Synthesis of Adaptive Side-Channel Attacks

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London E1 4NS, UK

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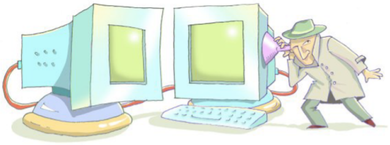
Overview



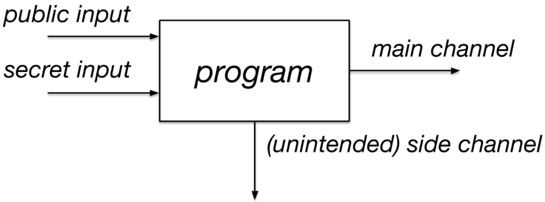


Figure: “RSA Key Extraction via Low-Bandwidth Acoustic Cryptanalysis”

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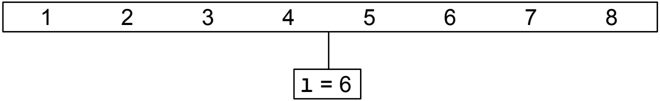

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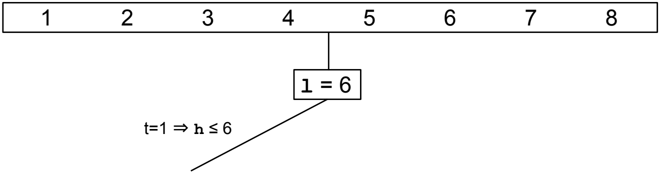

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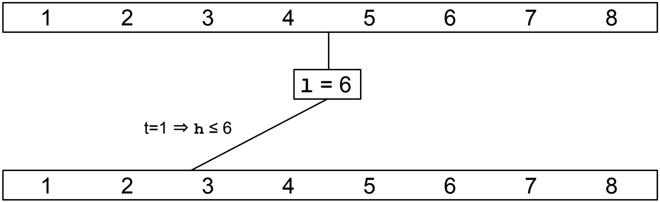

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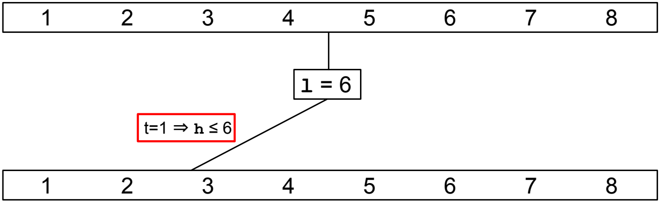

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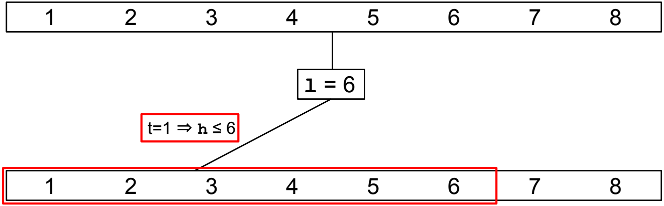

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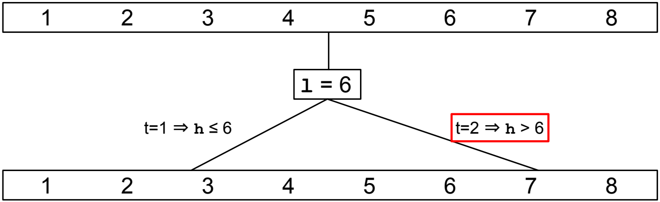

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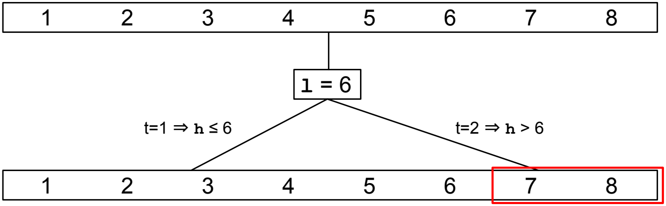

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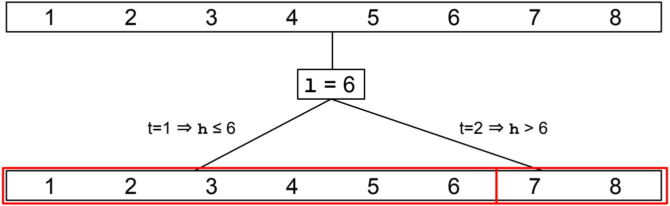

