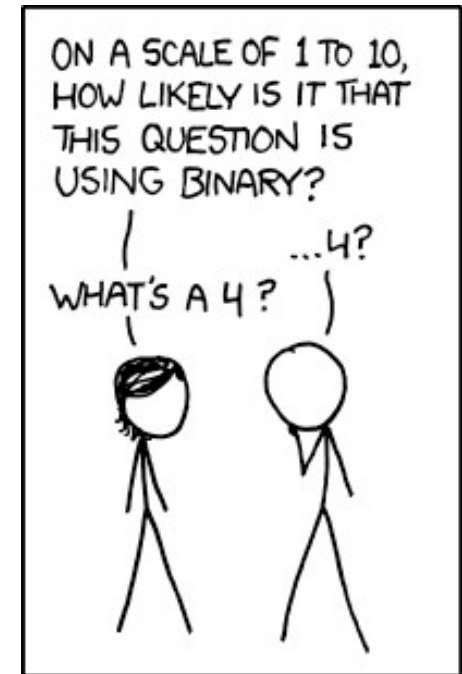


CS 101 Today...



Our top-10 list of binary jokes...

← **Looking Back**

Computing as composition
clay == **functions**

Looking Forward →

Computing as representation
clay == **data & bits**

Some legs to stand on... ?



This is heady stuff!

decipher

max

encipher

sScore

letScore

rot(c,n)

ord

chr

creating more and
more capable
compositions

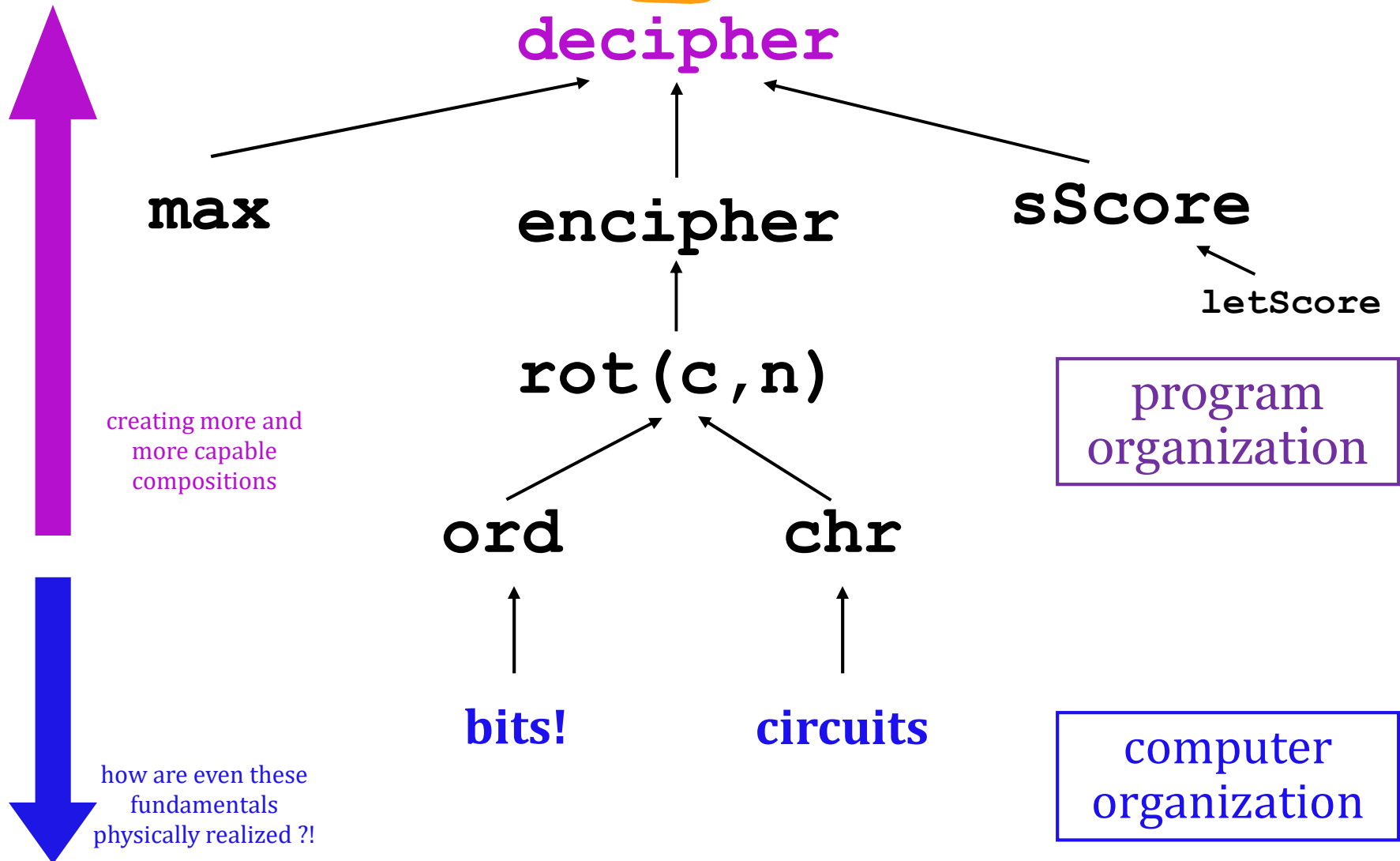
program
organization



Some legs to stand on!



It looks like I'm ahead of this...

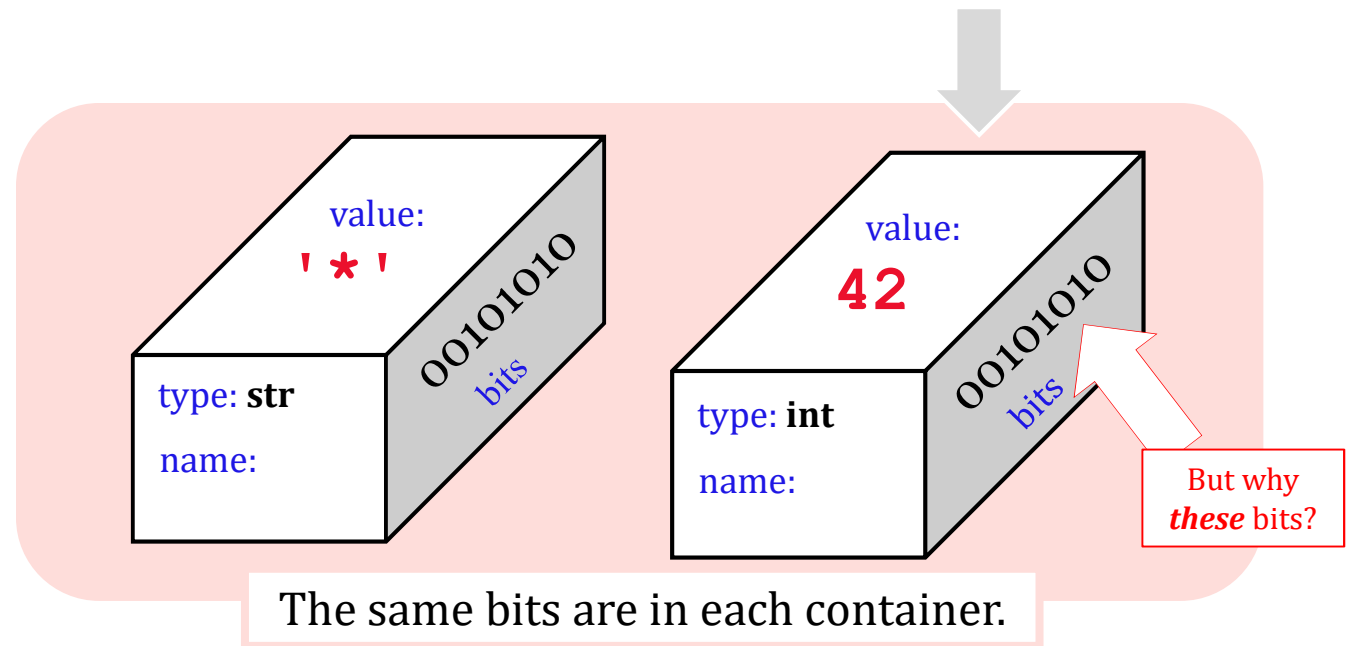


Binary Storage & Representation

Binary	Dec	Hex	Glyph
0010 0000	32	20	(blank) (space)
0010 0001	33	21	!
0010 0010	34	22	"
0010 0011	35	23	#
0010 0100	36	24	\$
0010 0101	37	25	%
0010 0110	38	26	&
0010 0111	39	27	'
0010 1000	40	28	(
0010 1001	41	29)
0010 1010	42	2A	*
0010 1011	43	2B	+

The SAME bits can represent different pieces of data, depending on **type**

8 bits = 1 byte = 1 box



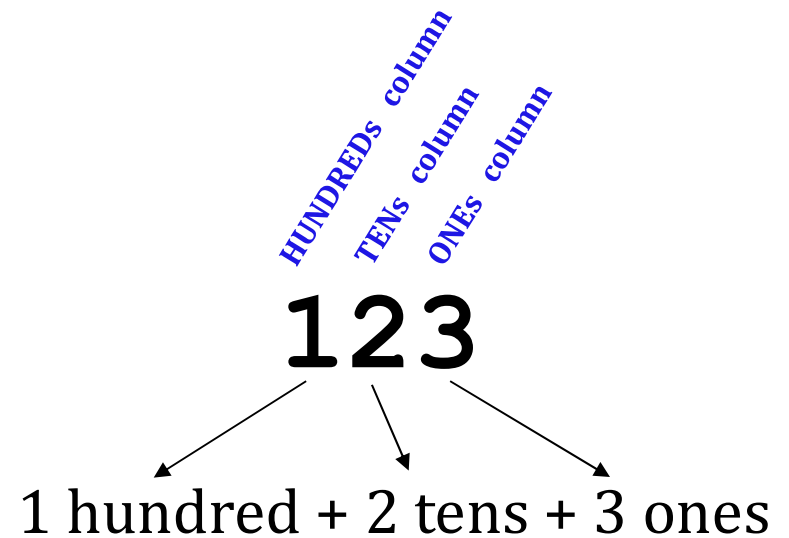
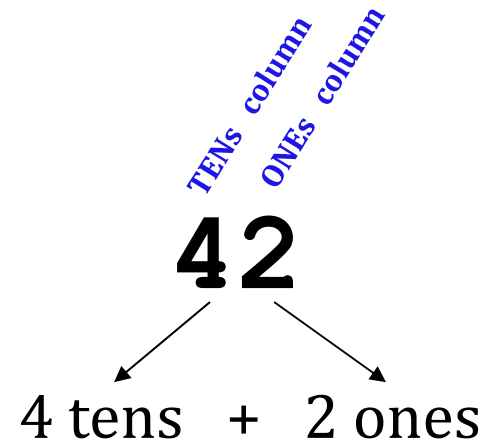
The same bits are in each container.

What *is* 42 ?

42

Base 10

What *is* 42 ?





42

Value (semantics)

stuff we care about (what things *mean*)

Syntax

stuff we need *to communicate*



SAME!

Value (semantics)

stuff we care about (what things *mean*)

101010

different

Syntax

stuff we need *to communicate*

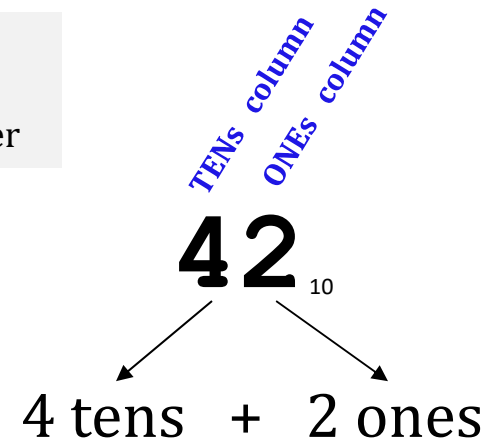
Base 2



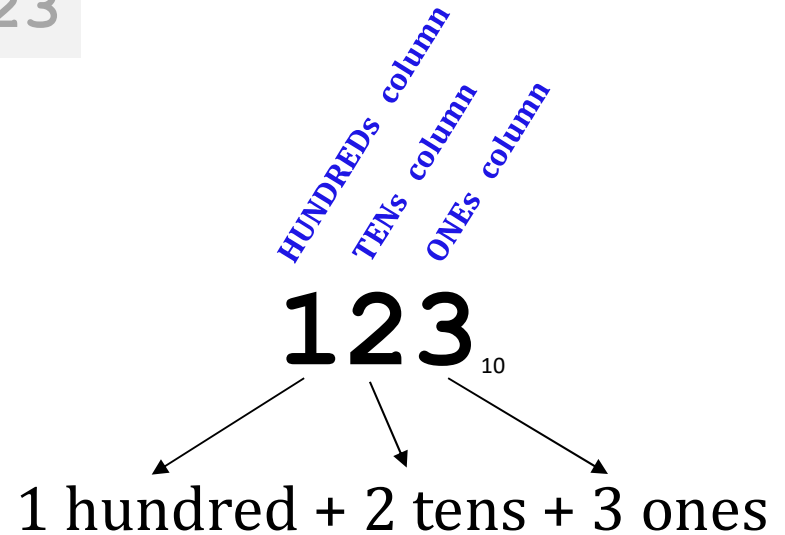
Write 123 in binary...

Base 10

each column represents the base's next power

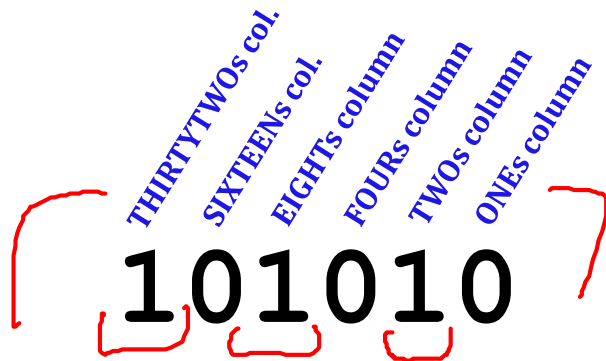


123

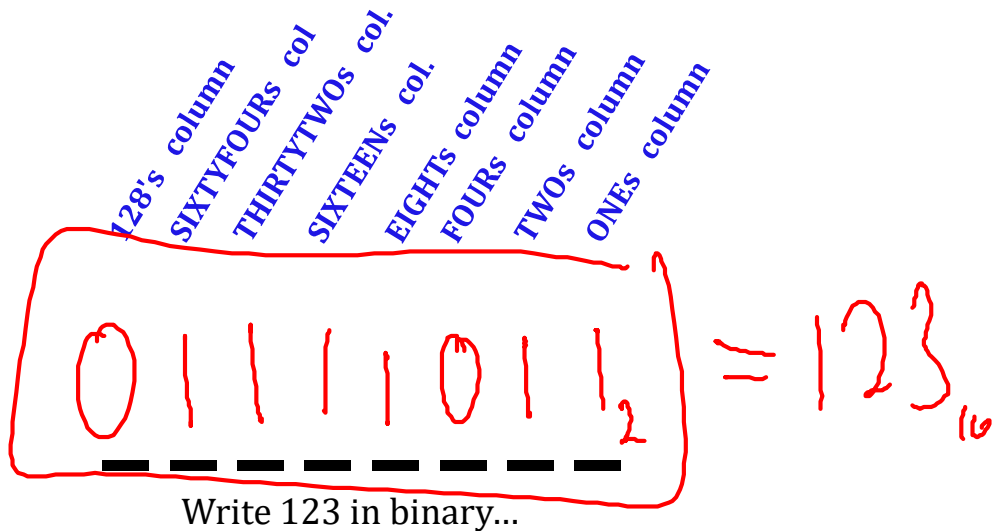


Base 2

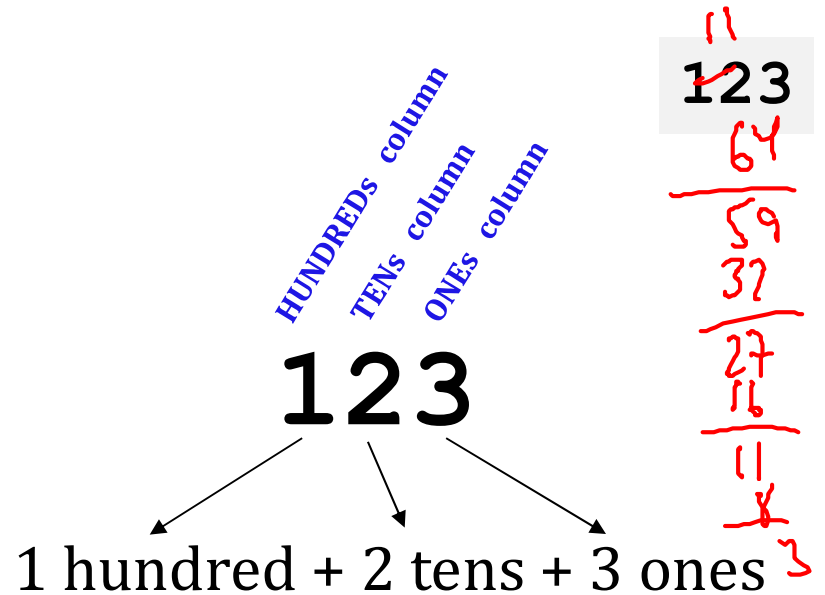
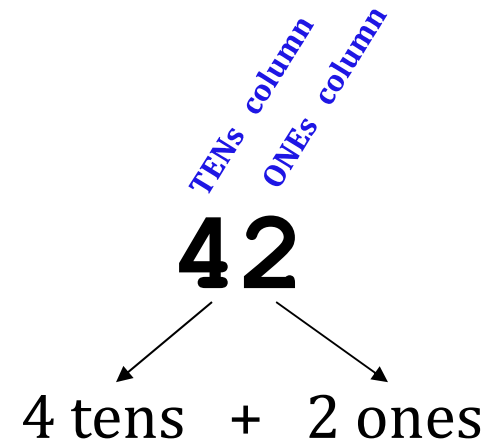
each column represents the base's next power



$$32 + 8 + 2 = 42_{10}$$



Base 10



Binary math

Decimal math

tables of
basic facts

+

Addition

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

*

Multiplication

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2		4	6	8	10	12	14	16	18	20
3			9	12	15	18	21	24	27	30
4				16	20	24	28	32	36	40
5					25	30	35	40	45	50
6						36	42	48	54	60
7							49	56	63	70
8								64	72	80
9									81	90
10										100

Binary math

+	0	1
0	0	1
1	1	10

two's ones

tables of basic facts

+

Addition

Decimal math

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

*

Multiplication

*	0	1
0	0	0
1	0	1

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2		4	6	8	10	12	14	16	18	20
3			9	12	15	18	21	24	27	30
4				16	20	24	28	32	36	40
5					25	30	35	40	45	50
6						36	42	48	54	60
7							49	56	63	70
8								64	72	80
9									81	90
10										100

Name(s): _____

Quiz

In binary, I'm an 11-eyed alien!



