String Analysis for Side Channels with Segmented Oracles

Lucas Bang[1] , Abdulbaki Aydin[1] , Quoc-Sang Phan[2] ,
Corina S. Păsăreanu[2,3] , Tevfik Bultan[1]

[1]University of California, Santa Barbara
Santa Barbara, CA, USA

[2]Carnegie Mellon University
Moffet Field, CA, USA

[3]NASA Ames Research Center
Moffet Field, CA, USA

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Overview

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Program

(Segmented Oracle)

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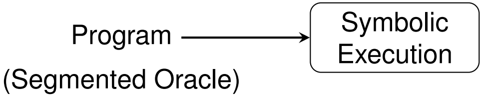

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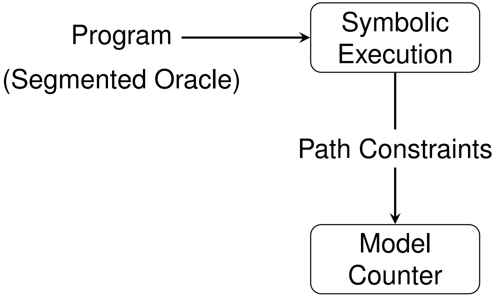
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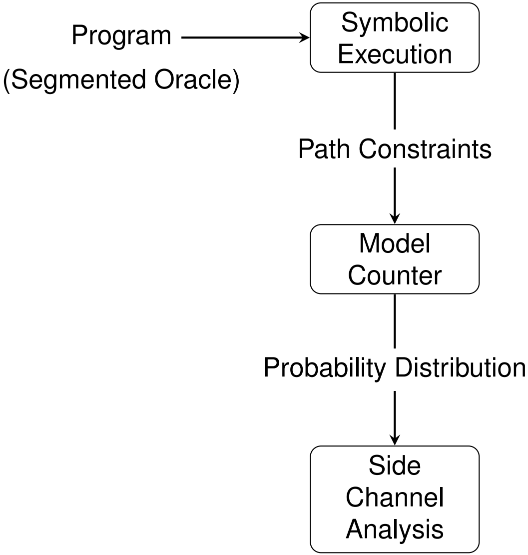
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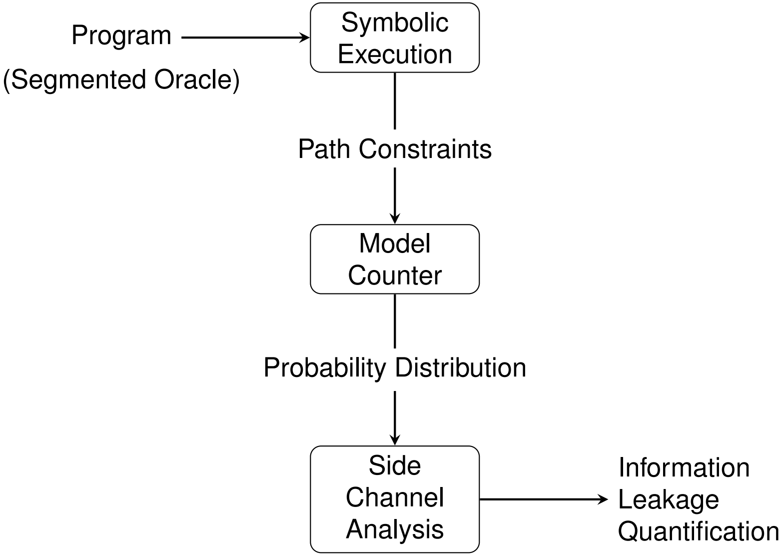
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- **side channels** that reveal information about
- **segments** (single characters, bytes, bits, array slice) of a
- **secret** program value.

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passcheck(char[] pw, char[] guess)
  for (int i = 0; i < length; i++)
    if (pw[i] != guess[i]) return false
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Using the program main channel (true, false), and brute force needs

$$(\text{alphabet size})^L = (128\ \texttt{ASCII chars})^L$$

guesses in the worst case $=$ thousands of years.

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- 1 observable time unit = 1 loop execution.
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	seaaaaaaaaaaaa	false	3 loops

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	seatacaaaaaaaa	false	7 loops

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Using the program timing channel, adversary needs

$$\text{(alphabet size)} \times L = (128) \times 15 \text{ guesses} = \text{a few seconds.}$$

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Goal: quantify information leakage for these types of vulnerabilties.