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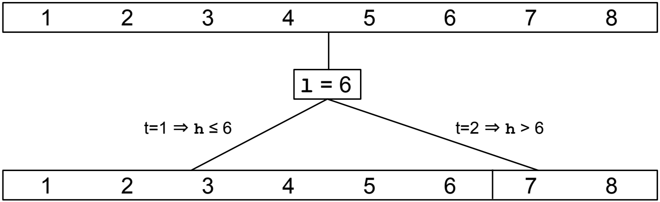

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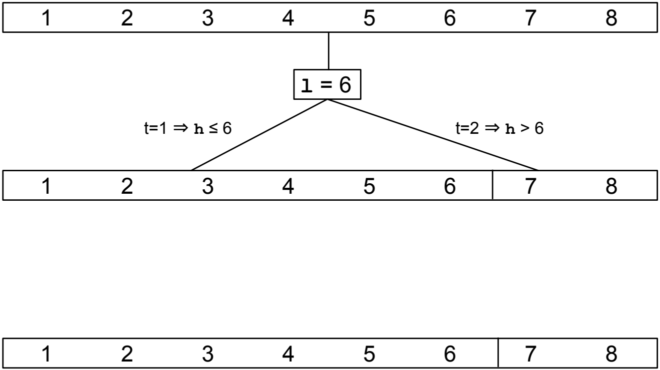

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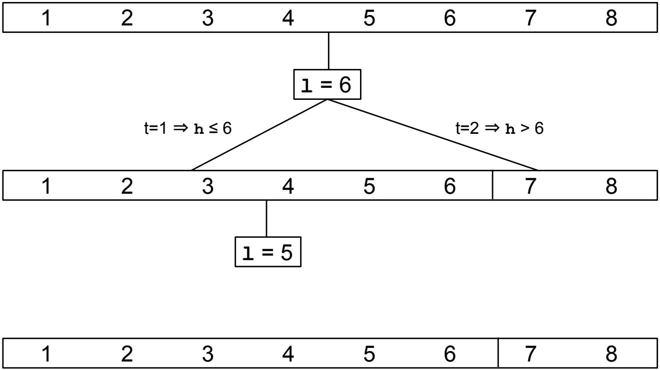

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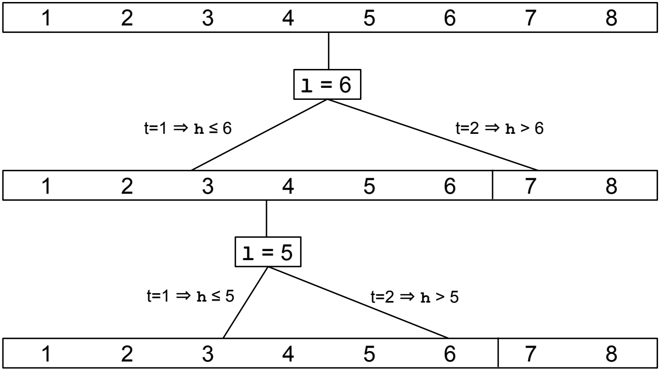

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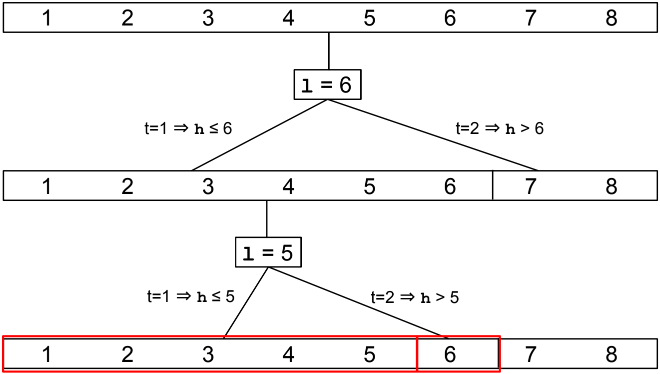

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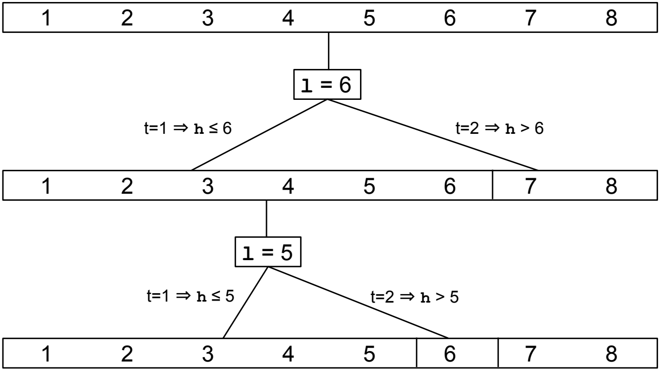