Convert these two binary numbers *to decimal*:

32 16 8 4 2 1
110011

10001000

Convert these two decimal numbers *to binary*:

32 16 8 4 2 1

28₁₀

101₁₀

Add these two binary numbers:

101101
+ **1110**

Multiply
these binary numbers:

101101
* **1110**

WITHOUT
converting
to decimal!

+ -----

1
529
+ 742

1271

Hint: Remember these algorithms? They're the same in binary!

529
* 42

1058
+ 2116

22218

Extra! Can you figure out the last binary digit (bit) of 53 *without determining any other bits*? The last two? 3?

Convert these two binary numbers *to decimal*:

32 16 8 4 2 1
110011

32 + 16 + 2 + 1

51

values in blue

128 64 32 16 8 4 2 1
10001000

128 + 8

136

Convert these two decimal numbers *to binary*:

28

32 16 8 4 2 1

011100

syntax in orange

101₁₀

128 64 32 16 8 4 2 1

01100101

Extra! Can you figure out the last binary digit (bit) of 53 *without determining any other bits*? The last two? 3?

We'll return to this *in a bit...*

Add these two binary numbers
WITHOUT converting to decimal !

32 16 8 4 2 1

$$\begin{array}{r} 101101 \\ + 1110 \\ \hline \end{array}$$

45

14

59

32 16 8 4 2 1

Hint:

$$\begin{array}{r} 529 \\ + 742 \\ \hline 1271 \end{array}$$

Do you remember this algorithm? It's the same!

Multiply these two binary numbers
WITHOUT converting to decimal !

		32	16	8	4	2	1				
		101101									
		1110									
*											
		000000									
		1011010									
		10110100									
+		101101000									
		1001110110									
		512	256	128	64	32	16	8	4	2	1

45
14

Hint:

	529	
*	42	←
	1058	
+	2116	
	22218	

Do you remember this algorithm? It's the same!

"partial products"

630
Goal

A machine could -
 and probably *should*
 - be doing this !

Beyond Binary

base 1 digits: 1

base 2 ——— ^{32 16 8 4 2 1} 101010 digits: 0, 1

base 3 ————— ^{27 9 3 1} digits: 0, 1, 2

42 ?

There are 10 kinds of "people" in the universe:

those who know ternary,
those who don't, and
those who think this is a binary joke!



Beyond Binary

base 1 digits: 1

base 2 ——— ^{32 16 8 4 2 1} 101010 digits: 0, 1

base 3 ——— ^{27 9 3 1} 1120 _{16 4 1} digits: 0, 1, 2

base 4

base 5

base 6

base 7

base 8

base 9

base 10 42 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

base 11

base 12

. . .

base 16

Which of these *isn't* 42...?

222 60 54 46 39

and what are the *bases* of the rest?

Beyond Binary

base 1 digits: 1

base 2 ——— 101010 digits: 0, 1

base 3 ——— 1120 digits: 0, 1, 2

base 4

base 5

base 6

base 7

base 8

base 9

base 10

base 11

base 12

...

base 16

Which of these *isn't* 42...?
and what are the *bases* of the rest?

222

60₇

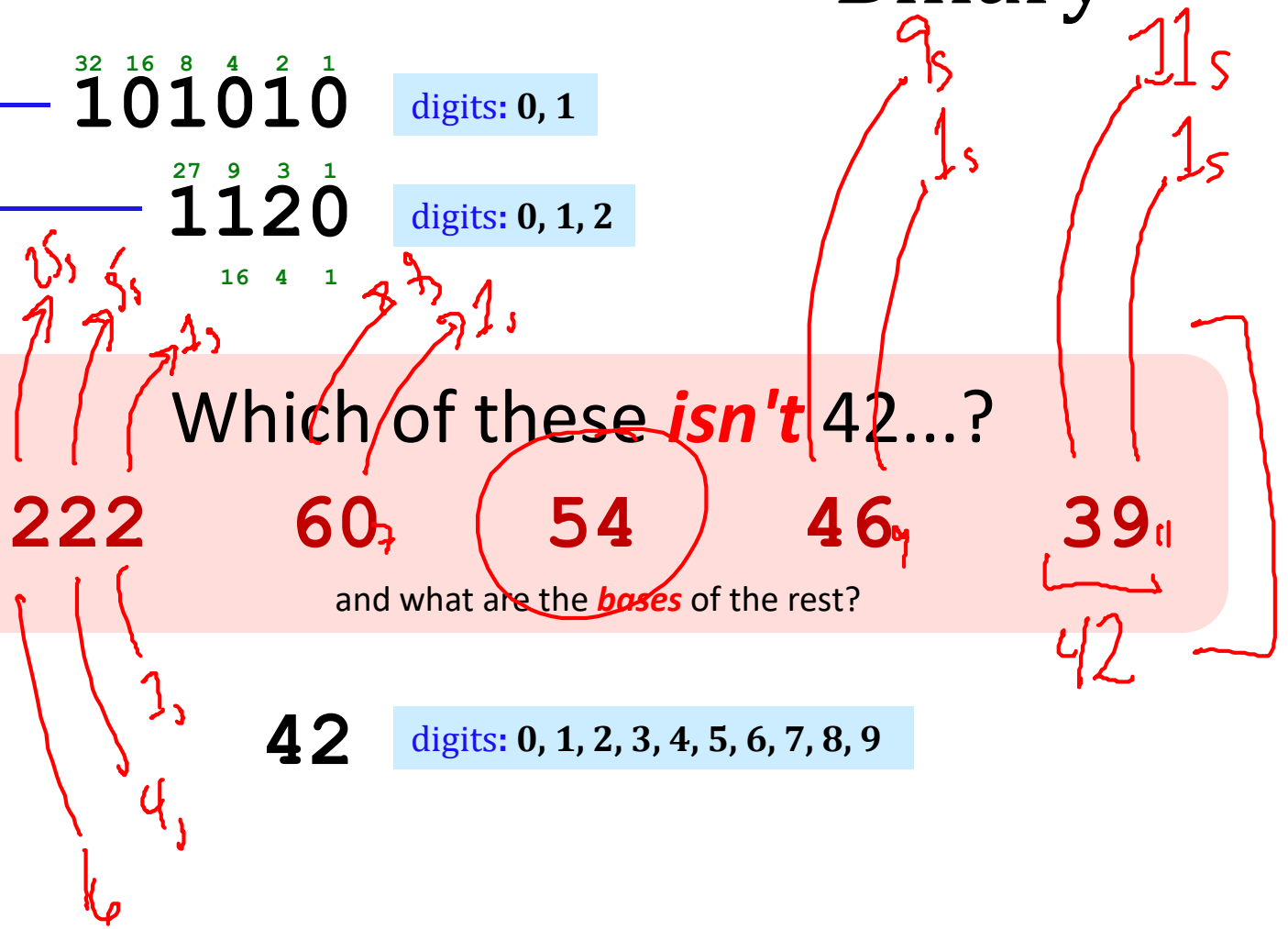
54

46₉

39₁₁

42

digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9



Our Mascot, the Panda ←

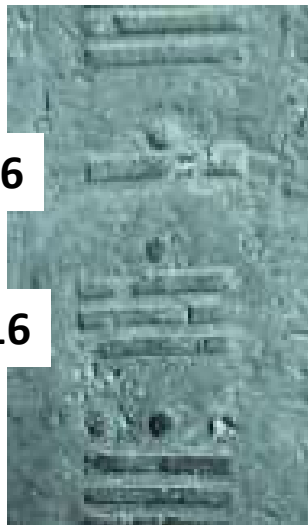


Off base?

Base 12 –

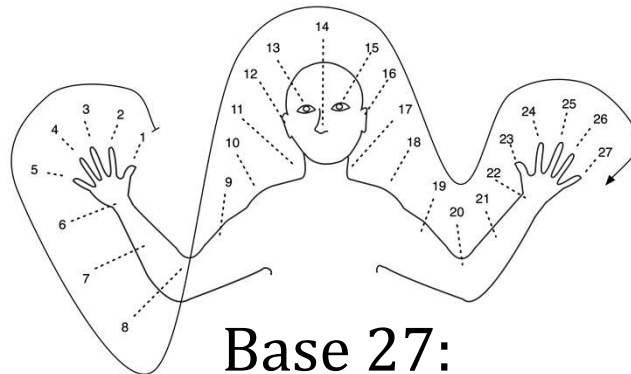
"Duodecimal Society"
 "*Dozenal* Society"

Base 20:
 Americas



Olmec base-20 numbers
 E. Mexico, ~ 300 AD

Telefol is a language spoken by the Telefol people in Papua New Guinea, notable for possessing a base-27 numeral system.



Base 27:
 New Guinea

Base 60 – Ancient Sumeria

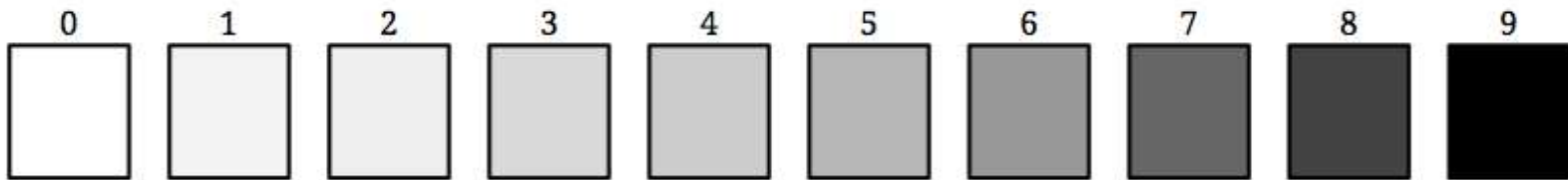
1	∩	11	∩∩	21	∩∩∩	31	∩∩∩∩	41	∩∩∩∩∩	51	∩∩∩∩∩∩
2	∩∩	12	∩∩∩	22	∩∩∩∩	32	∩∩∩∩∩	42	∩∩∩∩∩∩	52	∩∩∩∩∩∩∩
3	∩∩∩	13	∩∩∩∩	23	∩∩∩∩∩	33	∩∩∩∩∩∩	43	∩∩∩∩∩∩∩	53	∩∩∩∩∩∩∩∩
4	∩∩∩∩	14	∩∩∩∩∩	24	∩∩∩∩∩∩	34	∩∩∩∩∩∩∩	44	∩∩∩∩∩∩∩∩	54	∩∩∩∩∩∩∩∩∩
5	∩∩∩∩∩	15	∩∩∩∩∩∩	25	∩∩∩∩∩∩∩	35	∩∩∩∩∩∩∩∩	45	∩∩∩∩∩∩∩∩∩	55	∩∩∩∩∩∩∩∩∩∩
6	∩∩∩∩∩∩	16	∩∩∩∩∩∩∩	26	∩∩∩∩∩∩∩∩	36	∩∩∩∩∩∩∩∩∩	46	∩∩∩∩∩∩∩∩∩∩	56	∩∩∩∩∩∩∩∩∩∩∩
7	∩∩∩∩∩∩∩	17	∩∩∩∩∩∩∩∩	27	∩∩∩∩∩∩∩∩∩	37	∩∩∩∩∩∩∩∩∩∩	47	∩∩∩∩∩∩∩∩∩∩∩	57	∩∩∩∩∩∩∩∩∩∩∩∩
8	∩∩∩∩∩∩∩∩	18	∩∩∩∩∩∩∩∩∩	28	∩∩∩∩∩∩∩∩∩∩	38	∩∩∩∩∩∩∩∩∩∩∩	48	∩∩∩∩∩∩∩∩∩∩∩∩	58	∩∩∩∩∩∩∩∩∩∩∩∩∩
9	∩∩∩∩∩∩∩∩∩	19	∩∩∩∩∩∩∩∩∩∩	29	∩∩∩∩∩∩∩∩∩∩∩	39	∩∩∩∩∩∩∩∩∩∩∩∩	49	∩∩∩∩∩∩∩∩∩∩∩∩∩	59	∩∩∩∩∩∩∩∩∩∩∩∩∩∩
10	A	20	∩∩∩∩∩∩∩∩∩	30	∩∩∩∩∩∩∩∩∩∩∩	40	∩∩∩∩∩∩∩∩∩∩∩∩	50	∩∩∩∩∩∩∩∩∩∩∩∩∩∩		

Some of these bases are still echoing around...

But *why* binary?

Ten symbols is too many!

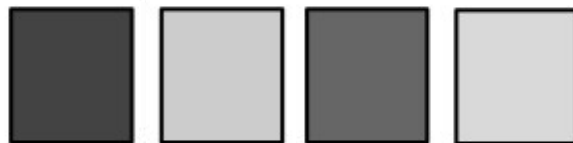
A computer has to differentiate *physically* among all its possibilities.



ten symbols ~ ten different voltages

This is too difficult to replicate billions of times

engineering!

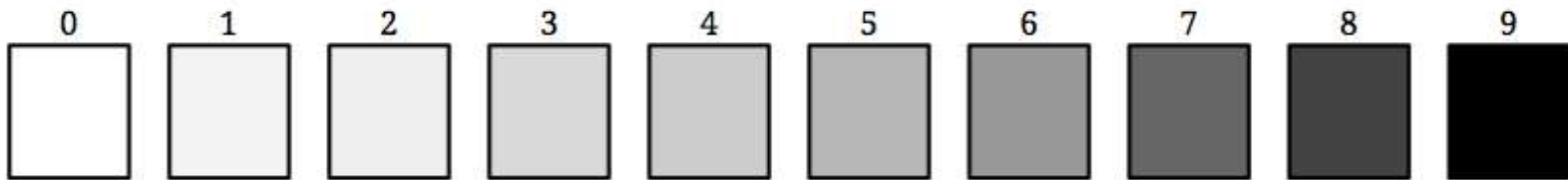


What digits are these?

Ouch!

Ten symbols is too many!

A computer has to differentiate *physically* among all its possibilities.



ten symbols ~ ten different voltages

This is too difficult to replicate billions of times engineering!

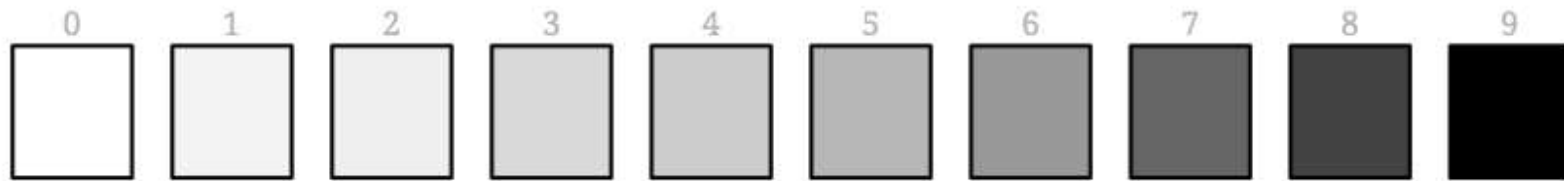


What digits are these?

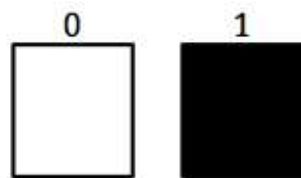
Ouch!

Two symbols is easiest!

A computer has to differentiate *physically* among all its possibilities.



ten symbols ~ ten different voltages



two symbols ~ two different voltages

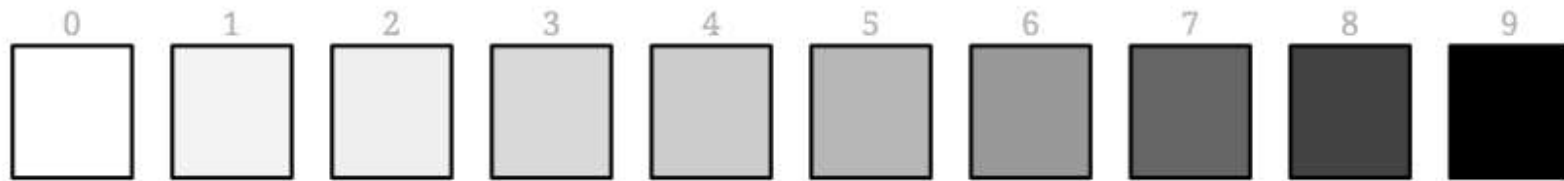


What digits are these?

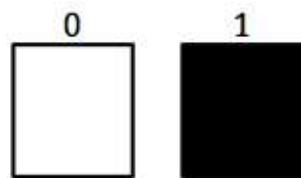
Easy!