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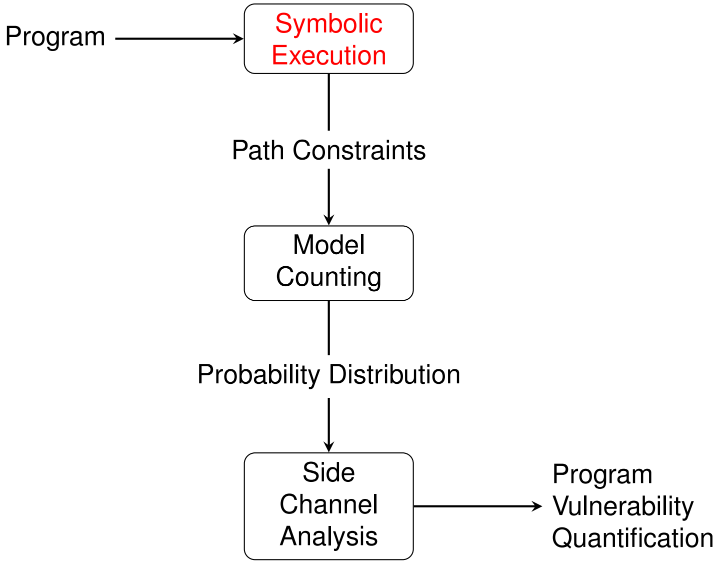

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P: pw, G: guess

$o_i =$ lines of code

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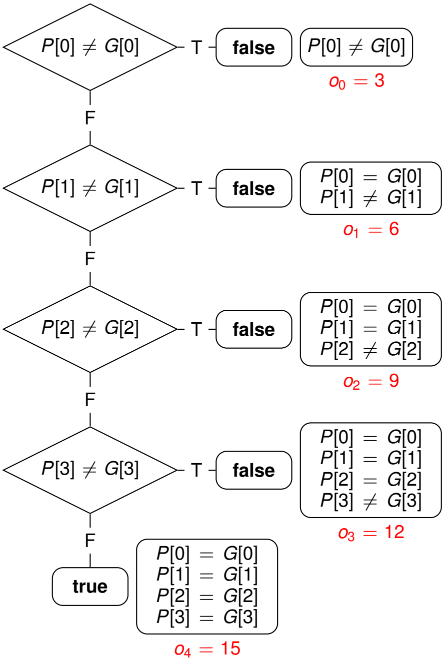

Segmented Oracle Path Constraints Pattern

$$(o_i, PC_i) : P[0] = G[0] \ldots \wedge P[i-1] = G[i-1] \wedge P[i] \neq G[i]$$

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A criterion for segmented oracles: path constraints grouped by observable are logically equivalent to this pattern (up to reordering).

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Adversary learns more with multiple invocations.

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Symbolic execution of *S*: **all possible** observable sequences.

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What is the the probability of a particular program execution path?

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How do you compute the number of solutions $|PC|$ automatically?

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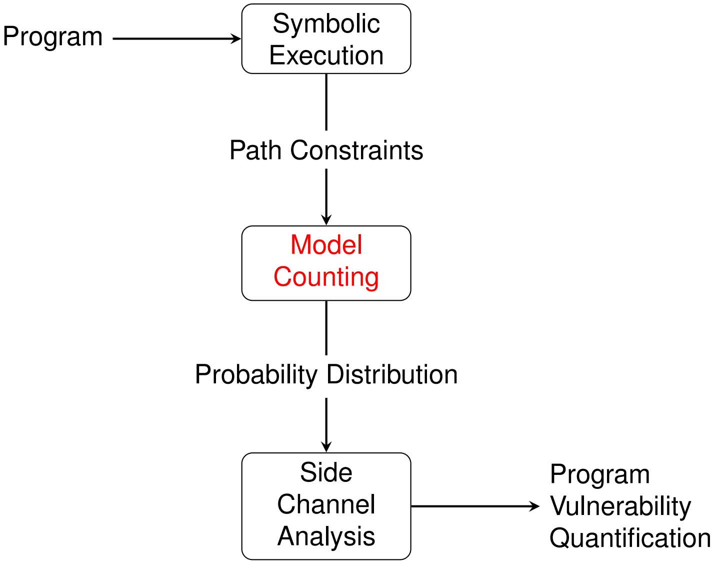
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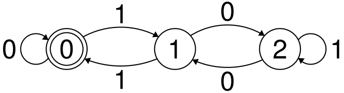

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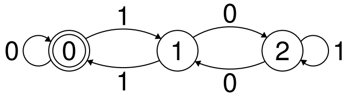

- $|PC|$ is number of accepting paths in automaton.

