	T	T	T	T

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F	F	F	F	F	F
⋮	⋮	⋮	⋮	⋮	⋮
T	F	F	T	T	F
T	F	T	F	F	F
T	**F**	**T**	**F**	**T**	**T**
T	F	T	T	F	F
T	**F**	**T**	**T**	**T**	**T**
T	T	F	F	F	F
T	T	F	F	T	F
T	T	F	T	F	F
T	T	F	T	T	F
T	**T**	**T**	**F**	**F**	**T**
T	**T**	**T**	**F**	**T**	**T**
T	**T**	**T**	**T**	**F**	**T**
T	**T**	**T**	**T**	**T**	**T**

ϕ has 6 models.

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⋮	⋮	⋮	⋮	⋮	⋮
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T	F	T	F	F	F
T	**F**	**T**	**F**	**T**	**T**
T	F	T	T	F	F
T	**F**	**T**	**T**	**T**	**T**
T	T	F	F	F	F
T	T	F	F	T	F
T	T	F	T	F	F
T	T	F	T	T	F
T	**T**	**T**	**F**	**F**	**T**
T	**T**	**T**	**F**	**T**	**T**
T	**T**	**T**	**T**	**F**	**T**
T	**T**	**T**	**T**	**T**	**T**

ϕ has 6 models.

Truth table method is $\theta(2^n)$.

Davis-Putnam-Logemann-Loveland (DPLL) Algorithm

DPLL can be converted into a procedure for $\#$CNF-SAT.

Function : DPLL(ϕ, t)
Input : CNF formula ϕ over *n* variables; $t \in \mathbb{Z}$
Output : $\#\phi$, the model count of ϕ
begin
  UnitPropagate(ϕ)
  if ϕ has false clause **then return** *false*
  if all clauses of ϕ satisfied **then return** *true*
  x $\leftarrow$ SelectBranchVariable(ϕ)
  return DPLL($\phi[x \mapsto true]$, $t-1$) $\vee$ DPLL($\phi[x \mapsto true]$, $t-1$)
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  if all clauses of ϕ satisfied **then return** 2^t
  x $\leftarrow$ SelectBranchVariable(ϕ)
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  x $\leftarrow$ SelectBranchVariable(ϕ)
  return DPLL($\phi[x \mapsto \mathit{true}], t - 1$) $+$ DPLL($\phi[x \mapsto \mathit{true}], t - 1$)
end

Counting with DPLL

$$\phi = \{x \vee y, \neg x \vee z, z \vee w, x, y \vee v\}, n = 5$$

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$x \mapsto F$

0 $\{z, F, y \vee v\}$ $t = 4$

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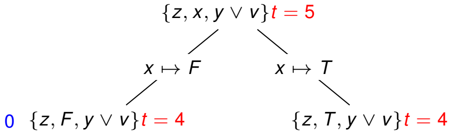

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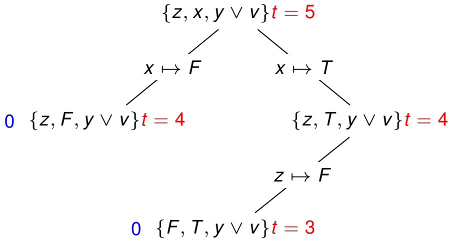

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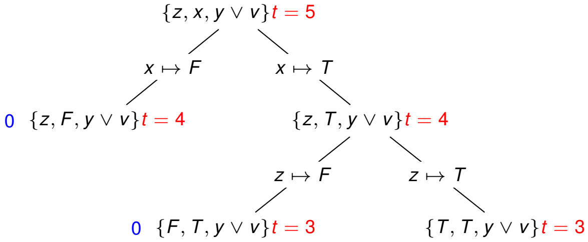

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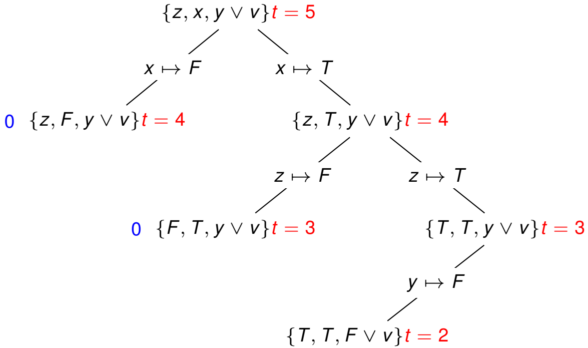