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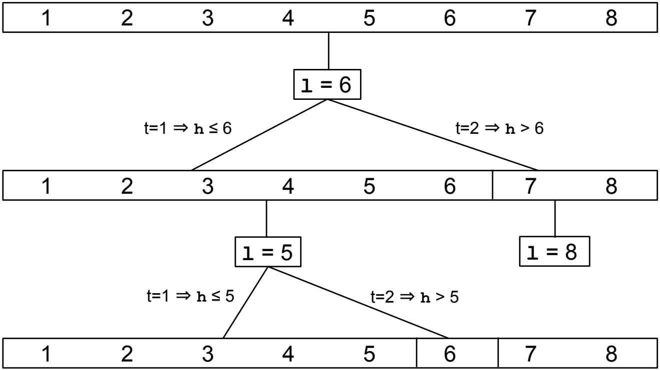

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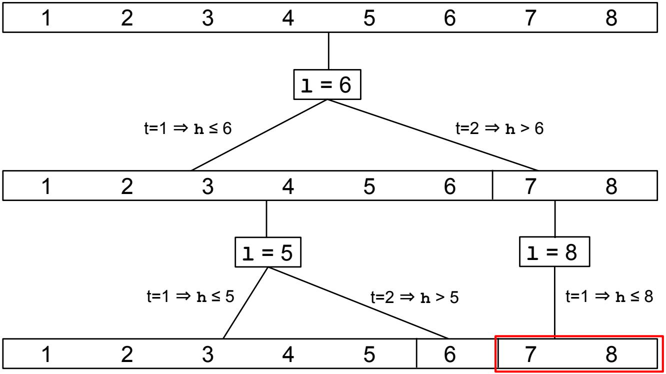

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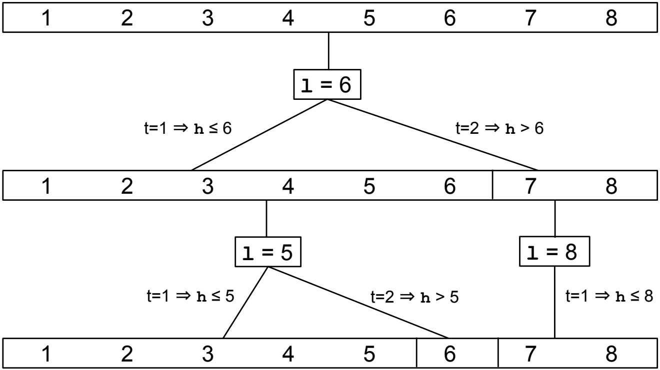

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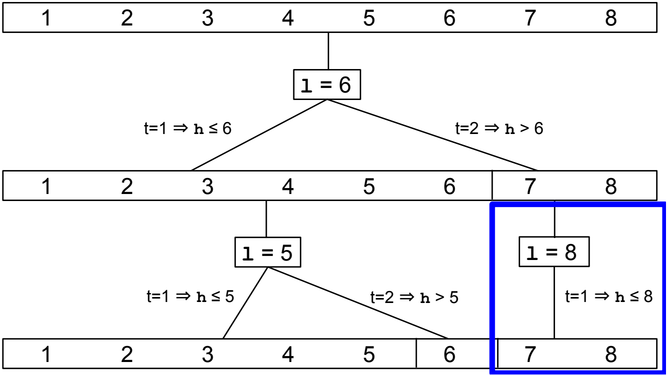

Too few divisions.

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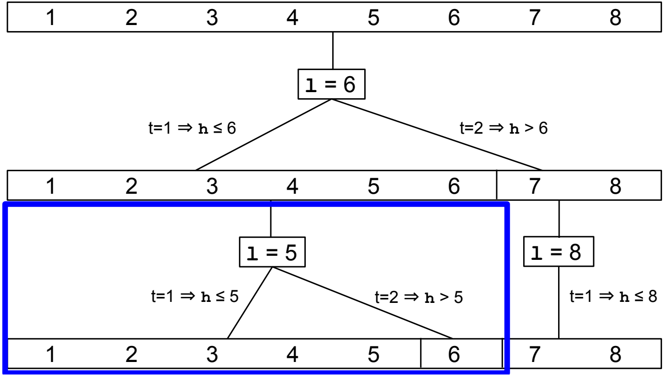

Unbalanced divisions.