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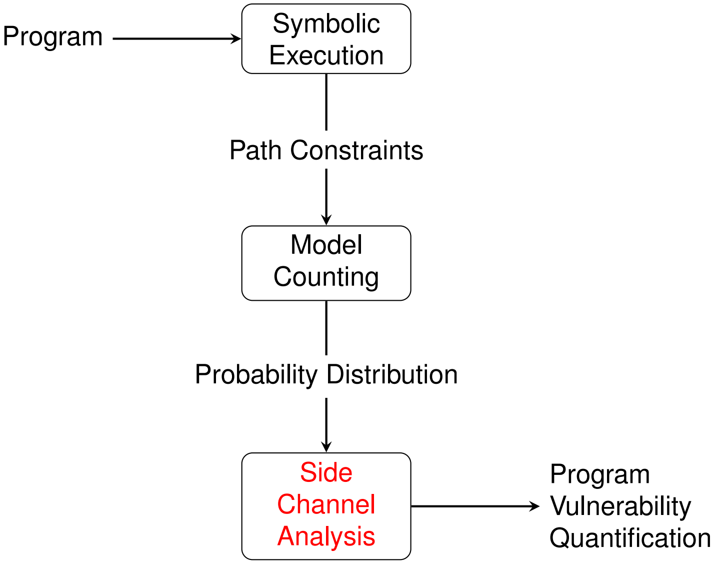

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Eventually, $H = 0$, no uncertainty, secret revealed.

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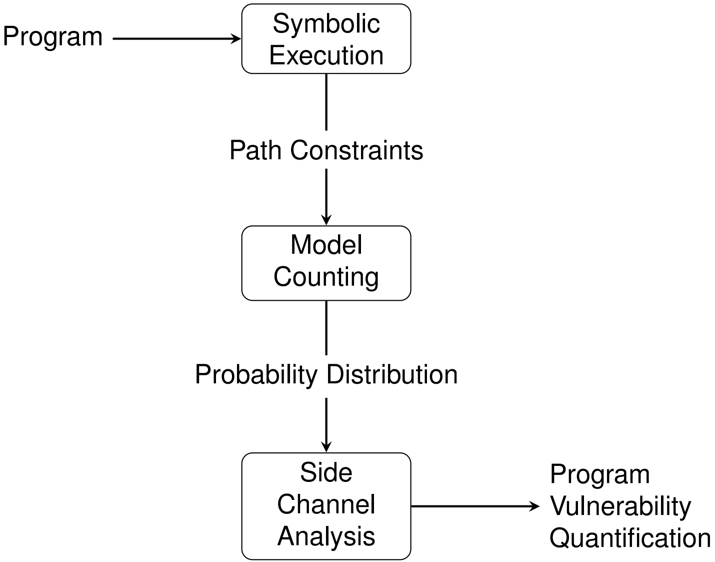
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Derive recurrence relating segment sizes $|D_i|$ to $|PC_i|$:

$$\begin{cases} \prod |\mathbf{D}| = |PC_n| \\ \prod |\mathbf{D}| \cdot (|D_i| - 1) \cdot \prod |\mathbf{D}|_{i+1:n-1} = |PC_i| \end{cases}$$

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and probablity recurrence:

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Efficiently compute $p(\overrightarrow{o})$ using standard dynamic programming and memoization techniques.

Implementation

- Java Symbolic Pathfinder (JPF / SPF), symbolic execution.
- Specialized listeners for tracking observables.
- ABC and Latte for model counting path constraints.
- SPF packages to quantify information leakage.

Experiments

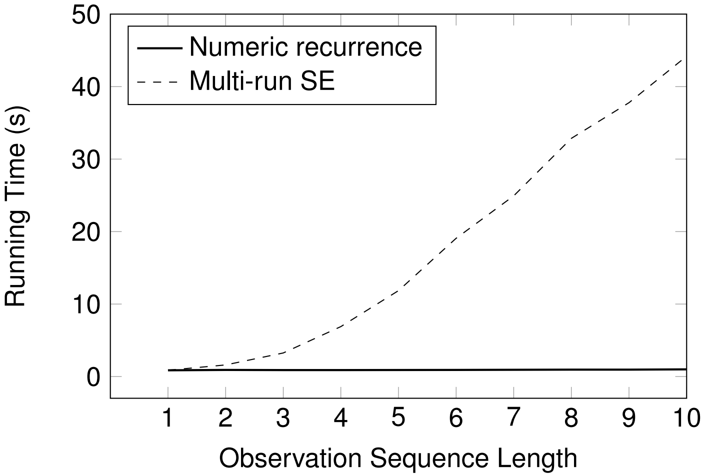

Figure : Time for multi-run and single-run SE.