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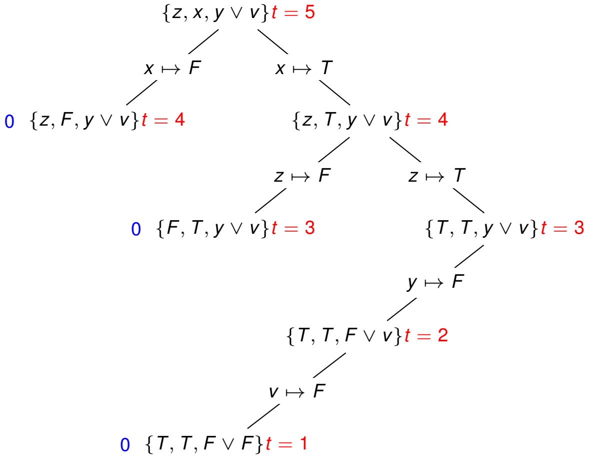

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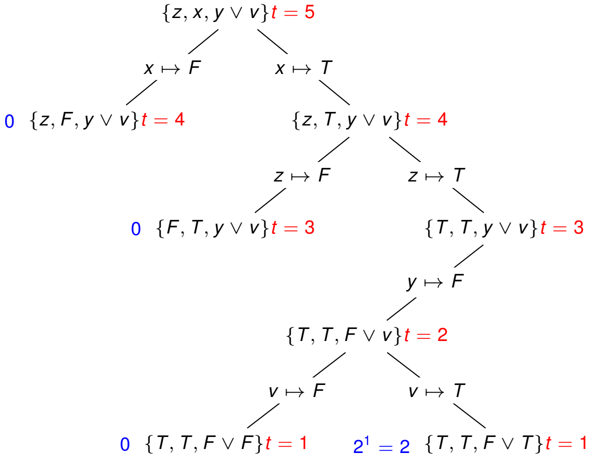

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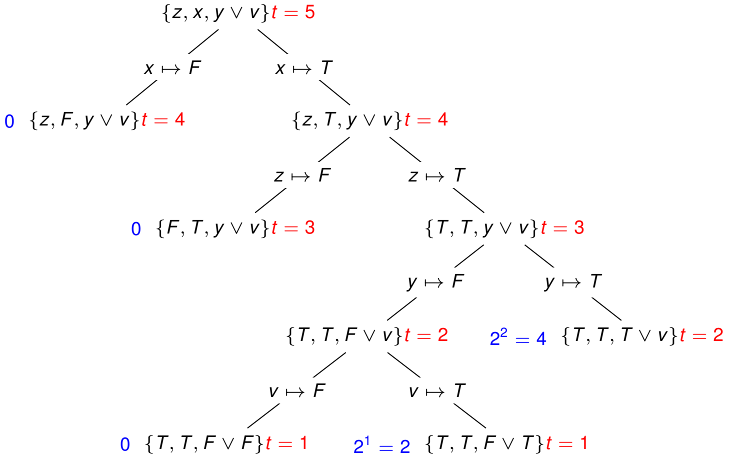

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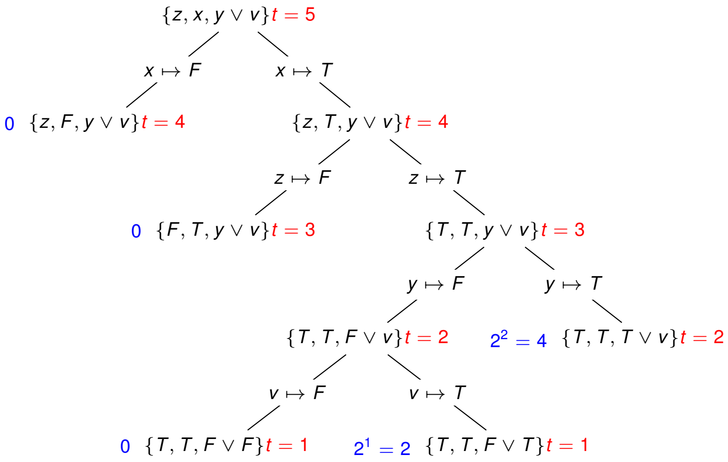

Result: $0 + 0 + 0 + 2 + 4 = 6$ models

Model Counting for Other Theories

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$$g(z) = a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + a_4 z^4 + \dots$$

Outline

Model Counting Strings

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$$X \in (0|(1(01^*0)^*1))^*$$

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A generating function for language $\mathcal{L}$ encodes

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$$g(z) = 1z^0$$

k	X	a_k
0	ε	1

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Q: How many solutions for X? A: Infinitely many!