Two symbols is easiest!

A computer has to differentiate *physically* among all its possibilities.



ten symbols ~ ten different voltages



two symbols ~ two different voltages



What digits are these?

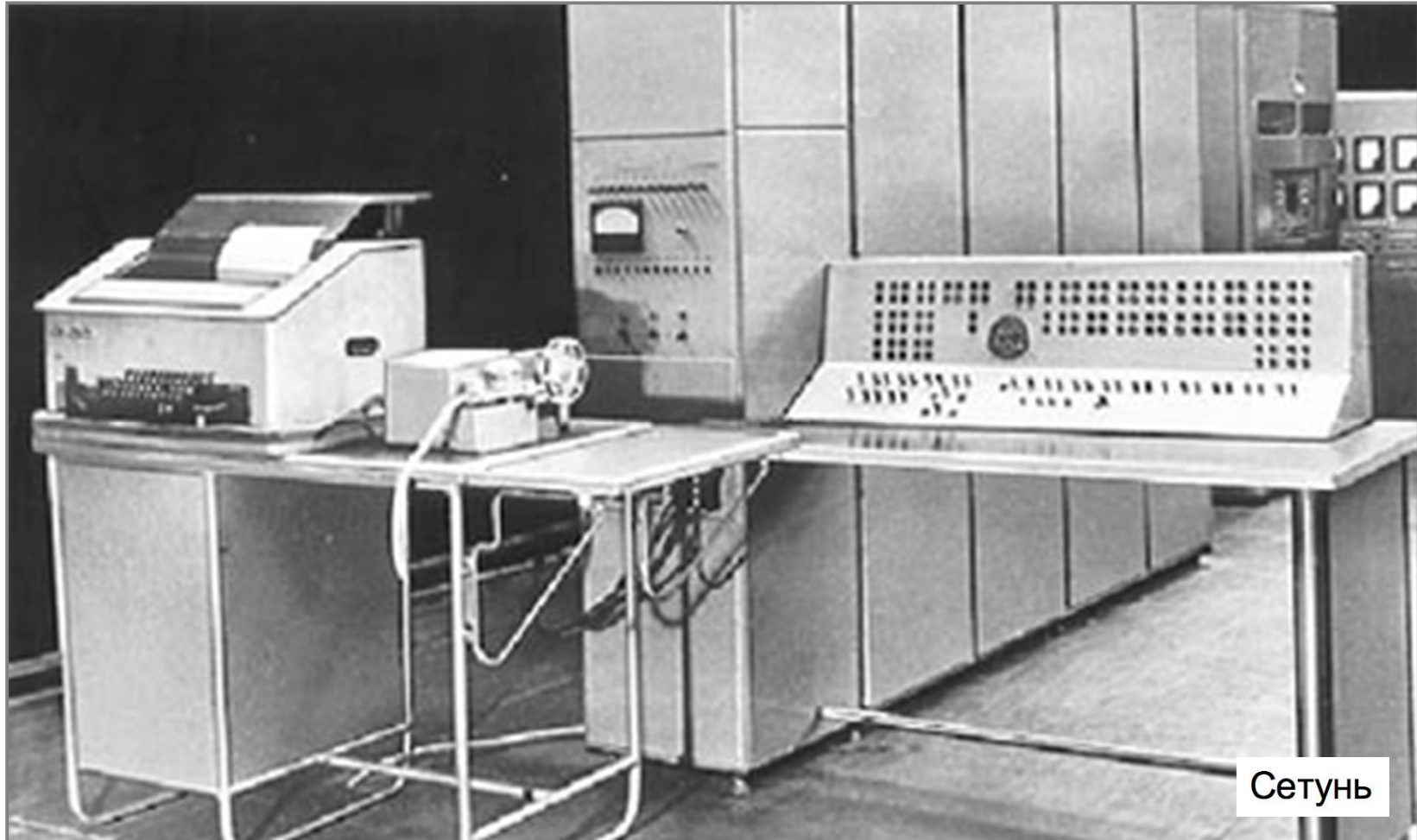
Easy!

Ternary computers?

Everything should
be base-3!



50 of these *Setun* ternary machines were made at Moscow U. ~ 1958



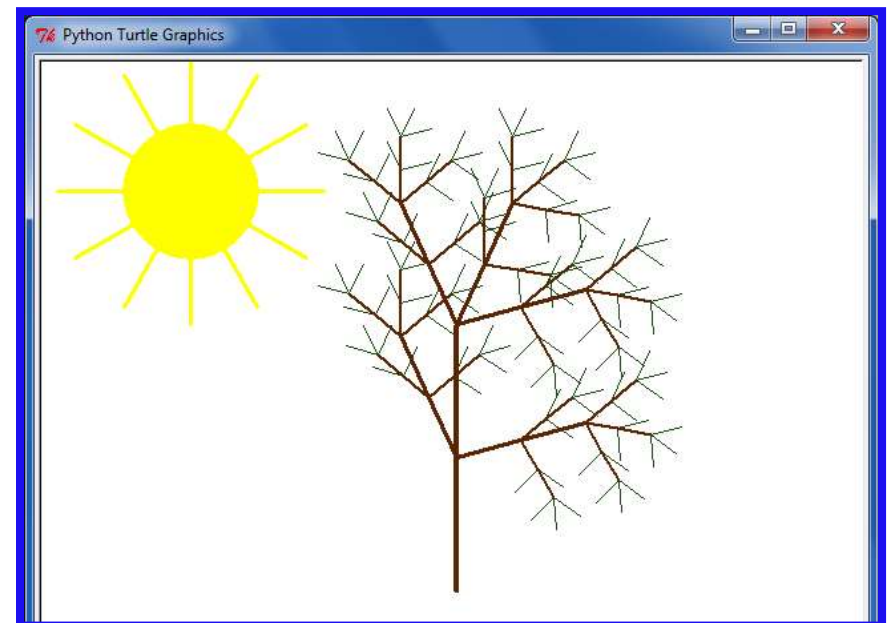
This project was discontinued in 1970... *though not because of the ternary design!*

Eye-catching
submissions...

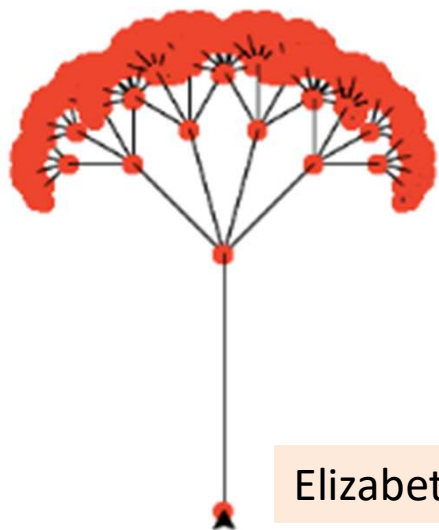
and turtle art

```
C:\Windows\system32\cmd.exe - python -i hw2pr2.py
-----[ ] [ ]
-----[ ] [ ]
-----[ ] [ ]
-----[ ]
-----[POLICE BOX
-----[ #] [#]
-----[[]][o]
-----[ ] [ ]
-----[ ] [ ]
-----[ ] [ ]
-----[ ] [ ]
-----[ ]
-----[POLICE BOX
-----[ #] [#]
-----[[]][o]
-----[ ] [ ]
-----[ ] [ ]
-----[ ] [ ]
-----[ ] [ ]
-----[ ]
-----[POLICE BOX
-----[ #] [#]
-----[[]][o]
-----[ ] [ ]
-----[ ] [ ]
-----[ ] [ ]
-----[ ] [ ]
-----[ ]
```

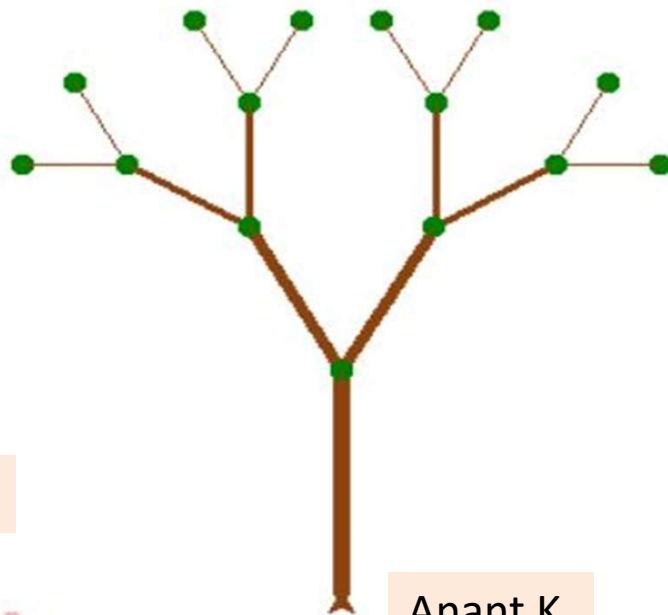
ASCII wanderings...



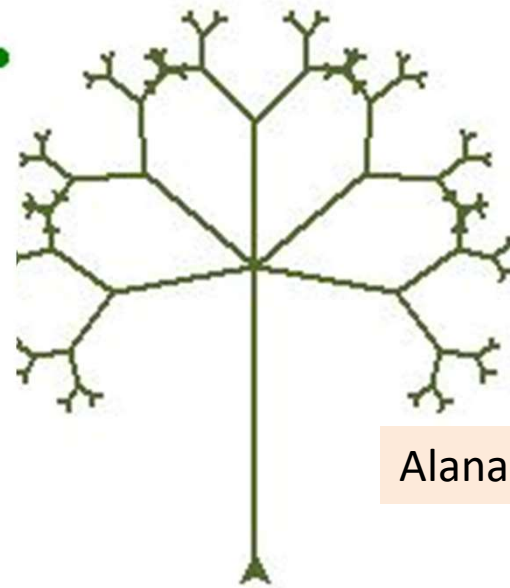
Trees...



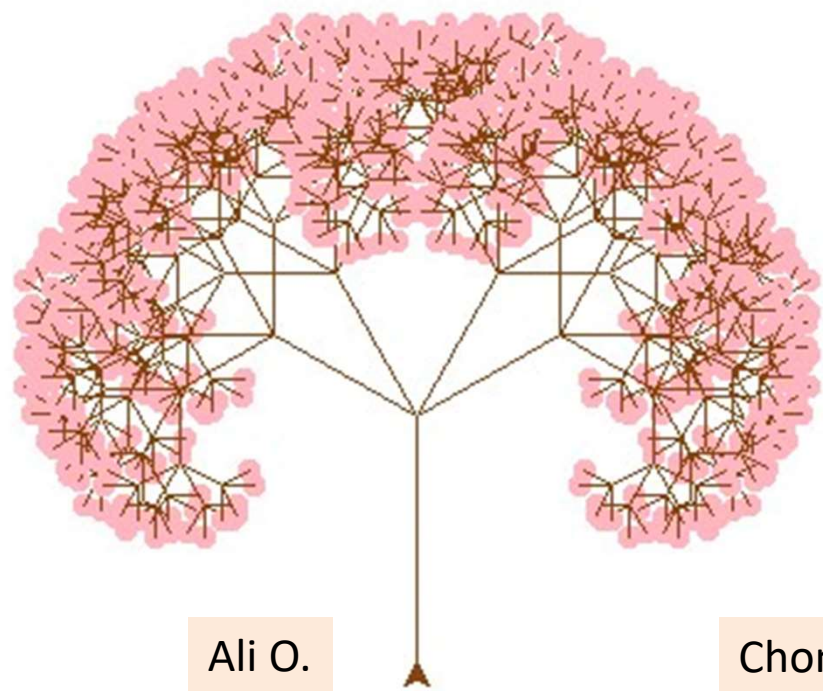
Elizabeth M.



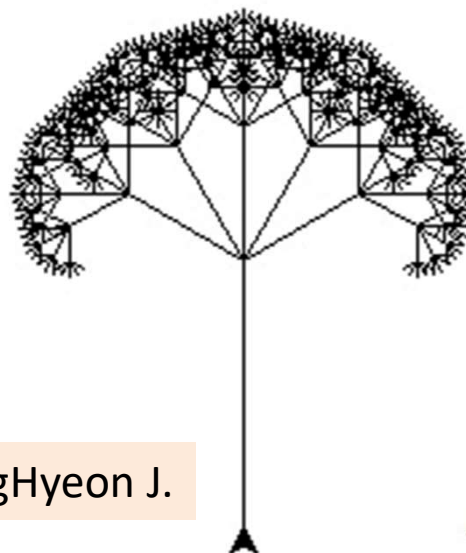
Anant K.



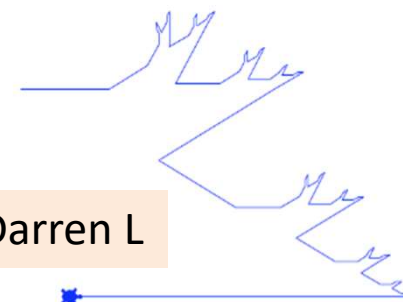
Alana G.



Ali O.



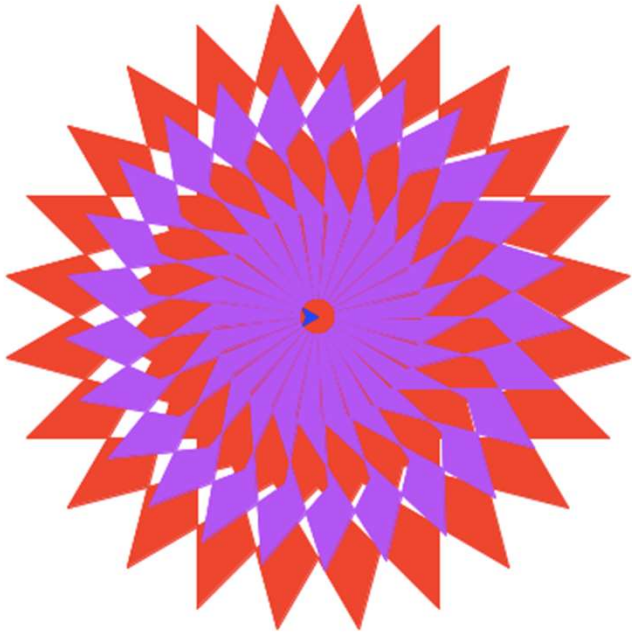
ChongHyeon J.



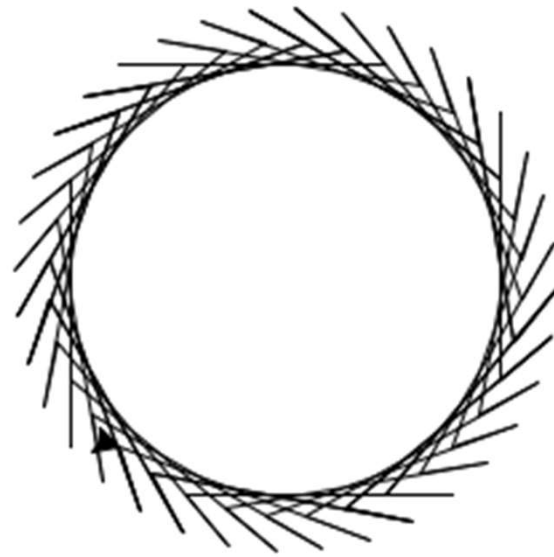
Darren L



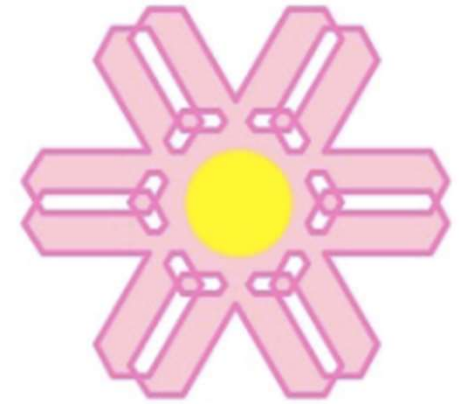
Yuexi L.



Yinfeng H.

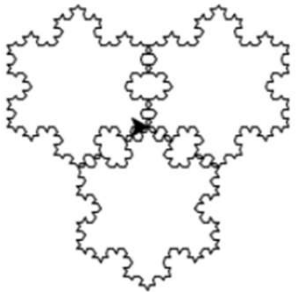


Arthur C.

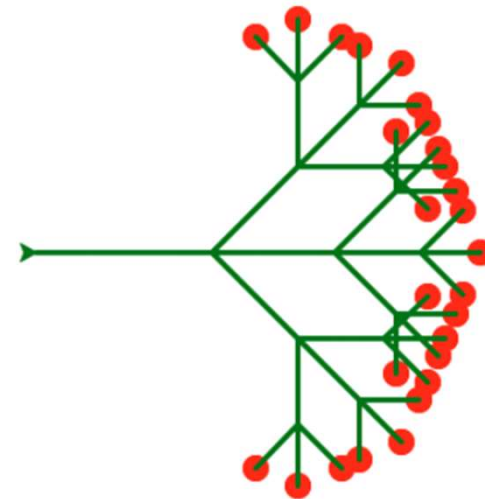
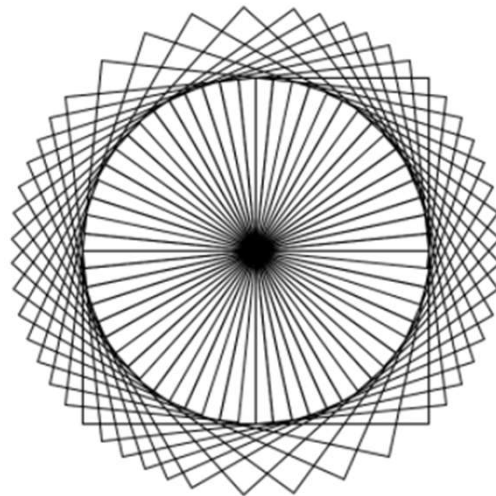


Roxy K.