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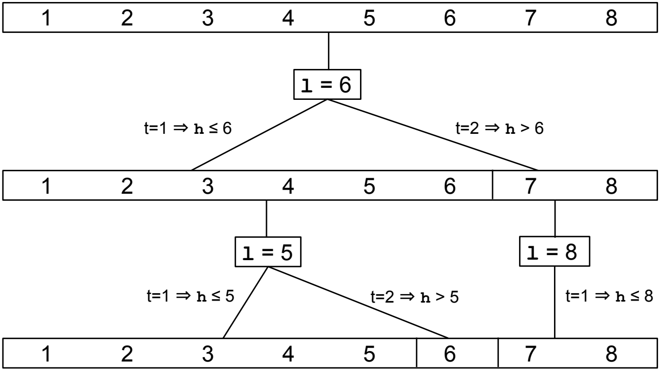

Best tree induces **maximum # divisions**

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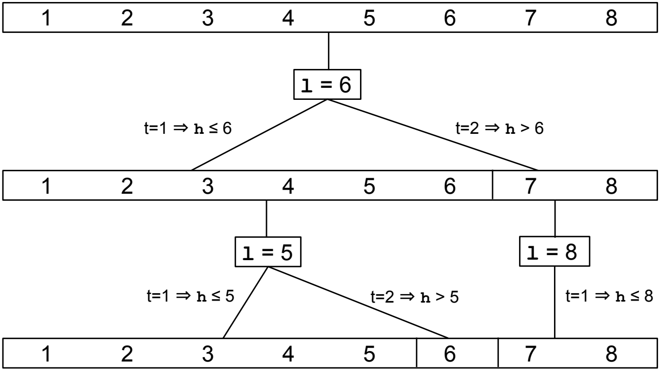

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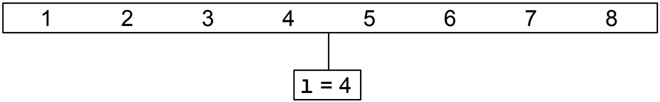

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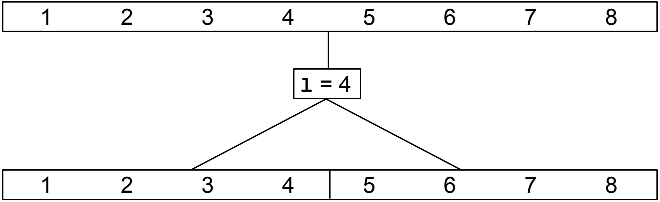

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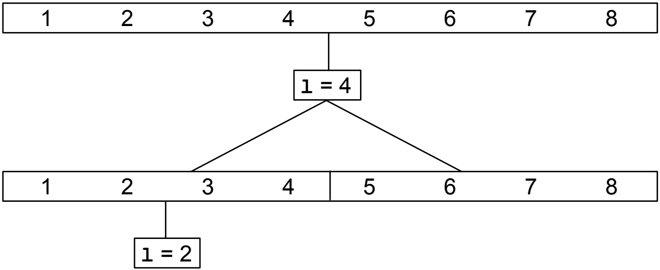

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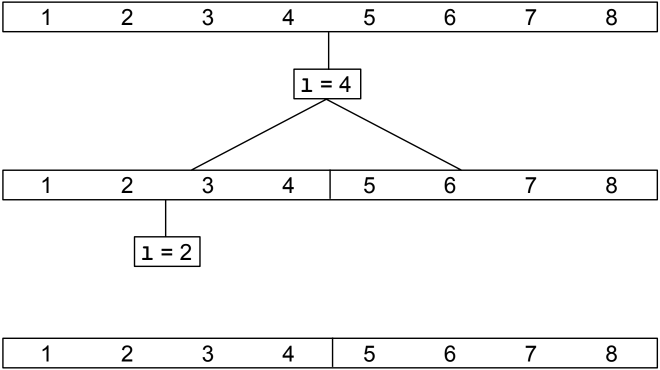

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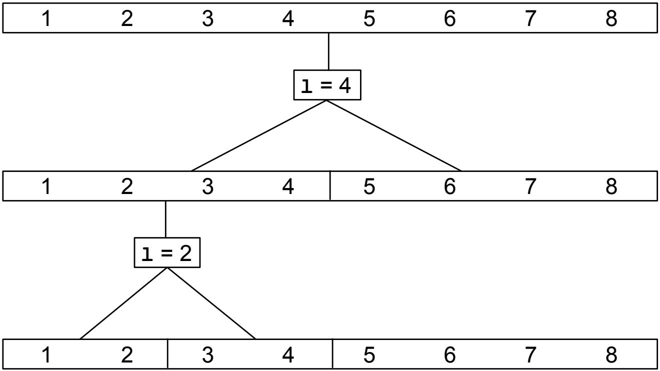