Q: How many solutions for X of length k?

A generating function for language $\mathcal{L}$ encodes

$$a_k = |\{s : s \in \mathcal{L}, \text{len}(s) = k\}|$$

$$g(z) = 1z^0 + 1z^1$$

k	X	a_k
0	ε	1
1	0	1

Regular Expressions

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k	X	a_k
0	ε	1
1	0	1
2	11	1

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$$g(z) = 1z^0 + 1z^1 + 1z^2 + 1z^3$$

k	X	a_k
0	ε	1
1	0	1
2	11	1
3	110	1

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$$a_k = |\{s : s \in \mathcal{L}, \text{len}(s) = k\}|$$

$$g(z) = 1z^0 + 1z^1 + 1z^2 + 1z^3 + 3z^4$$

k	X	a_k
0	ε	1
1	0	1
2	11	1
3	110	1
4	1001, 1100, 1111	3

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k	X	a_k
0	ε	1
1	0	1
2	11	1
3	110	1
4	1001, 1100, 1111	3
5	10010, 10101, 11000, 11011, 11110	5

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$$\begin{array}{lcl} \varepsilon & \mapsto & 1z^0 \\ c & \mapsto & 1z^1 \\ A|B & \mapsto & A(z) + B(z) \end{array}$$

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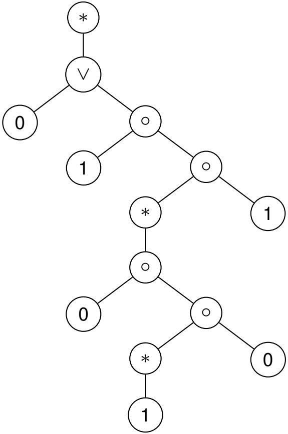

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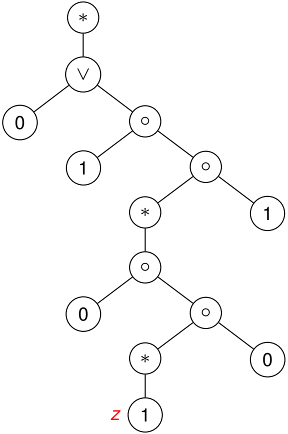

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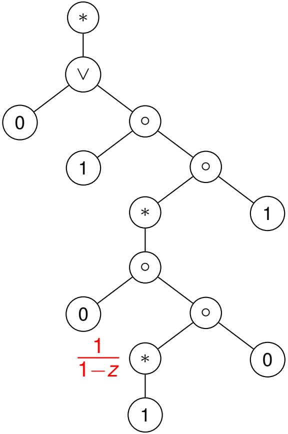

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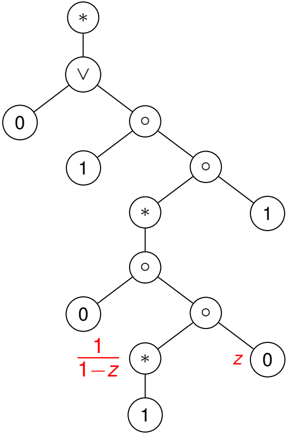

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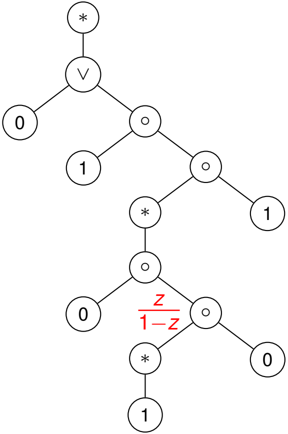

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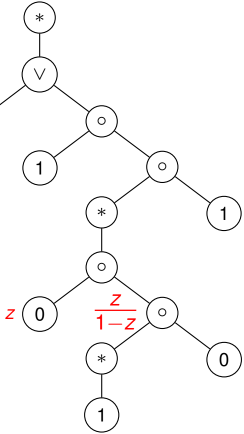

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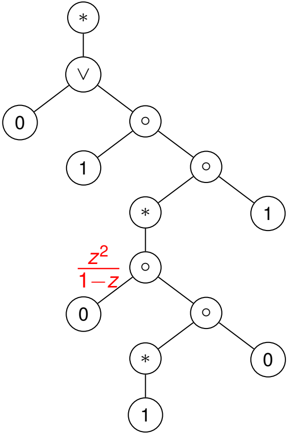