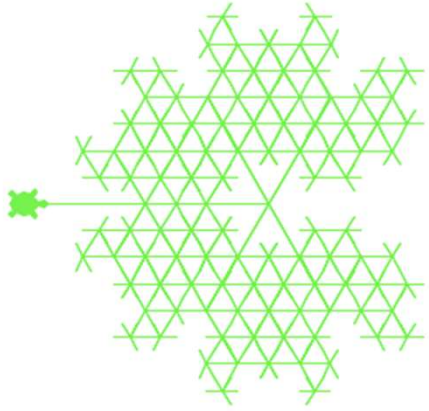
Spirals...



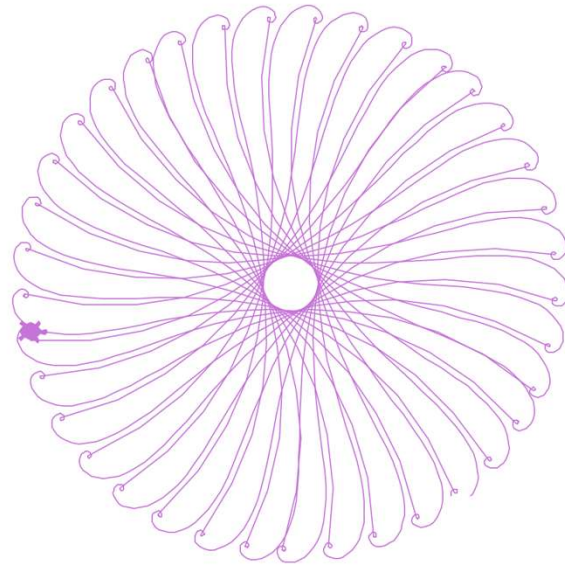
Indu S.



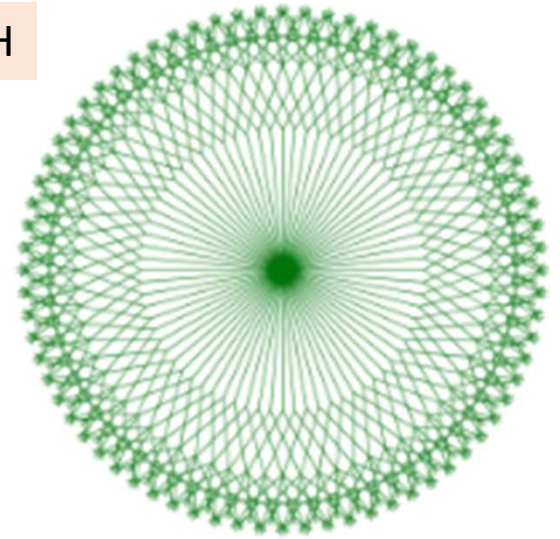
Shruthi T



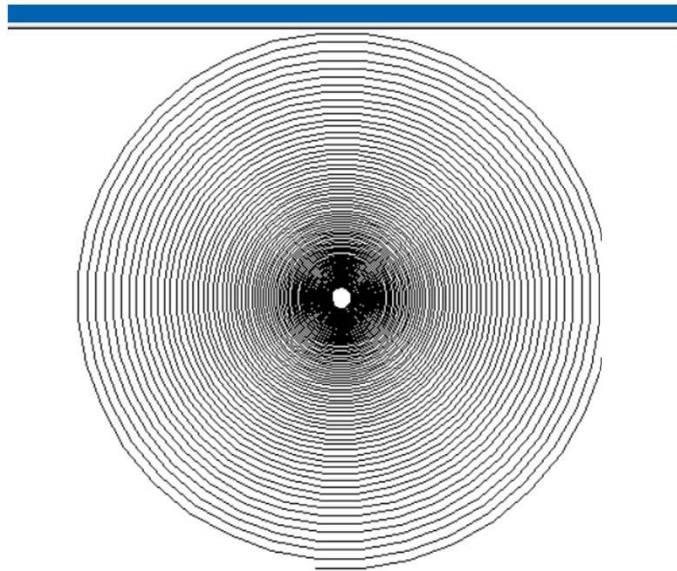
Shaneli J



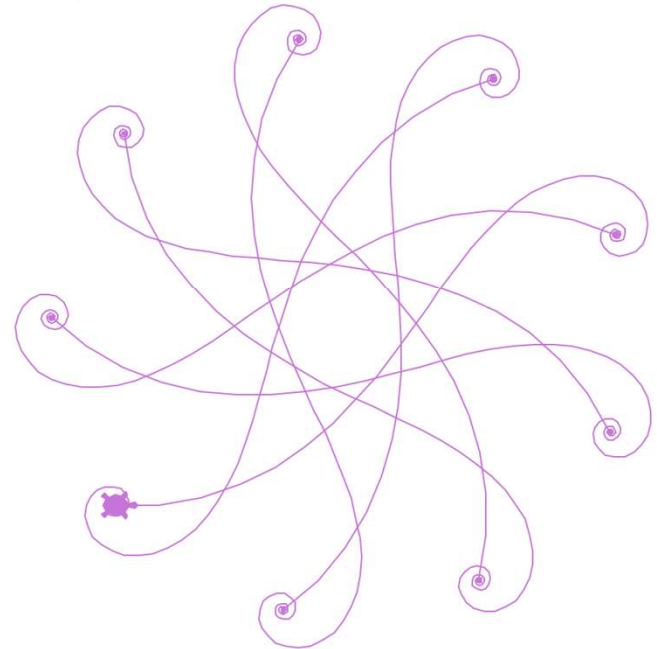
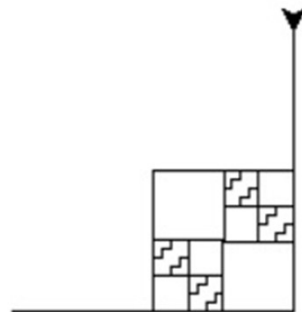
Madeline H

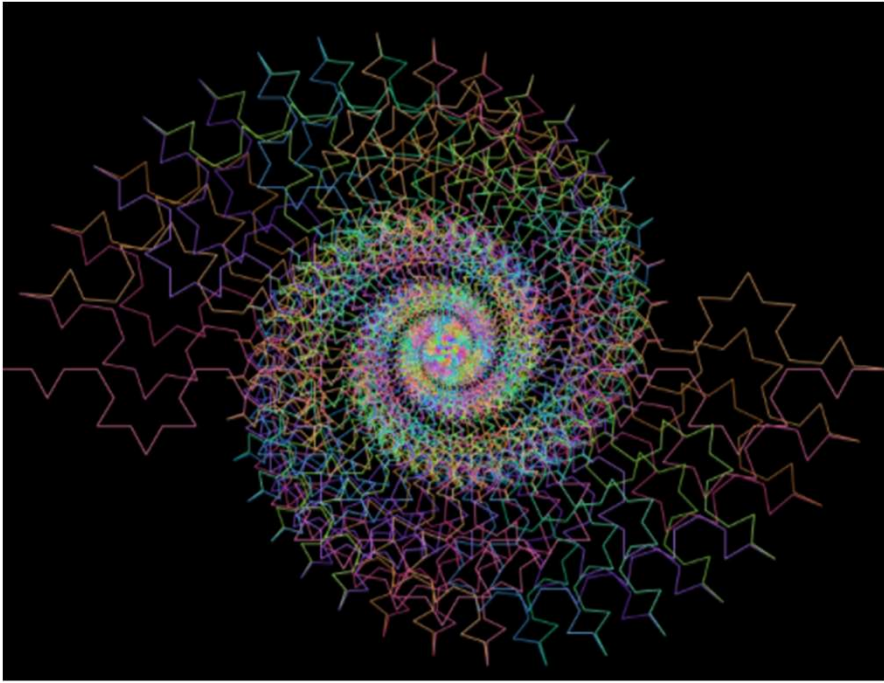


Daniel C.

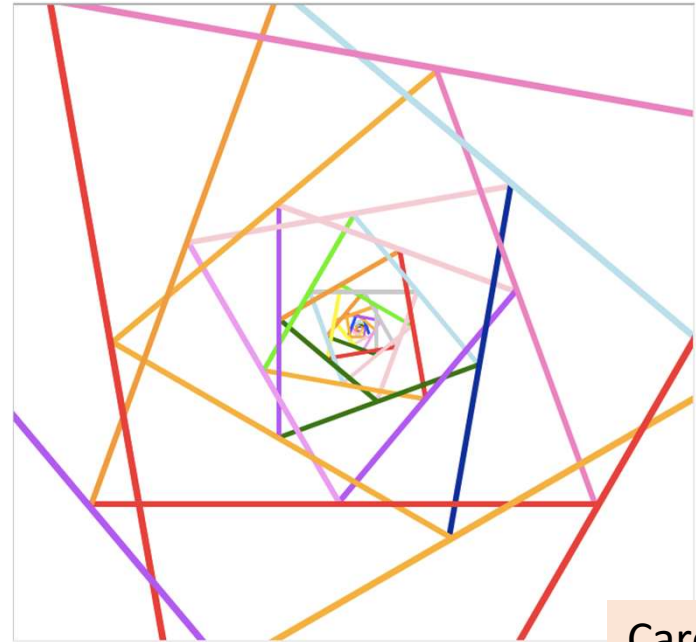


Jessica Y.

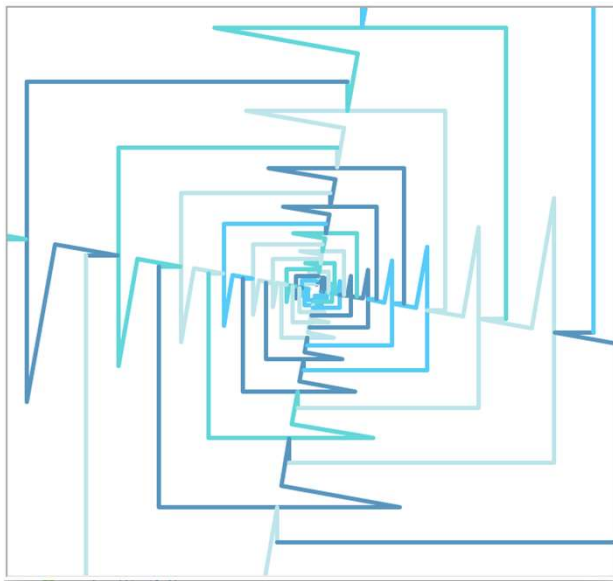




Seth T.B.



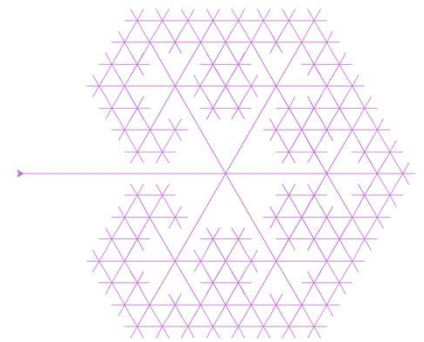
Caroline T

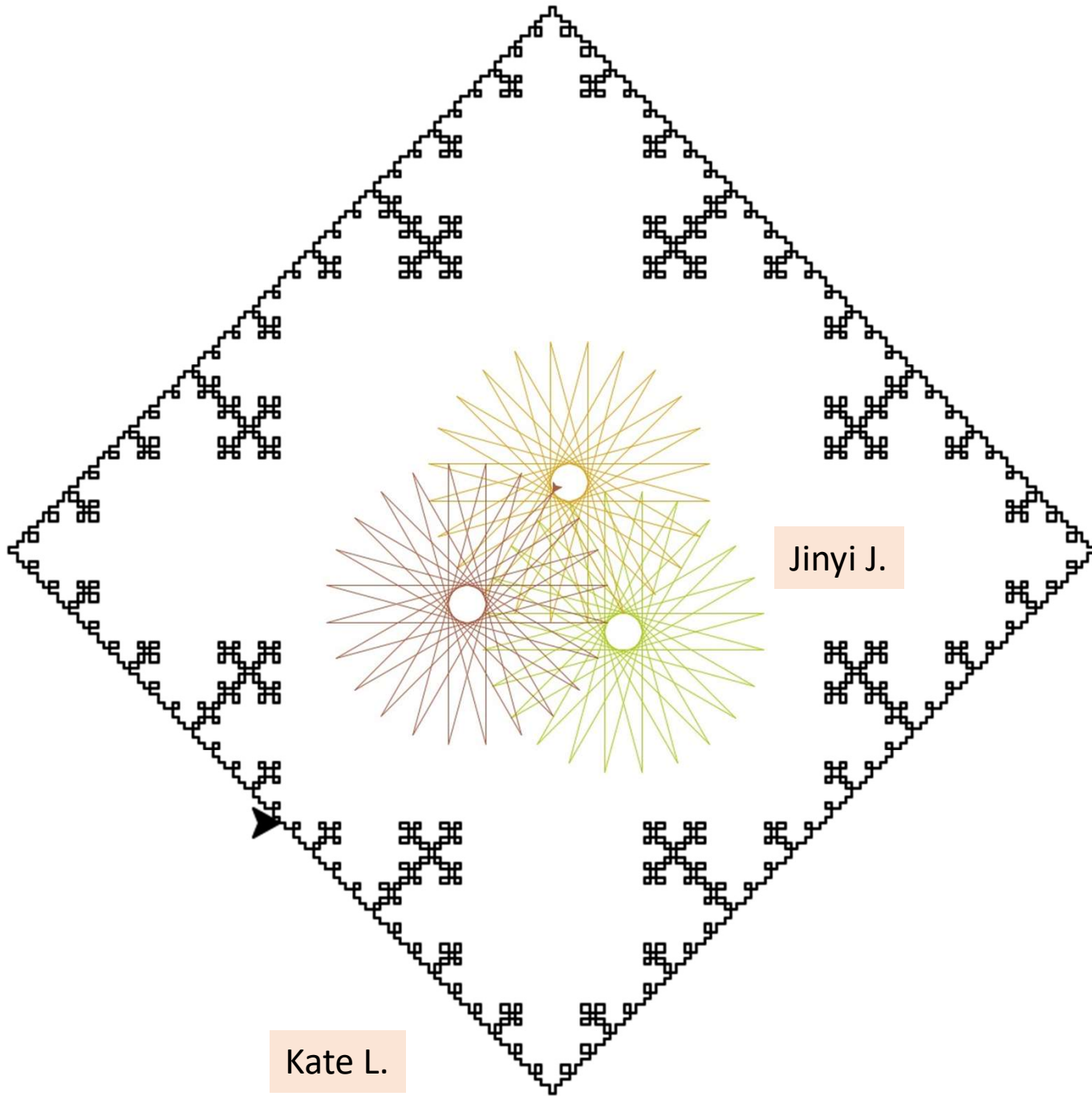


Emily Z



Both!?

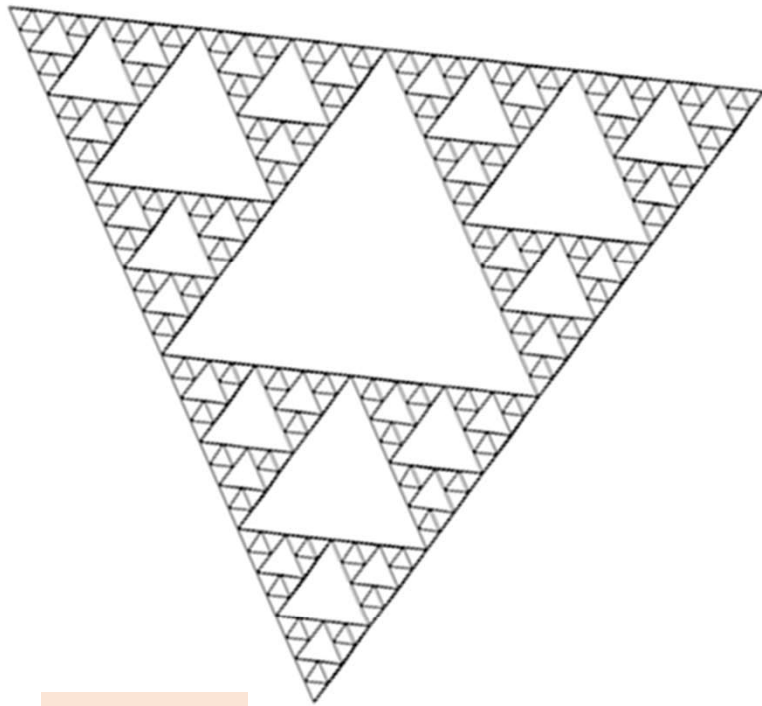




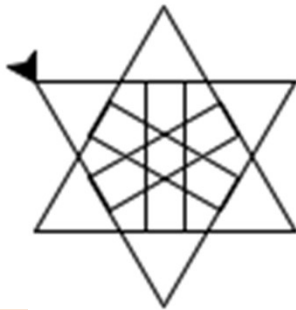
Jinyi J.

Kate L.