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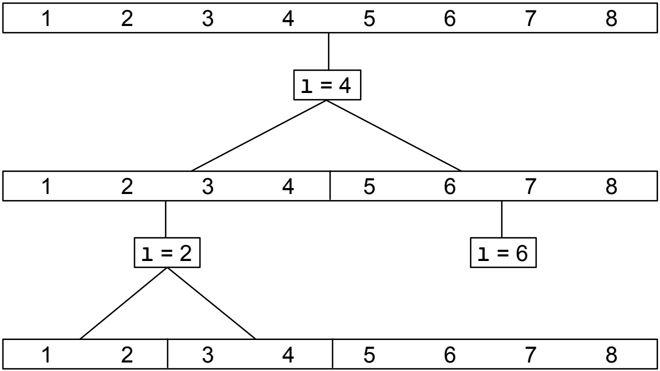

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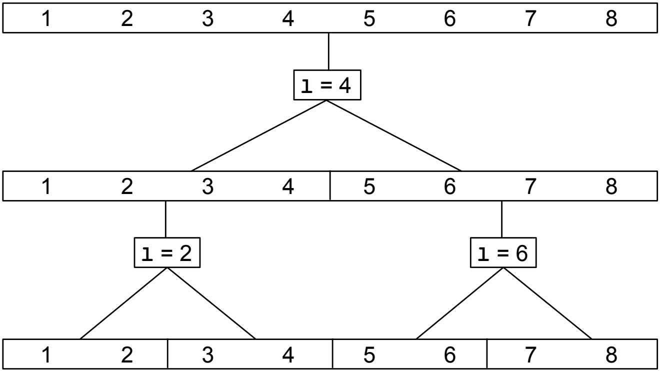

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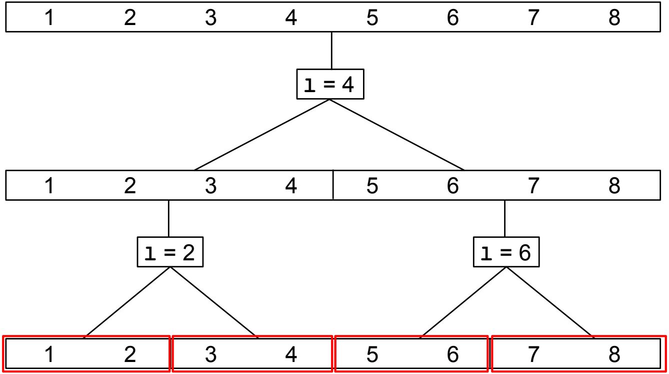

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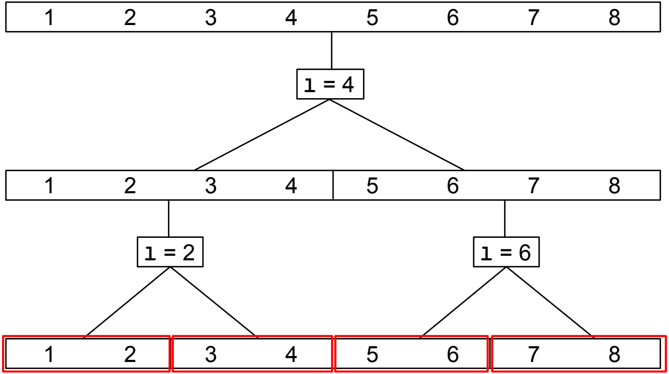

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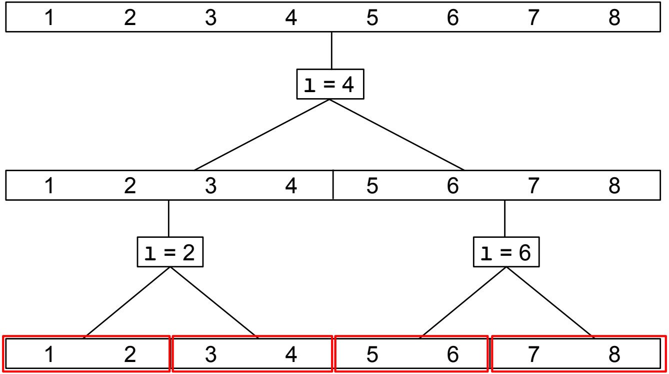

