Handshake and Fencepost Activity

For each problem, complete the following information.

Understanding the problem:
- What data or information is known?
- What is unknown?
- What are the conditions?

Plan the solution: Show your plan for solving this problem.

Carry out the plan: Using your plan, show your work and your solution.

Review and discuss your solution: Reflect on your solution.

Complete problems #1 and #2 individually.

1. Handshake Problem #1: Assume there are 20 people in a room, including you. You must shake hands with everyone else in the room. How many hands will you shake? If there are N (where N > 0) people in the room, how many hands will you shake?

2. Fence Post Problem: You need to build one side of a fence that is 12 yards long. This fence will be built with fence posts and rails that connect one fence post to another. If each fence post is 1 yard away from the next fence post, how many fence posts will be needed for this side of the fence? How many fence posts will be needed for a side of a fence that is N (where N > 0) yards long?

Read and begin planning your solution for problems #3 and #4. These problems will be completed in class tomorrow with your group. Each group will present their solutions to the class.

3. Handshake Problem #2: Assume there are 10 people in a room, including you. Each person in the room must shake hands one time, and only time, with all the other people in the room. How many handshakes will occur? If there are 20 people in the room, how many handshakes will occur? If there are N (where N > 0) people in the room, how many handshakes will occur?

4. Reflections: Why are problems like these important to learn how to solve? How could this type of solution be of benefit to a carpenter, a chef, a teacher?
Handshake Activity #2 Sample Solution

The sample solution is only one possibility. Student groups may have a wide variety of strategies. Ask questions that probe their understanding of the steps of the problem-solving process they used.

**Understanding the problem:**

- What data or information is known? *There are 10 people or N people in the room.*
- What is unknown? *Total number of handshakes*
- What are the conditions? *Each person must shake hands only one time with all others in the room. All of the handshakes must be added together.*

**Plan the solution:** A sample plan could be to describe the plan in words or use a chart or draw a picture and then act it out.

*Have the people line up in the room. The first person in the line walks down the line and shakes hands with all of the people in the line and then leaves the room. Count the number of handshakes and add to the total.*

*The next person in line walks down the line and shakes hands with all of the people left in the line and then leaves the room. Count the number of handshakes and add to the total.*

*This continues until there are only 2 people left. They shake hands and leave together. Increase the total by one.*

*Once the answer is known for 10 people, look for a pattern. Try the process for 5 people, 2 people. See if the pattern holds.*

**Carry out the plan:** Using your plan, show your work and your solution.

<table>
<thead>
<tr>
<th>Person</th>
<th>Shakes Hands with # of people left in line</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>7</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td>4</td>
</tr>
<tr>
<td>G</td>
<td>3</td>
</tr>
<tr>
<td>H</td>
<td>2</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>J</td>
<td>0 (last person in line– no one left to shake hands with)</td>
</tr>
</tbody>
</table>
Now add up the number of handshakes: $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 = 45$

For 10 people, the answer is the sum of the numbers from 1 to 9, which is 45. 9 is $10 - 1$.

For 5 people, the answer is the sum of the numbers from 1 to 4, which is 10. 4 is $5 - 1$.

For 2 people, the answer is the sum of the numbers from 1 to 1, which is 1. 1 is $2 - 1$.

For $N$ people, the answer is the sum of the numbers from 1 to $(N-1)$.

**Review and discuss your solution:** Each person shakes hands with $N - 1$ other people. The answer is not $N(N-1)$, though, because each handshake counts as the one handshake for each person, but only one handshake for the total. The Hershey Bar problem helped to start the plan for this problem, but I needed to adjust the plan to only allow one handshake between each pair in the room.

So the 10 people make 9 handshakes each, but each handshake happens between 2 people, and can only be counted once. I could "divide" the handshake and let each person count the handshake as a 1/2 handshake. So 10 people make 9 half-handshakes each = 45 handshakes.

$N$ people make $N-1$ half-handshakes each = $N(N-1)/2$

The sum of the numbers from 1 to $N-1$ = $N(N-1)/2$