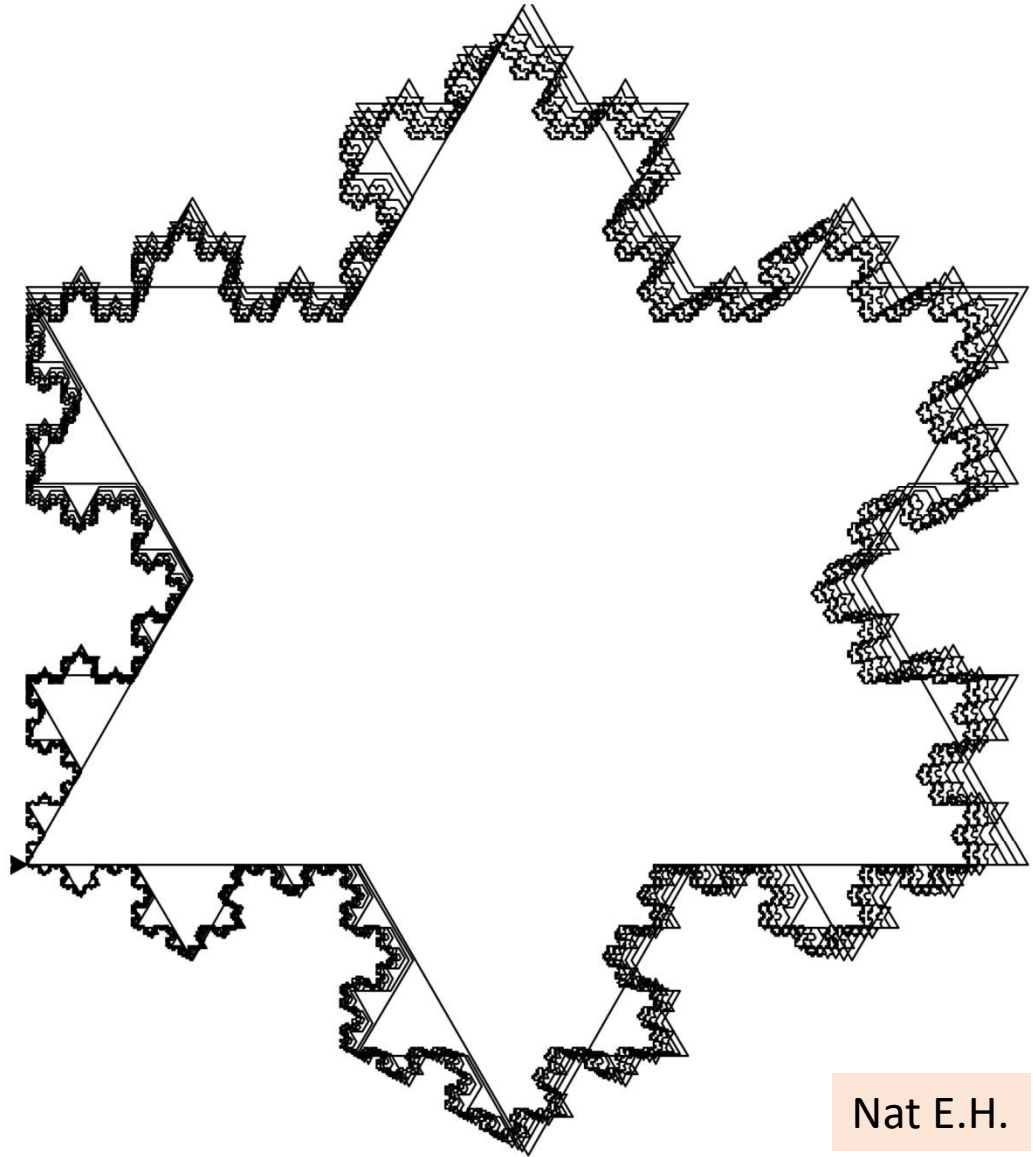
Fractals!



Emma Q



Hailey L

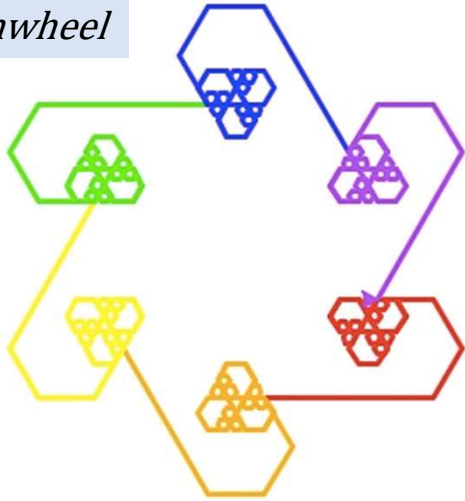


Nat E.H.



city_harbour

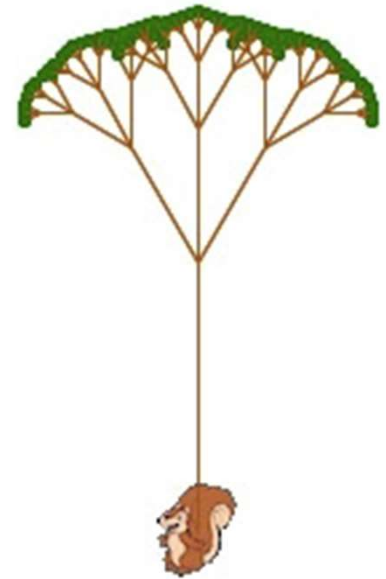
rainbow pinwheel



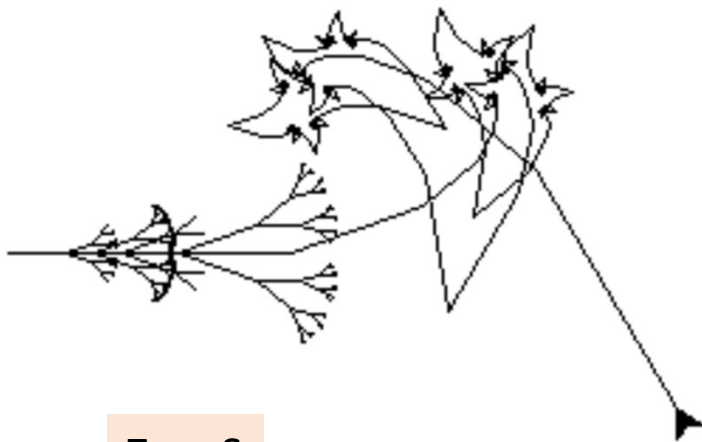
Well-named images!

Roxy K.

tree squirrel

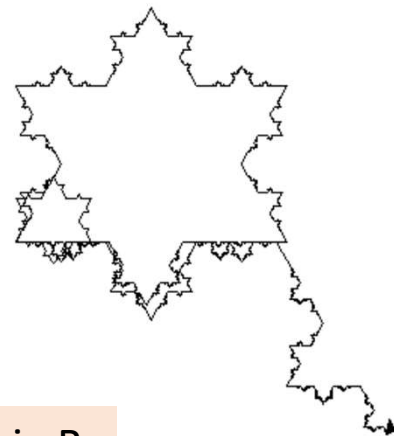


dandelion_s

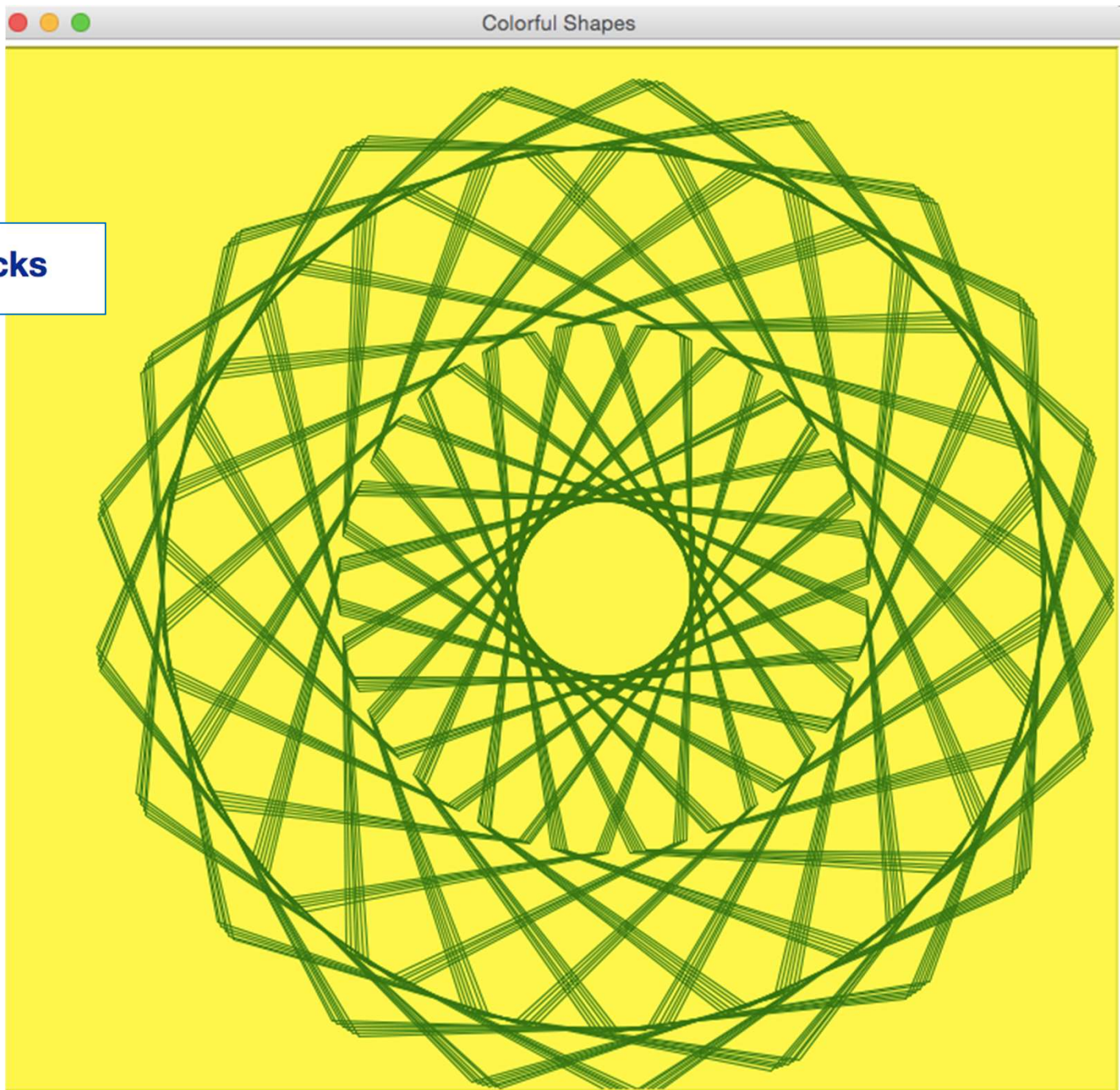


Zara S

Gloria B.



star_kite

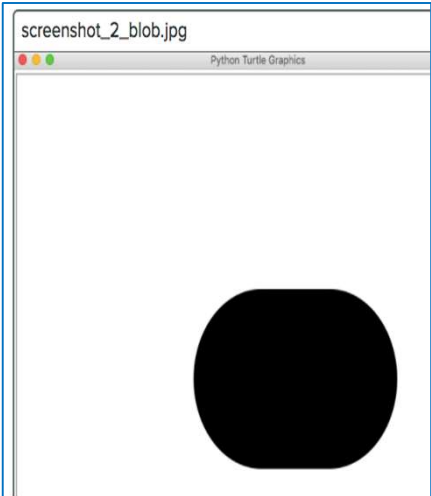


comic sans sucks

comic sans sucks

Dina R.

Joe. K



Veronica M

screenshot_4_tree_gone_wrong.png



Python Turtle Graphics



screenshot_5_turtles_all_the_way_down.png



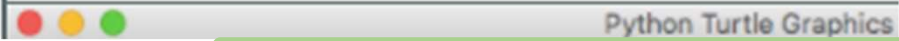
Python Turtle Graphics



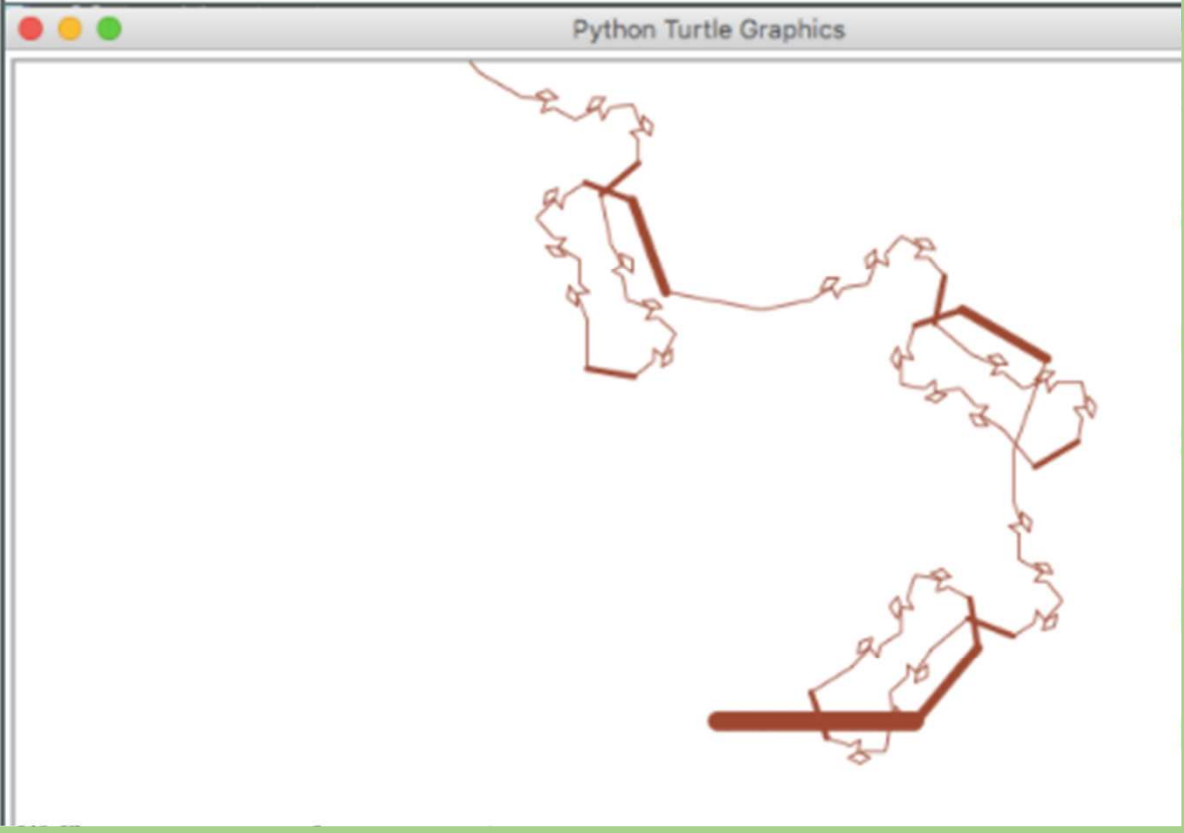
screenshot_1_lobster.jpg

Veronica M

screenshot_4_tree_gone_wrong.png



screenshot_3_velmas_glasses_by_now.png



screenshot_1_lobster.jpg

Veronica M

badha

i call this one, "kid on a leash on Halloween"

g

Jeni Z.

lie K.

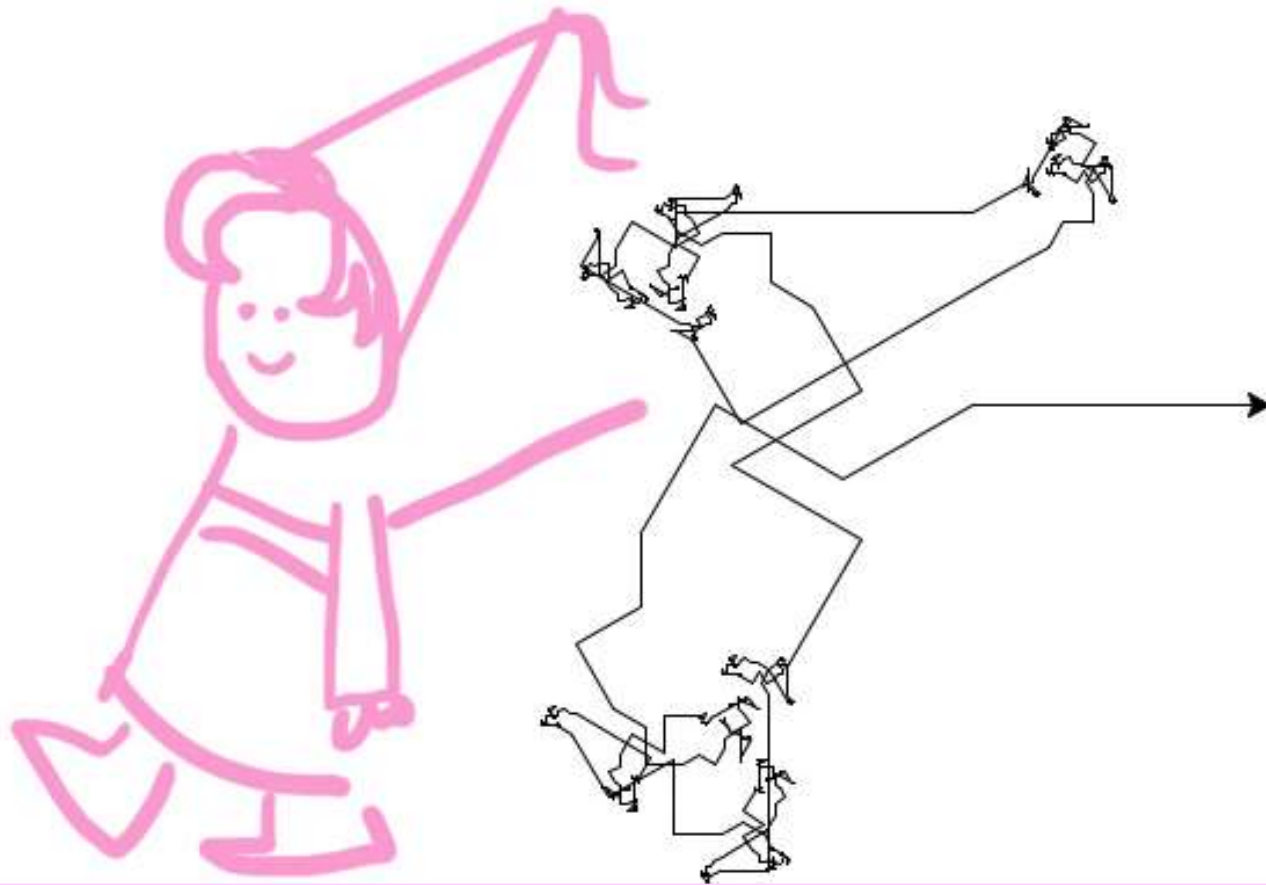
see?