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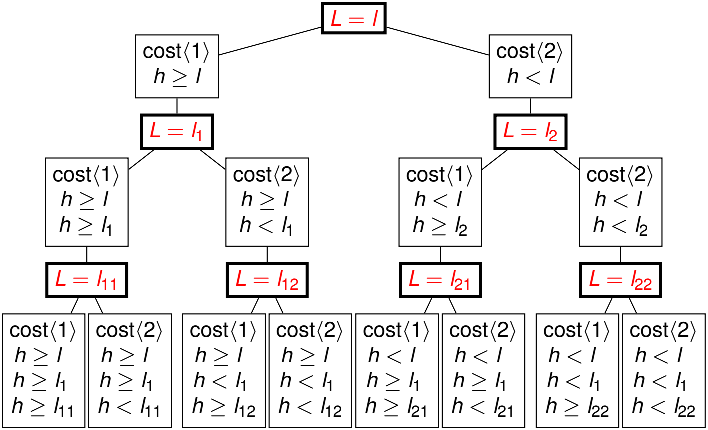

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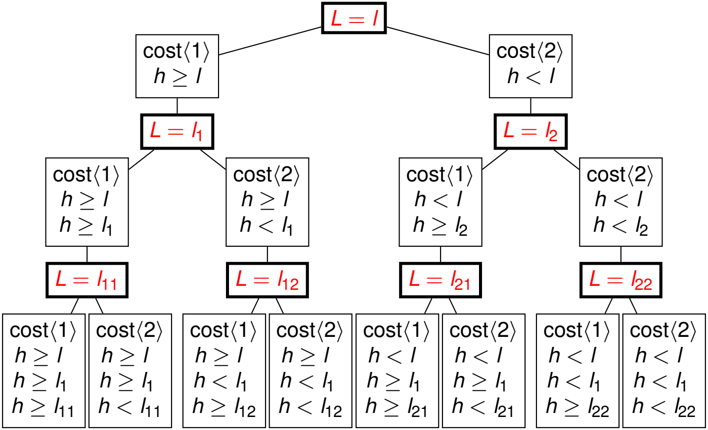

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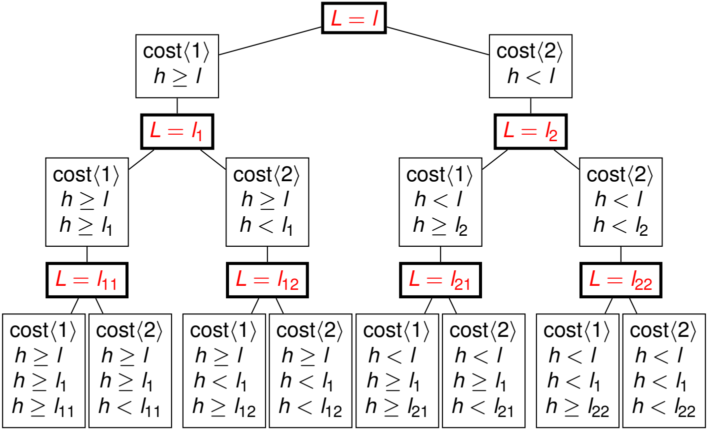

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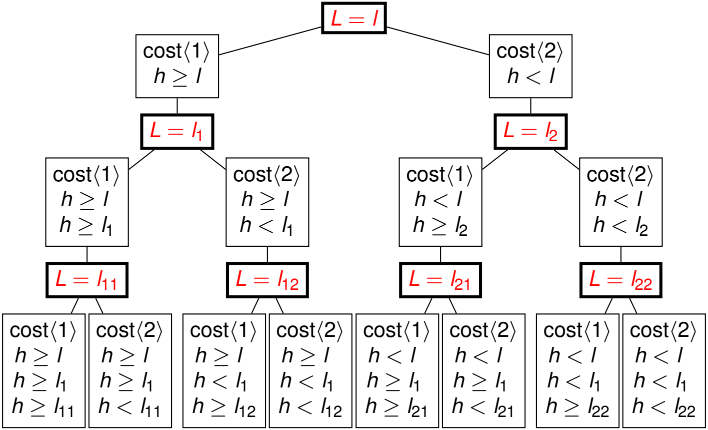

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Finding Best Attack Tree
Method 1