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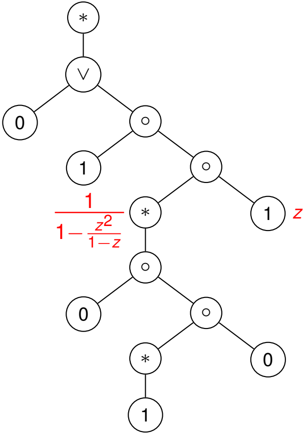

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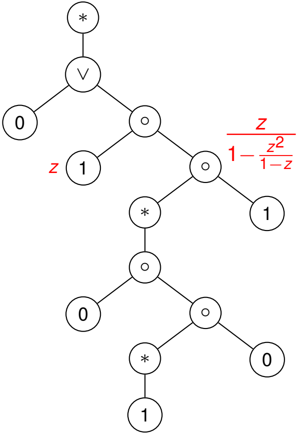

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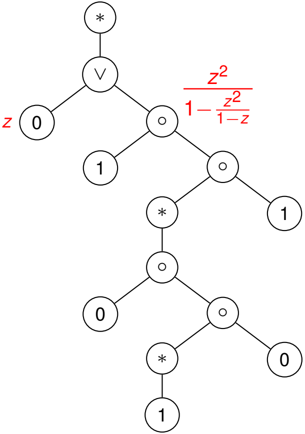

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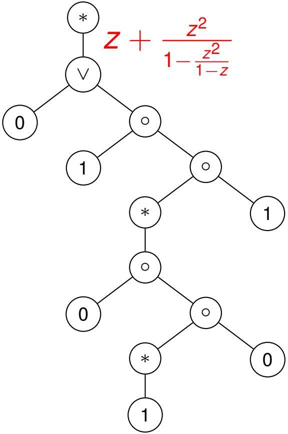

$$z + \frac{z^2}{1-\frac{z^2}{1-z}}$$

Regular Expressions

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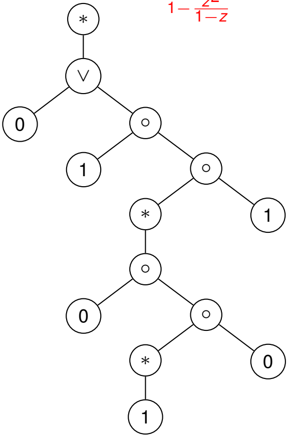
$$\frac{1}{1-z-\frac{z^2}{1-\frac{z^2}{1-z}}}$$



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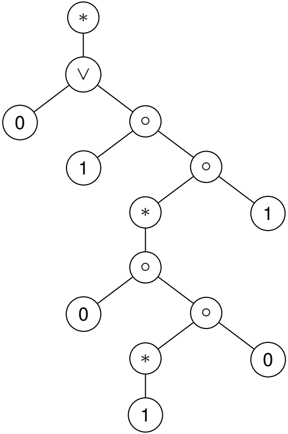

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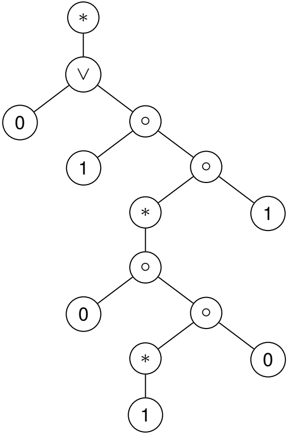

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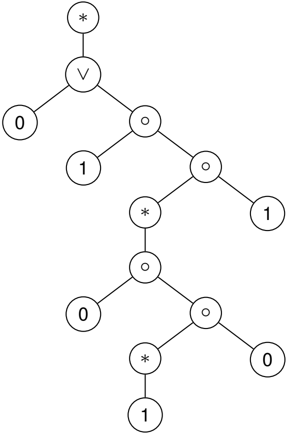

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$$g(z) = 1z^0 + 1z^1 + 1z^2 + 1z^3 + 3z^4 + 5z^5 + \dots$$

Deterministic Finite Automata

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$$X \in (0|(1(01^*0)^*1))^*$$

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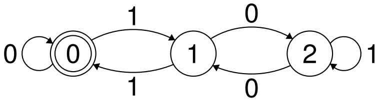

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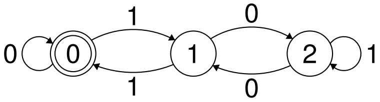

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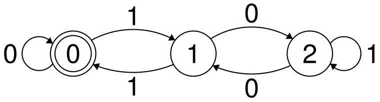

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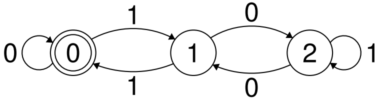
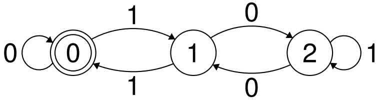
String counting $\equiv$ path counting

Deterministic Finite Automata



How to count paths of length k?