ill

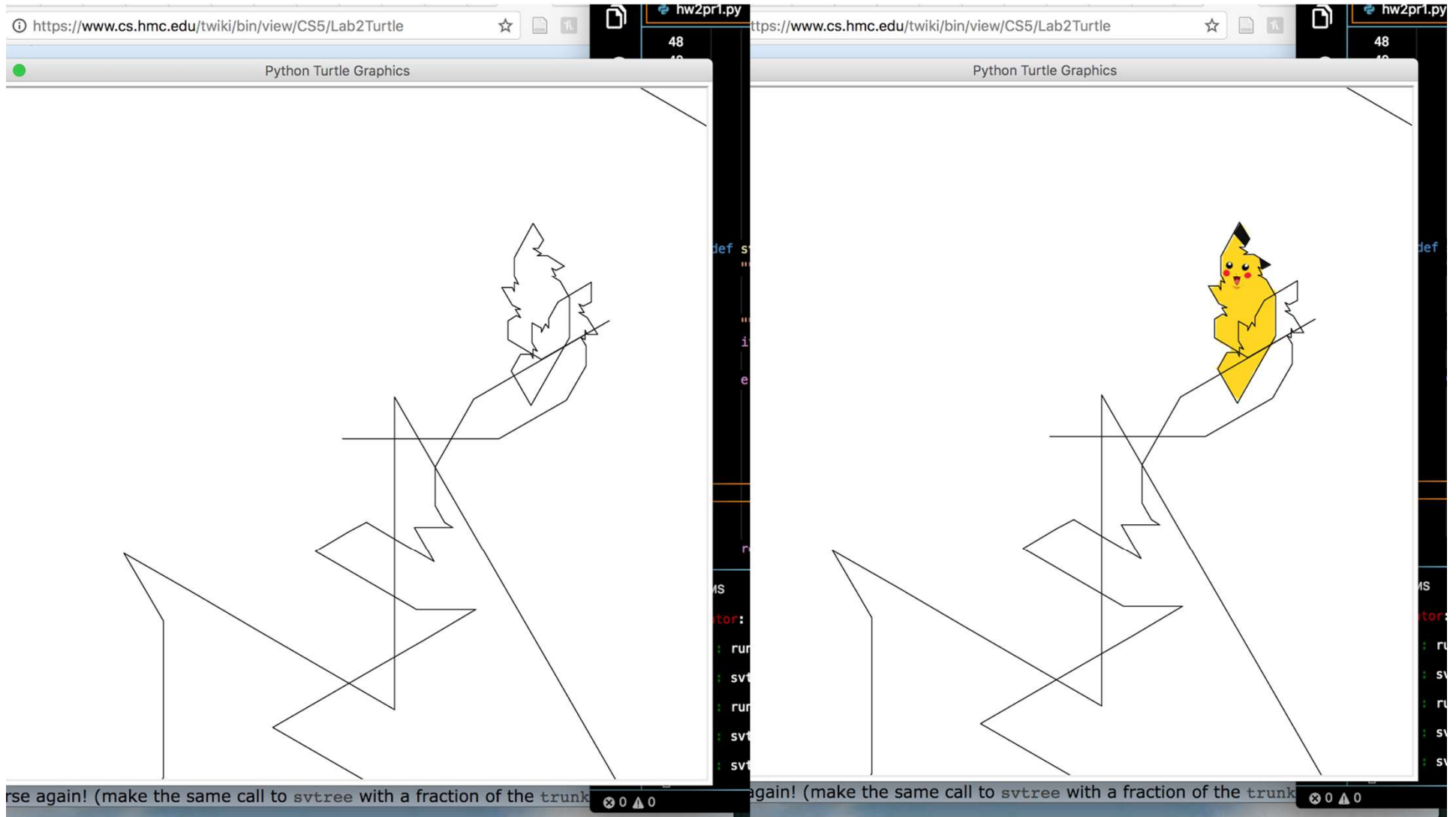
ee.PNG

Pet

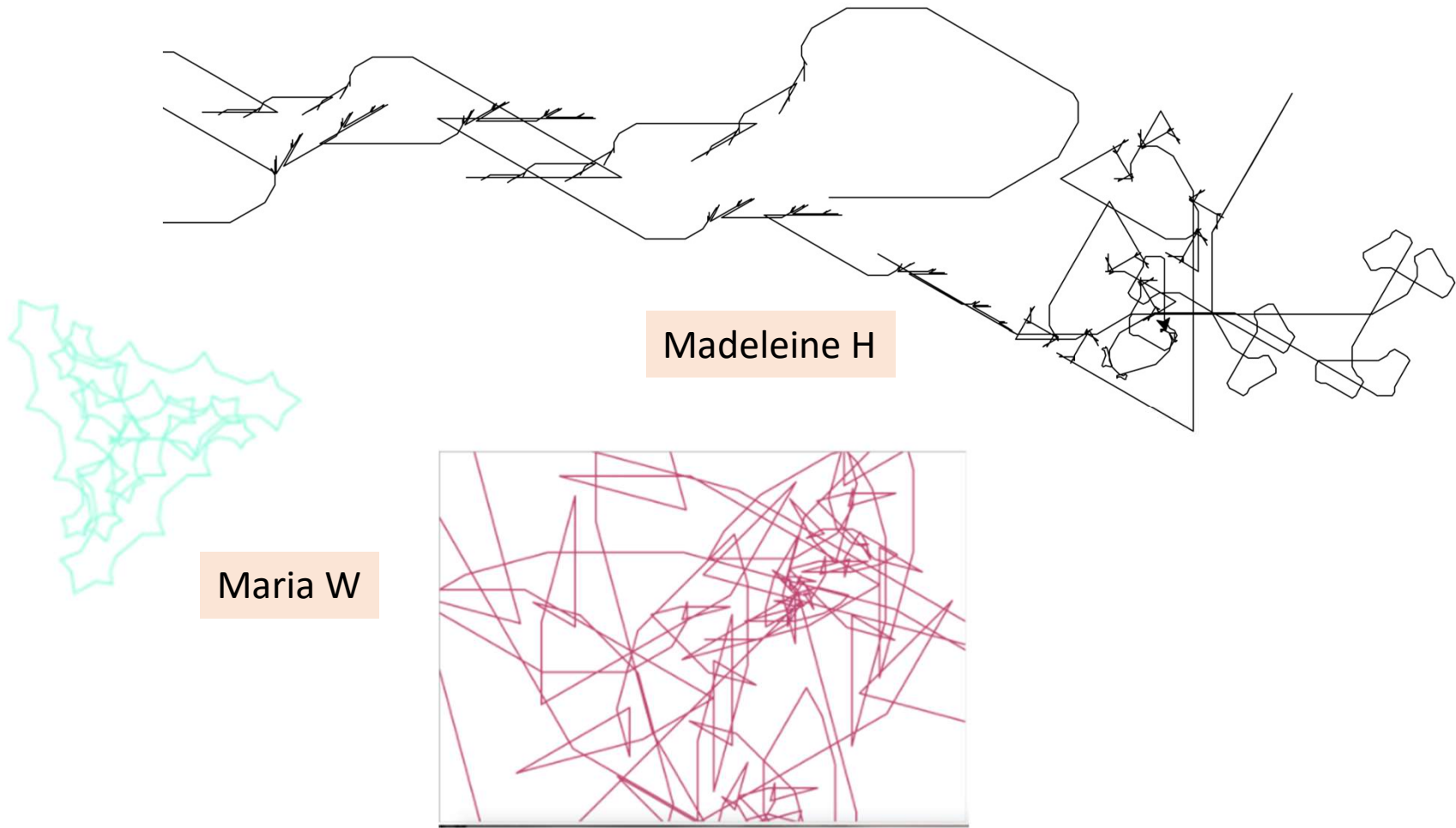
Vaiva P.



Please find enclosed my extra credit assignment from Lab. I have included the original screenshot of my failure, followed by a badly rendered **Pikachu** version.



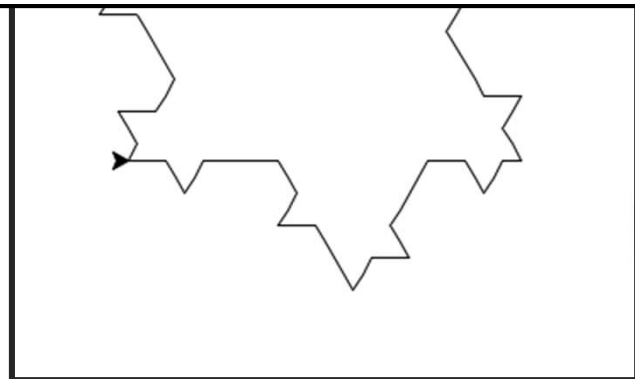
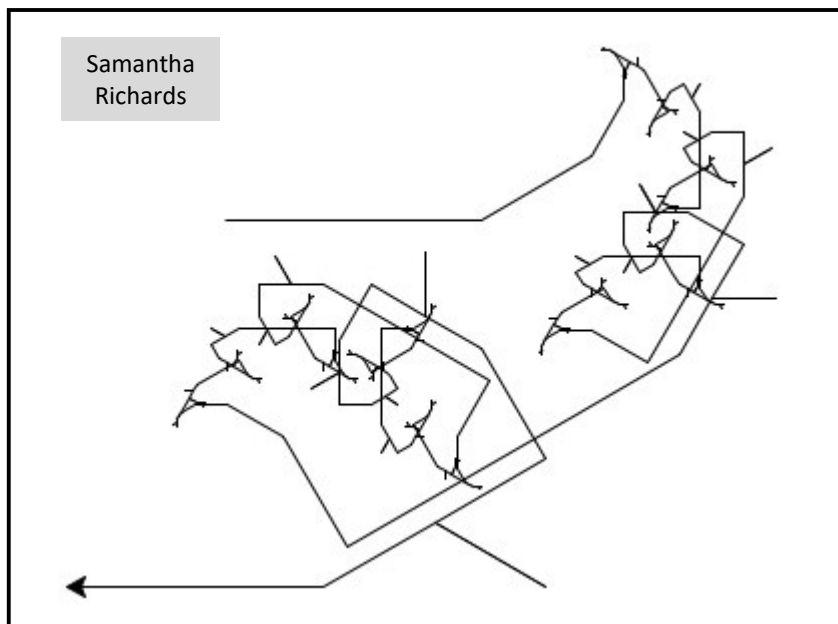
Rachel L



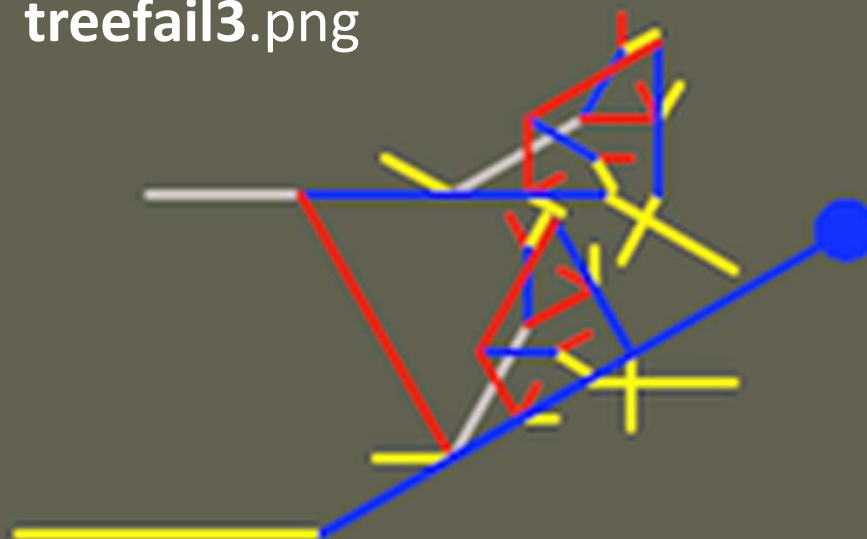
Maria W

Madeleine H

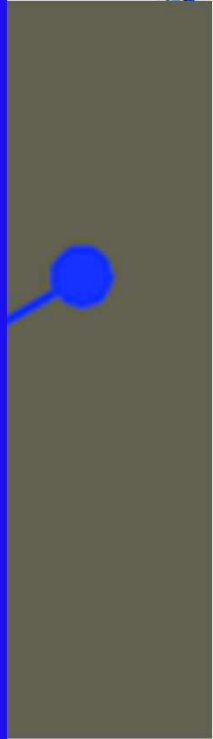
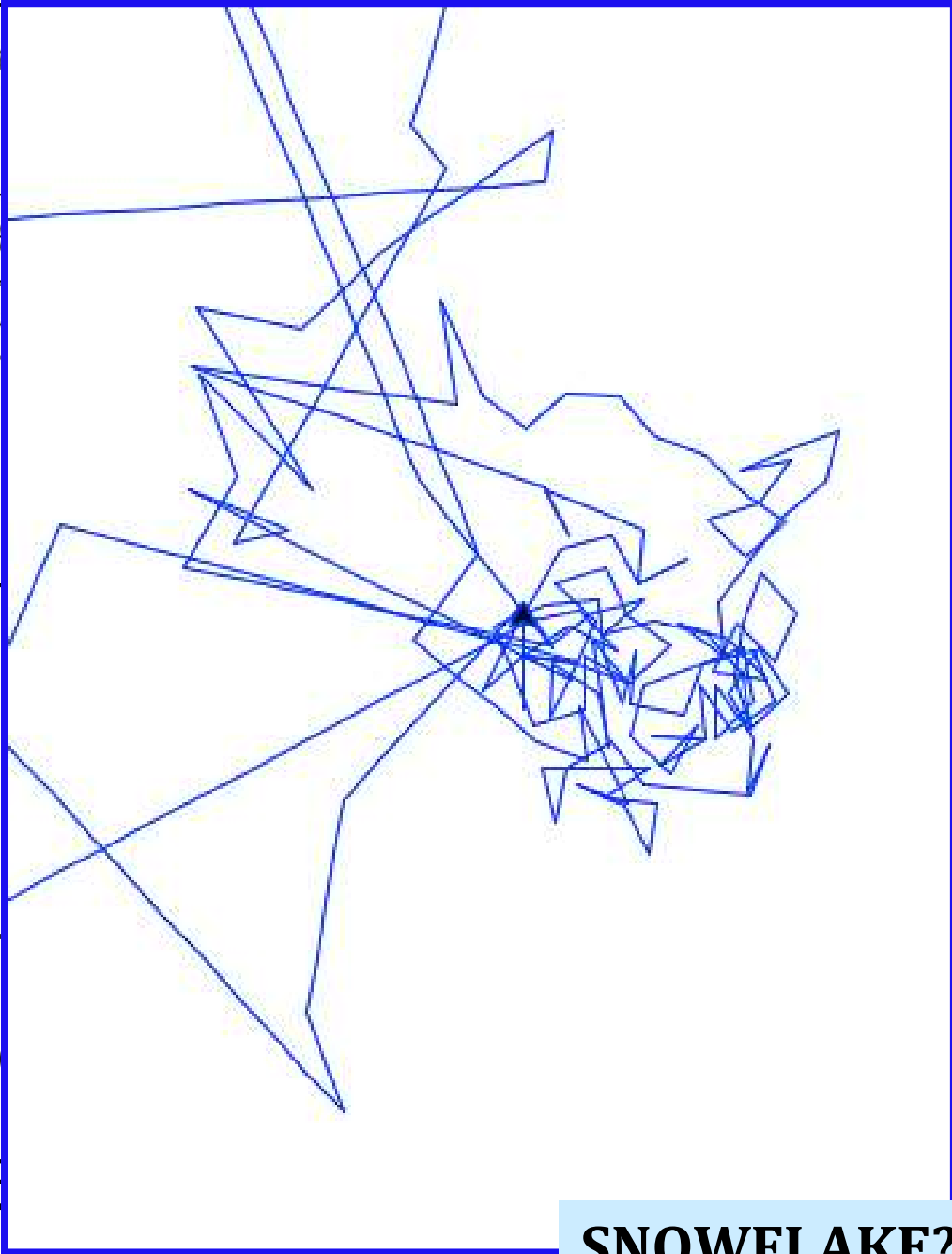
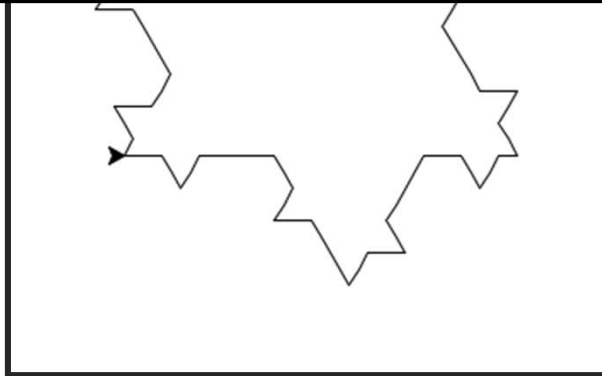
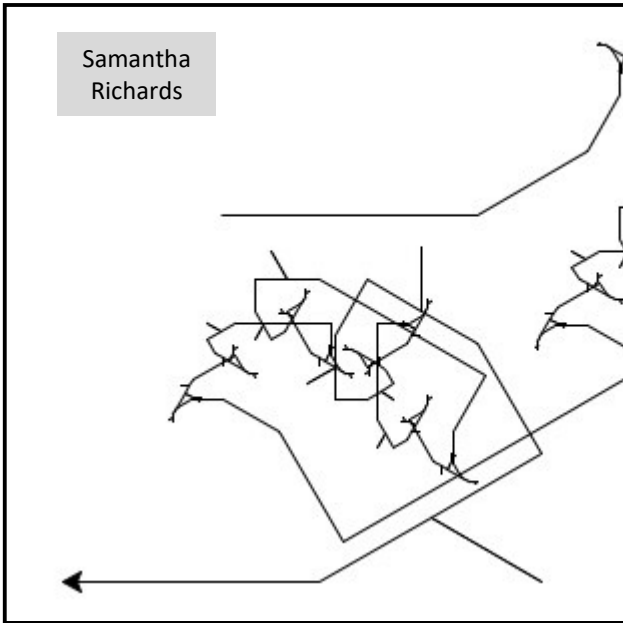
It all depends on how you
define *success* ...



treefail3.png



It all depends on how you
define *success* ...