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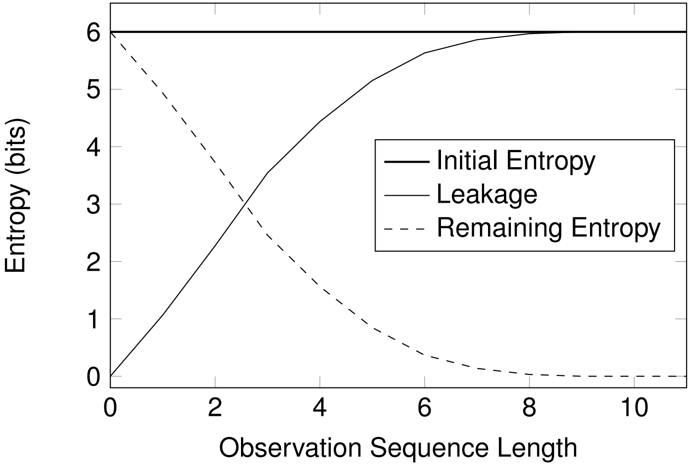

Figure : Information leakage and remaining entropy for password checking function. Length = 3, alphabet size = 4.

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- Leaks all information after 10 executions by the adversary.
- Running time: 8.695 seconds

Conclusions

In this talk:

- Segmented oracles.
- Multi-run symbolic exection of adversary model to get leakage.
- Infer multi-run leakage from a singel run of symbolic execution.
- Model counting for string manipulating programs.
- Experimentally validated our appraoch.

Future work:

- Extend analsysis to more general oracles.
- Incorporate model of system noise.
- Automatically generate adversary strategies.

Closing Remark

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"Premature optimization is the root of all evil." -Tony Hoare

Important tradeoff: efficiency vs. security.

Important problem to address: we need tools for automatically measuring this tradeoff.

Questions?

Thank you.

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procedure $S = (A_B, F)$

vars

s: the current segment of h being searched
b: the first time s is searched
$o^0, o^1, \ldots o^k$: observations of the adversary

begin

$s \leftarrow 1$, $b \leftarrow 1$, $o^0 \leftarrow 0$
for all $i \in [1..k]$ {
  for all $j \in [b..i)$ { ***assume*** $(l^i[s] \neq l^j[s])$ }
  $o^i \leftarrow F(h, l^i)$
  if $(o^i = |h|)$ { **return** }
  if $(o^i > o^{i-1})$ {
    for all $j \in [i+1..k]$ {
      for all $n \in [s..o^i]$ { ***assume*** $(l^j[n] = l^i[n])$ }
    }
    $s \leftarrow o^i + 1$, $b \leftarrow i + 1$
  }
}

end

Information Theory Intuition

Information Entropy:

$$H = \sum p_i \log \frac{1}{p_i}$$

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$$H = \sum p_i \log \frac{1}{p_i} = E\left[\log \frac{1}{p_i}\right]$$

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Seattle Weather, Always Raining
$p_{rain} = 1, p_{sun} = 0$

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Seattle Weather, Always Raining

$p_{rain} = 1, p_{sun} = 0$ $\qquad H = 0$

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Seattle Weather, Always Raining
$p_{rain} = 1, p_{sun} = 0$ $\qquad H = 0$

Costa Rica Weather, Coin Flip
$p_{rain} = \frac{1}{2}, p_{sun} = \frac{1}{2}$

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The expected amount of **"surprise"**.

Seattle Weather, Always Raining

$p_{rain} = 1, p_{sun} = 0$ $\quad$ $H = 0$

Costa Rica Weather, Coin Flip

$p_{rain} = \frac{1}{2}, p_{sun} = \frac{1}{2}$ $\quad$ $H = 1$

Information Theory Intuition

Information Entropy:

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The expected amount of information gain.
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Seattle Weather, Always Raining
$p_{rain} = 1, p_{sun} = 0$ $\qquad H = 0$

Costa Rica Weather, Coin Flip
$p_{rain} = \frac{1}{2}, p_{sun} = \frac{1}{2}$ $\qquad H = 1$

Santa Barbara Weather, Almost Always Sunny.
$p_{rain} = \frac{1}{10}, p_{sun} = \frac{9}{10}$

Information Theory Intuition

Information Entropy:

$$H = \sum p_i \log \frac{1}{p_i} = E\left[\log \frac{1}{p_i}\right]$$

The expected amount of information gain.
The expected amount of **"surprise"**.

Seattle Weather, Always Raining

$p_{rain} = 1, p_{sun} = 0$ $\quad H = 0$

Costa Rica Weather, Coin Flip

$p_{rain} = \frac{1}{2}, p_{sun} = \frac{1}{2}$ $\quad H = 1$

Santa Barbara Weather, Almost Always Sunny.

$p_{rain} = \frac{1}{10}, p_{sun} = \frac{9}{10}$ $\quad H = 0.4960$