Maximizing Number of Partition Divisions

```
foo(int l,int h)
  if (l<0)
    if (h<0)          sleep(1)
    else if (h<5)     sleep(2)
    else              sleep(3)
  else
    if (h>1)          sleep(4)
    else              sleep(5)
```

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\begin{array}{ll}
C_1: & l < 0 \wedge h_1 < 0 \\
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- Find an assignment for l and h_i
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Max-SMT: Maximum Satisfiablity Modulo Theories

$$\begin{array}{ll} C_1\colon & l < 0 \wedge h_1 < 0 \\ C_2\colon & l < 0 \wedge h_2 \geq 0 \wedge h_2 < 5 \\ C_3\colon & l < 0 \wedge h_3 \geq 5 \\ C_4\colon & l \geq 0 \wedge h_4 > 1 \\ C_5\colon & l \geq 0 \wedge h_5 \leq 1 \end{array}$$

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- Find an assignment for l and h_i that maximizes the number of satisfiable constraints.
- Optimal choice $l = -1$.
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MAX-SMT Problem: Find an assignment of values to variables that maximizes the number of simultaneously satisfied clauses.

Finding Best Attack Tree
Method 2

Finding Balanced Partitions

Finding Balanced Partitions

Find low inputs L for an attack tree with optimally balanced divisions

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Compared with MAX-SMT:

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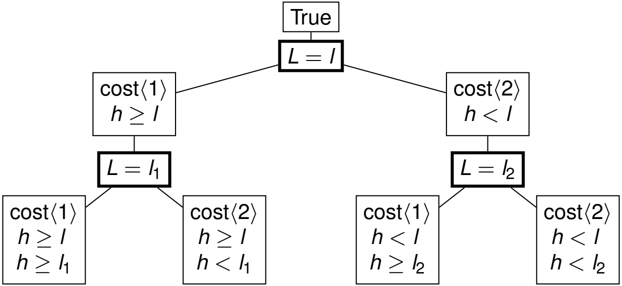
Compared with MAX-SMT:

Channel Capacity $= \log_2 \#$divisions

$$\mathcal{H} \leq CC$$

Maximizing Shannon Entropy Numerically



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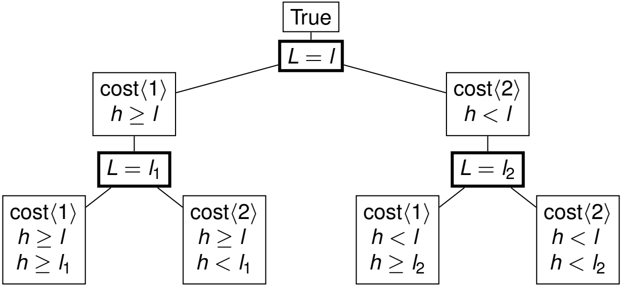

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\begin{aligned}
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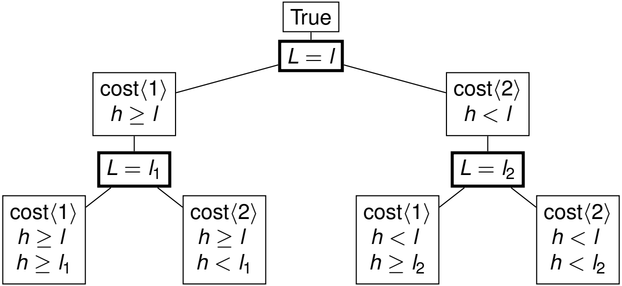

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Maximizing Shannon Entropy Numerically

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Symbolic model counting functions computed with Barvinok.

Barvinok gives piecewise multi-variate polynomial.

$$F_1(l, l_1, l_2) = \begin{cases} 6 & : l > 6 \wedge l_1 > 6 \\ l - 1 & : 1 \leq l \leq 6 \wedge l \leq l_1 \\ l_1 - 1 & : 1 \leq l_1 \leq 6 \wedge l_1 < l \end{cases}$$

$F_1(\bar{L})$ tells you the size of the partition cell for C_1, for given $\bar{L}$.