It all d
de

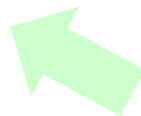
SNOWFLAKE?!



a turtle-drawn portrait from turtle graphics ...

Whoa!
'12

Back to bits...



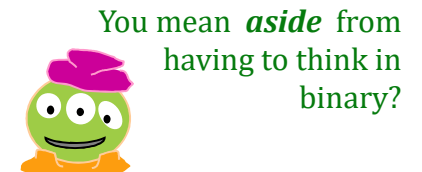
not the original name...

Lab 4: Computing in binary

base 2	=	base 10
... 4 2 1		100 10 1
		141

↑

This first step of **left-to-right** conversion into binary is tricky to program... *Why?*



Lab 4: Computing in binary

base 2	=	base 10
128 64 32 16 8 4 2 1		100 10 1
		141



This first step of **left-to-right** conversion into binary is tricky to program... *Why?*

It's tricky to find the largest power needed...

b.d. ~ binary digit ~ **bit**

"bit" first appeared in print in 1948 (Claude Shannon)

Orders Let f word (word) be 2 orders, each bit = $C(A) = \text{Control} + \text{Data}$

Address in M1	Address in M2	End of data	Address in M2	SHL, R	+	1-2	17	17	17
7	0	7	0	1	0	1	0	1	0
(1-8), 7	(10, 20)	(11-12)	(8-9), 9	(10, 11)	1-2	Allied	Attn		
(11, 11)			(11, 11)	2, 11		Prim	Prim		
(13-16)						2-10	for All		
17			17						

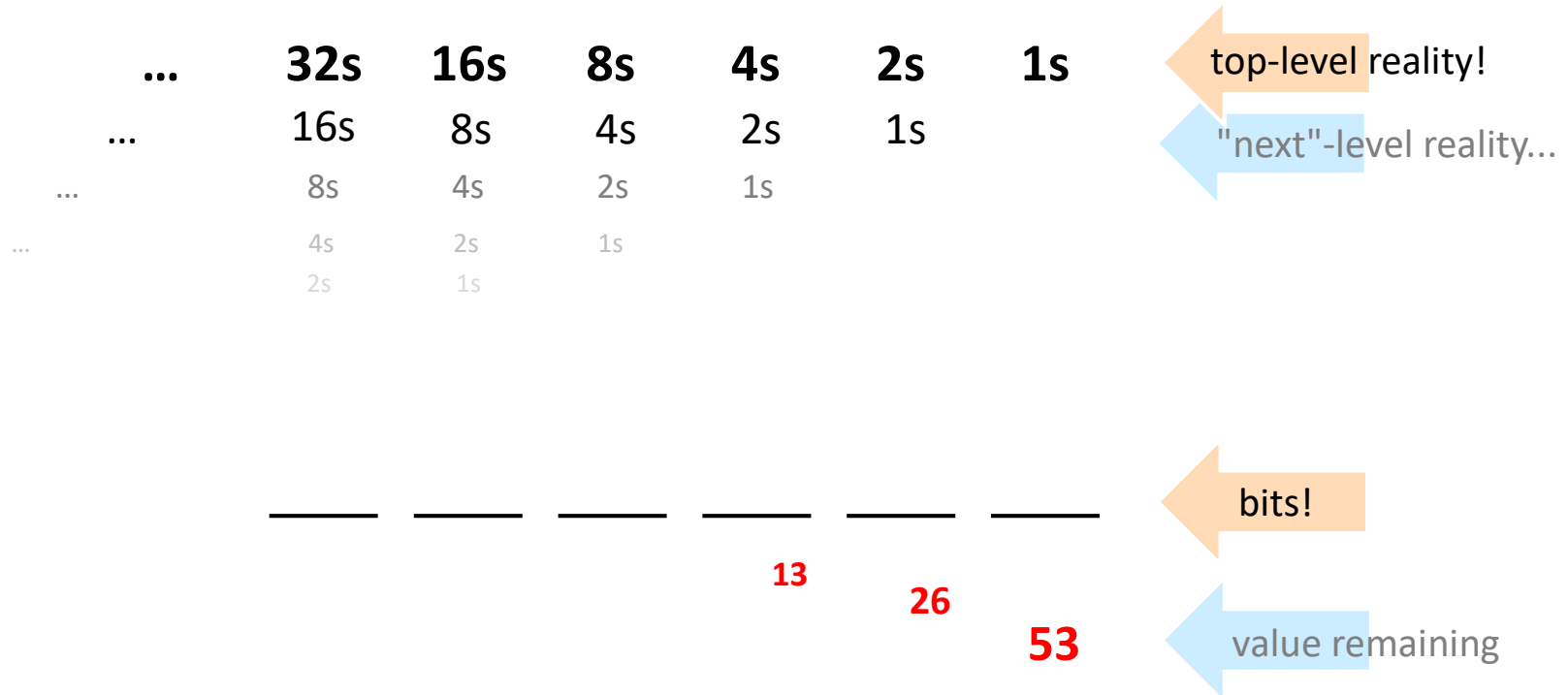
early document allocating different bits to control or data portions of a processor's work

Extra! Can you figure out the **last binary digit** (bit) of **53** without determining any earlier bits? The last **two**? **three**?

All of them?

53

in the end,
we need
"53"-worth
of value

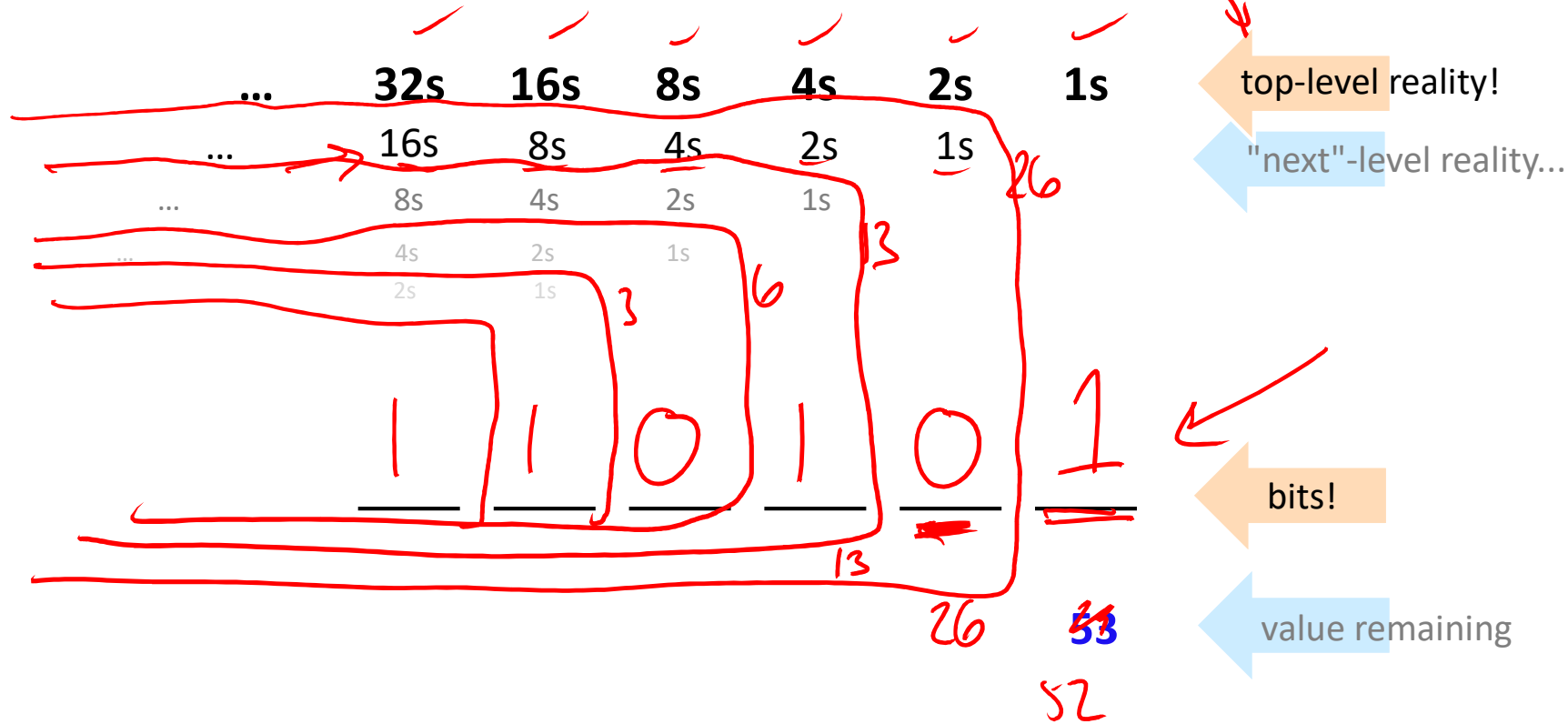


Extra! Can you figure out the last binary digit (bit) of **53** *without determining any earlier bits?* The last two? three?

All of them?

53 ✓

in the end,
we need
"53"-worth
of value



Extra! Can you figure out the last binary digit (bit) of **53** without determining any earlier bits? The last two? three?

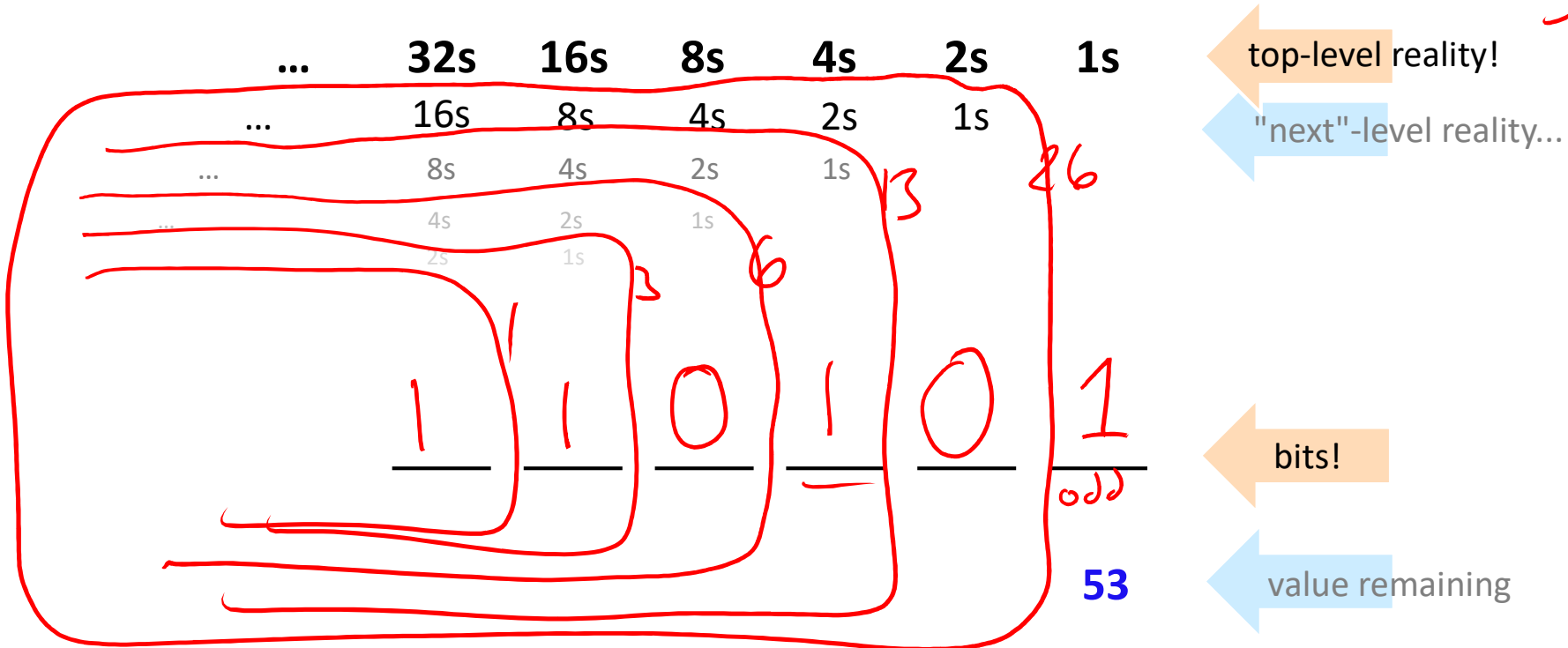
All of them?

1 1 0 1 0 1

53

in the end,
we need
"53"-worth
of value

52

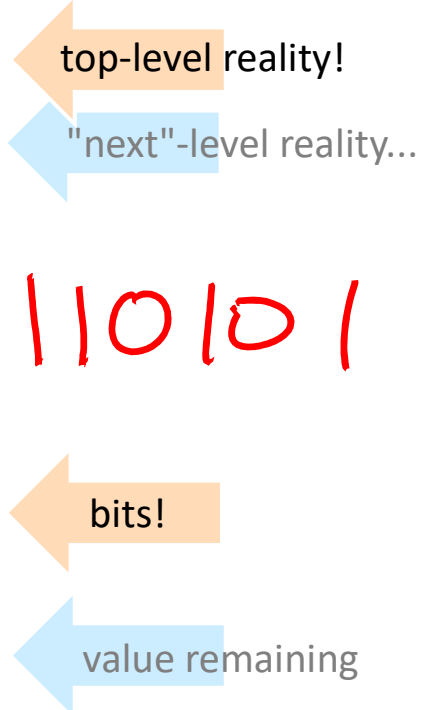
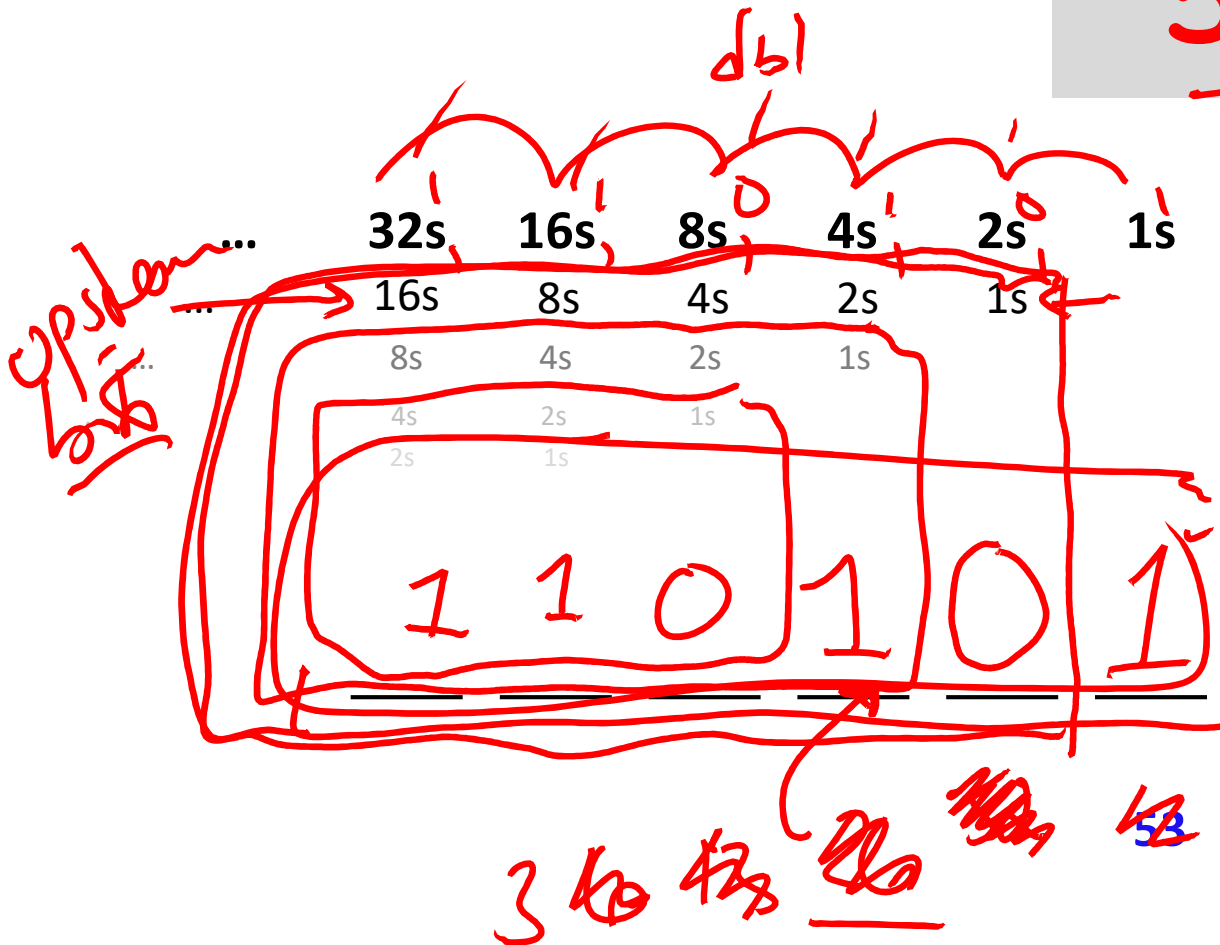


Extra! Can you figure out the last binary digit (bit) of **53** *without determining any earlier bits?* The last two? three?