Deterministic Finite Automata



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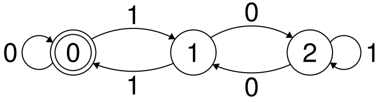
Dynamic Programming

Deterministic Finite Automata



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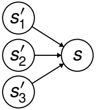
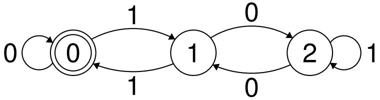


$\eta_s(k)$

Deterministic Finite Automata



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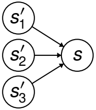

$$\eta_s(k) = \sum_{s' \to s} \eta_{s'}(k-1)$$

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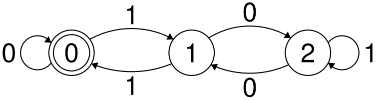
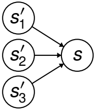


How to count paths of length k?

Dynamic Programming

Matrix Exponentiation



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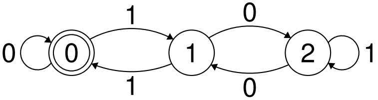

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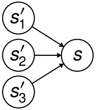

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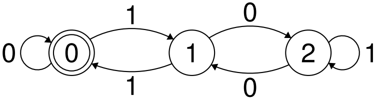

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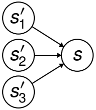

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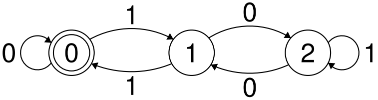

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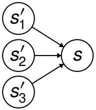

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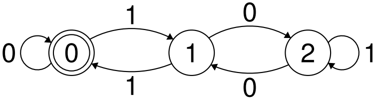

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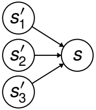

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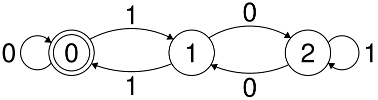
Generating Functions

Deterministic Finite Automata



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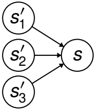

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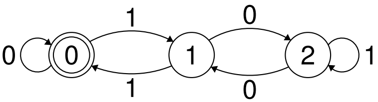

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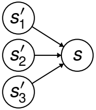

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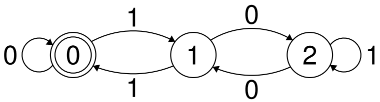

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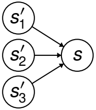

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Outline

Model Counting Linear Integer Arithmetic

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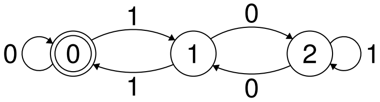

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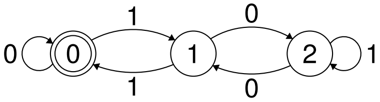

Idea: DFA can represent (some) relations on sets of binary integers. We can use similar techniques that we used for #String to solve #LIA.

Model Counting Linear Integer Arithmetic

Quantifier-Free Linear Integer Arithmetic $(\mathbb{Z}, +, <)$.

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Constraints of the form:

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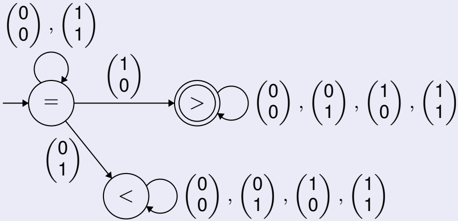
It is possible to represent the solutions to a set of LIA constraints as a binary multi-track DFA.

Binary Multi-track DFA

Solution DFA for LIA constraints.

- Read bits of x and y from most to least significant.
- Alphabet is a tuple of bits: $\begin{pmatrix} b_x \\ b_y \end{pmatrix}$

Solution DFA for the constraint $x > y$.