Maximizing Shannon Entropy Numerically

$C_1 = h < l \wedge h < l_1$	$F_1(\bar{l}) = \begin{cases} 8 & : l > 8 \wedge l_1 > 8 \\ l - 1 & : 1 \leq l \leq 8 \wedge l \leq l_1 \\ l_1 - 1 & : 1 \leq l_1 \leq 8 \wedge l_1 < l \end{cases}$
$C_2 = h < l \wedge h \geq l_1$	$F_2(\bar{l}) = \begin{cases} 8 & : l_1 < 1 \wedge 8 < l \\ l - l_1 & : 1 \leq l_1 \leq l \leq 8 \\ l - 1 & : l_1 < 1 \leq l \leq 8 \\ 9 - l_1 & : 1 \leq l_1 \leq 8 < l \end{cases}$
$C_3 = h \geq l \wedge h < l_2$	$F_3(\bar{l}) = \begin{cases} 8 & : l < 1 \wedge 8 < l_2 \\ l_2 - l & : 1 \leq l \leq l_2 \leq 8 \\ l_2 - 1 & : l < 1 \leq l_2 \leq 8 \\ 9 - l & : 1 \leq l \leq 8 < l_2 \end{cases}$
$C_4 = h \geq l \wedge h \geq l_2$	$F_4(\bar{l}) = \begin{cases} 8 & : l < 1 \wedge l_2 < 1 \\ 9 - l & : 1 \leq l \leq 8 \wedge l_2 < l \\ 9 - l_2 & : 1 \leq l_2 \leq 8 \wedge l \leq l_2 \end{cases}$

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$$\frac{F_1(\bar{L})}{8}$$

Maximizing Shannon Entropy Numerically

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$$\mathcal{H}(\bar{L}) = \frac{F_1(\bar{L})}{8}$$

Maximizing Shannon Entropy Numerically

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$$\mathcal{H}(\bar{L}) = \frac{F_1(\bar{L})}{8} \log_2 \frac{8}{F_1(\bar{L})} + \frac{F_2(\bar{L})}{8} \log_2 \frac{8}{F_2(\bar{L})} + \frac{F_3(\bar{L})}{8} \log_2 \frac{8}{F_3(\bar{L})} + \frac{F_4(\bar{L})}{8} \log_2 \frac{8}{F_4(\bar{L})}$$

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Numerically maximize $H(\bar{L})$

$$\bar{L} = \langle 4, 2, 6 \rangle$$

Maximizing Shannon Entropy Numerically

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First two steps of optimal binary search attack on 8 secrets.

Finding Best Attack Tree Method 3

Maximizing Shannon Entropy, Third Approach

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Maximum Satisfiable Subsets (MSS).

Optimization version of SAT.

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MaxH-MARCO algorithm:

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MSS solution $\Rightarrow$ maximize Shannon entropy.

Finding Best Attack Tree

Finding Best Attack Tree
3 Methods

Finding Best Attack Tree
3 Methods

Do they work?

Finding Best Attack Tree
3 Methods

Do they work?

Yes

Implementation

- Java Symbolic Pathfinder (JPF / SPF) for symbolic execution.
- Specialized listeners for tracking observables (time, space).
- Latte and Barvinok for model counting path constraints.
- Max-SMT (Z3), MARCO (java + Z3) MSS.
- Mathematica's NMaximize for numeric maximization.
- Heuristics: top-down greedy optimization.

Case study: Law Enforcement Employment Database

From DARPA Space-Time Analysis for Cybersecurity (STAC)

Server

- 41 classes, 2844 line of code.
- stores all employee records by ID in a database.
- Some employee IDs have restricted access.

Client

Commands available for users: SEARCH, INSERT, GET, PUT, . . .

`SEARCH a b` has a timing channel: adaptive range query attack.

Case study: Law Enforcement Employment Database

Domain: 100 possible IDs in database (6.541 bits)

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MAX-SMT

- Attack tree depth: 17 (complete attack)
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Numeric Entropy Maximization

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Max SAT Subsets

- Attack tree depth: 7 (complete attack)
- Running time: 2m 36s

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Numeric Entropy Maximization

- Attack tree depth: 11
- Incomplete attack: leaks 10.0 out of 19.9 bits
- Running time: 15m 8s

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- Attack tree depth: 11
- Incomplete attack: leaks 10.0 out of 19.9 bits
- Running time: 15m 8s

Max SAT Subsets

Does not scale to this domain.

More Case Studies

We synthesized attacks for:

- ModPow used in RSA
- Compression Ratio Information Leak Made Easy (CRIME)
- java.util.Arrays.equal() (segment oracle attack)

Conclusions

- Symbolic exection of adversary model to get constraint tree.
- Solve optimization problem to get low inputs to maximize leakage: attack tree.
- MAX-SMT
  Symbolic Model Counting + Numeric Maximization
  Max-SAT-Subsets
- Experimentally validated our approach.

Questions?

Thank you.