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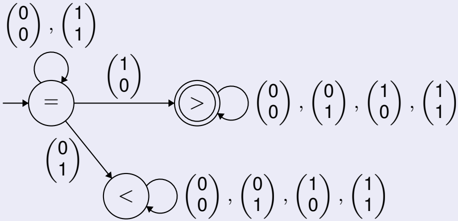
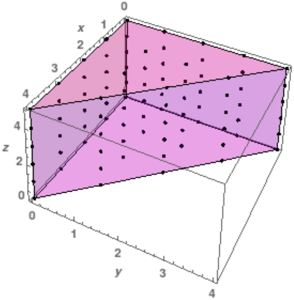


Solutions of length $n \equiv$ solutions within bound 2^n

Integer Grid Points Inside a Polytope, $\mathbb{Z}^n \cap P$



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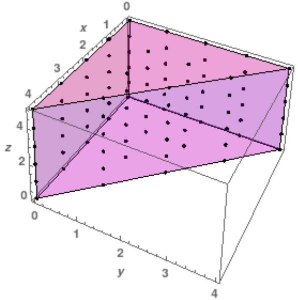

- Barvinok Algorithm
- LattE Integrale

Model Counting Summary

Counting Techniques for Different Theories

- Boolean

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  - Truth Table (Brute Force)
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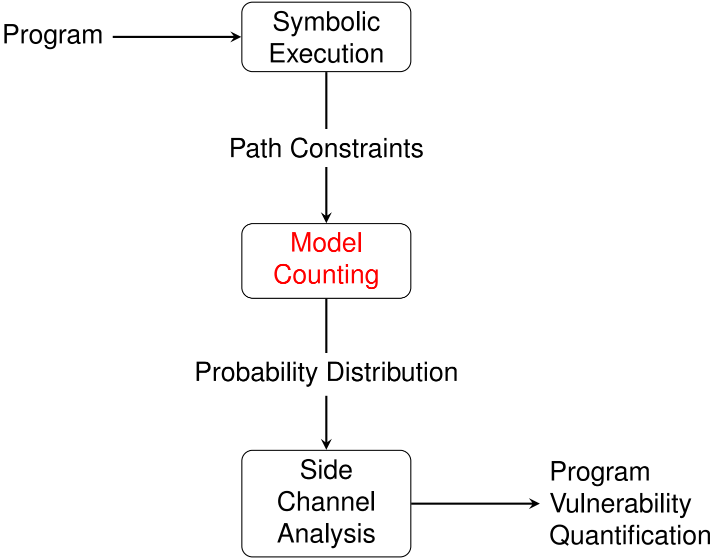
- Linear Integer Arithmetic
  - Binary Multi-track DFA
  - Polytope Methods

Review


Program
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Model Counting
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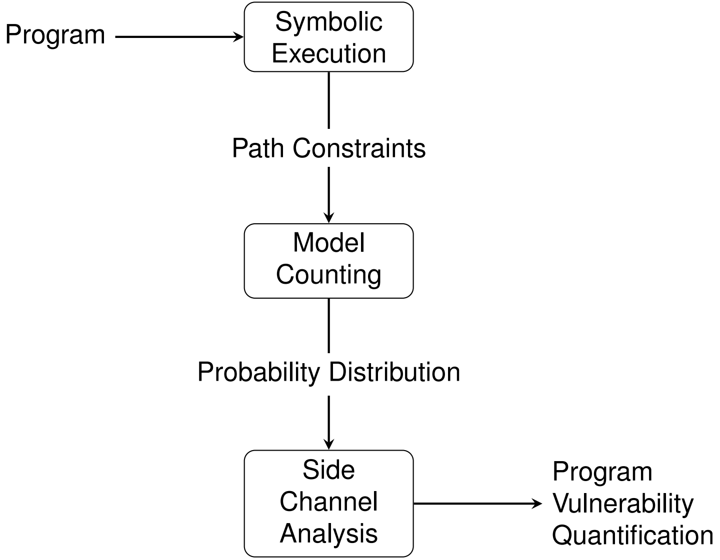
Review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My Recent Research

- CAV 2015: "Automata-based model counting for strings".
- FSE 2015: "Automatically computing path complexity of programs".
- Internship Summer 2015 Carnegie: Mellon University / NASA
  - Integration of string model counter with Java Symbolic Path Finder(SPF)
- 2015-2016: Side channel analysis using SPF.
- FSE 2016: "Side channel analysis of segmented oracles." (Submitted)

Questions?

Thank you.