All of them?

53

in the end,
we need
"53"-worth of
value

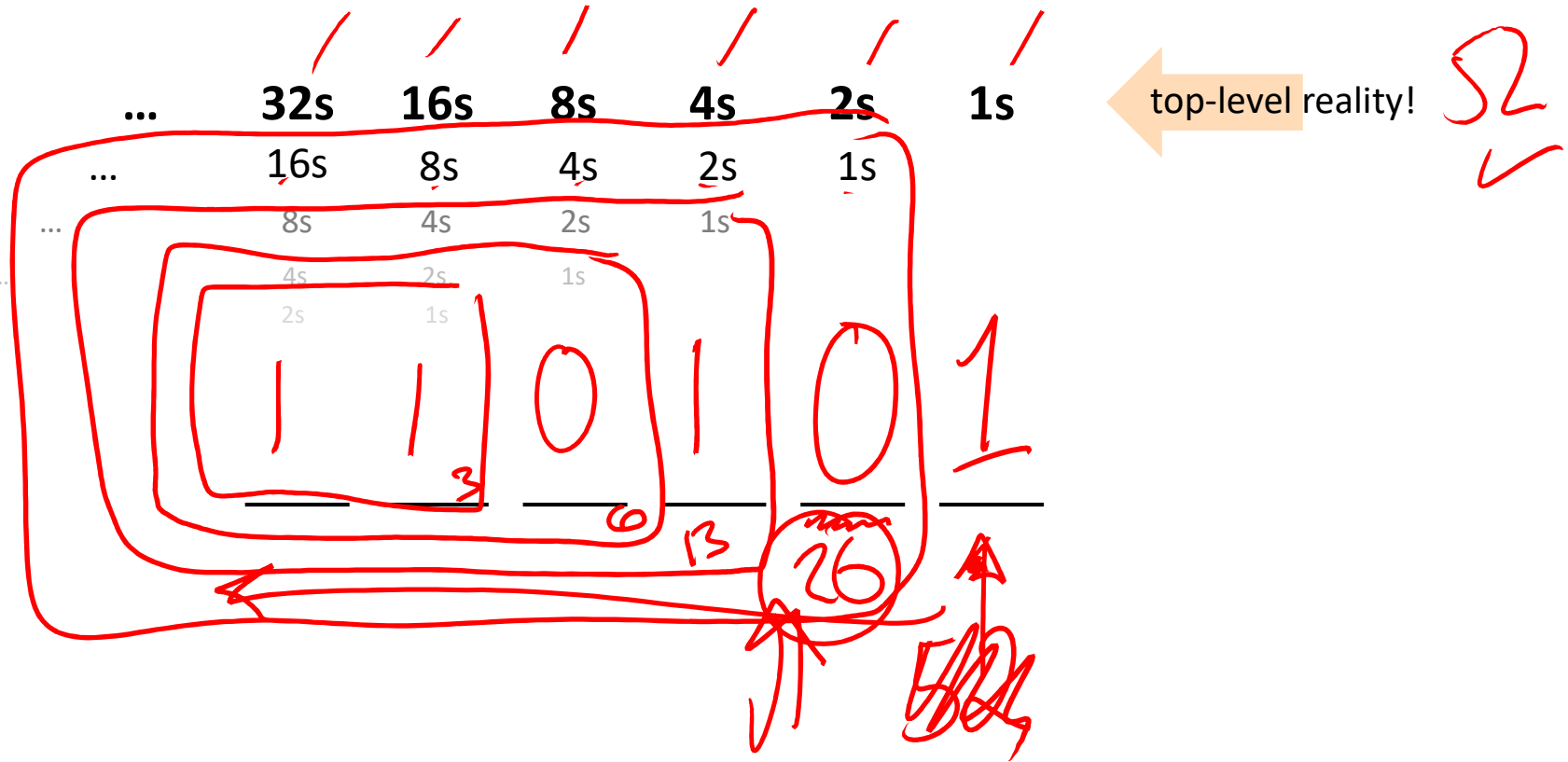


110101

Extra! Can you figure out the last binary digit (bit) of **53** without determining any earlier bits? The last two? three?

All of them?

Extra! Can you figure out the last binary digit (bit) of **53** ←
without determining any other bits? The last two? 3? All?

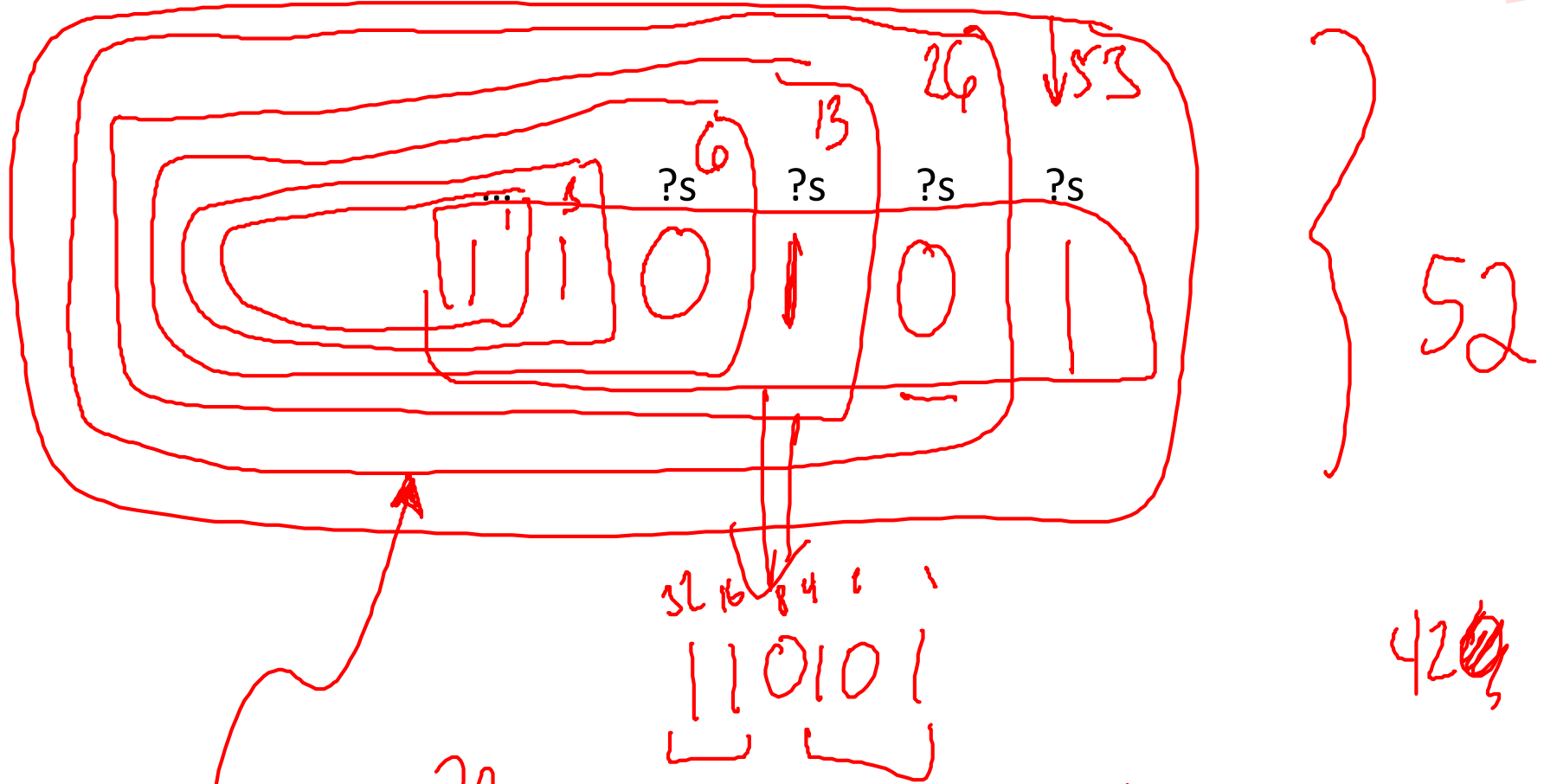


53

in the end,
 we need
 "53"-worth
 of value

Extra! Can you figure out the last binary digit (bit) of **53** without determining any other bits? The last two? three?

All?



$52/2 = 26$

32 16 8 4 2 1
110101

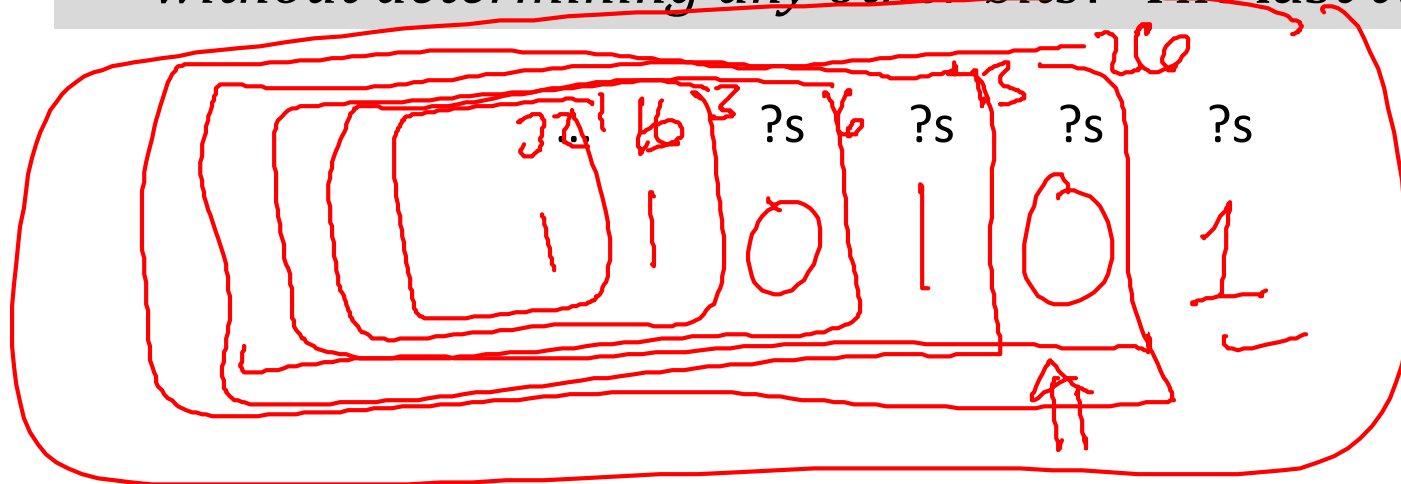
42

53

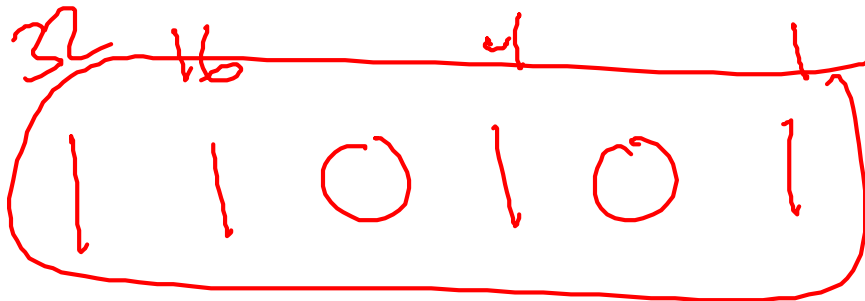
in the end, we need "53"-worth of value

Extra! Can you figure out the last binary digit (bit) of **53** without determining any other bits? The last two? three?

All?



53 value



32 value
Remaining

53

in the end,
we need
"53"-worth
of value

Lab 4: Converting to binary...

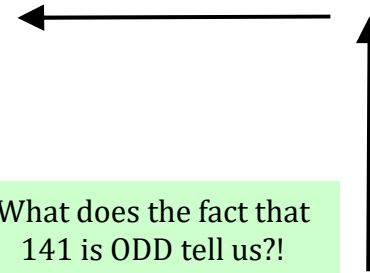
base 10

100 10 1
141

=

base 2

128 64 32 16 8 4 2 1



Try **right-to-left!**

141

=

10001101

128 64 32 16 8 4 2 1

answer

Lab 4: Converting to binary...

base 10

100 10 1
141 //2
70 //2
35 //2
17 //2
8 //2
4 //2
2 //2
1

base 2

128 64 32 16 8 4 2 1
10001101

What does the fact that 141 is ODD tell us?!

Try **right-to-left!**

141

10001101

answer

Lab 4: Computing in binary

base 10

100 10 1
141

=

base 2

128 64 32 16 8 4 2 1
'10001101'



Right-to-left works!

`numToBinary (N)`

You'll write these right! (-to-left)



`n2b (141)`

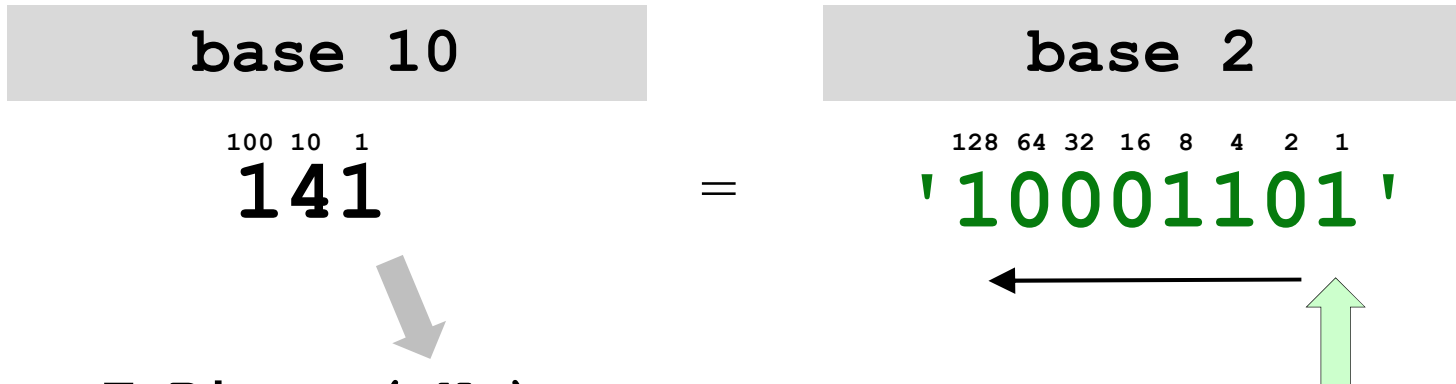
`binaryToNum (S)`

we need to *represent* binary numbers with **strings**



`b2n ('10001101')`

Lab 4: Computing in binary



```
def numToBinary( N ):
```

```
    if N == 0:
```

```
        return ''
```

```
    elif N%2 == 0:
```

```
        return numToBinary(  ) + 
```

```
    else:
```

```
        return numToBinary(  ) + 
```

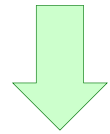
empty string means 0

If N is even, what is the final bit?

How much VALUE is left to convert!?

If N is odd, what is the final bit?

Lab 4: *Fleek* binary conversion !



```
def numToBinary( N ):  
    if N == 0: return ''  
    else: return numToBinary(N//2) +
```

```
def numToBinary( N ):
```

```
    if N == 0:
```

```
        return ''
```

```
    elif N%2 == 0:
```

```
        return numToBinary( N//2 ) + '0'
```

```
    else:
```

```
        return numToBinary( N//2 ) + '1'
```

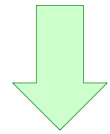
empty string means 0

If N is even, what is the final bit?

How much VALUE is left to convert!?

If N is odd, what is the final bit?

Lab 4: *Fleek* binary conversion !



```
def numToBinary( N ):  
    if N == 0: return ''  
    else: return numToBinary(N//2) + str(N%2)
```

```
def numToBinary( N ):  
    if N == 0: return ''  
    else:  
        return numToBinary( N//2 ) + '0'  
    else:  
        return numToBinary( N//2 ) + '1'
```

Use this page
in lab today!!

If N is even, what is the final bit?

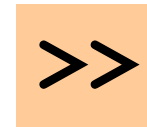
How much VALUE is left to convert!?

If N is odd, what is the final bit?

Reasoning, *bit by bit*



left-shift



right-shift



and



or

and
(both)

&

|

or
(either)

bitwise and

5:	101
6:	110
<hr/>	
&	100

5 & 6

4

bitwise and

11:	1011
5:	0101
<hr/>	
&	

11 & 5

bitwise or

5:	101
6:	110
<hr/>	
	111

5 | 6

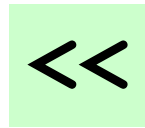
7

bitwise or

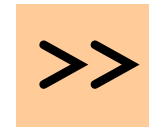
11:	1011
5:	0101
<hr/>	

11 | 5

Reasoning, *bit by bit*



left-shift



right-shift



and



or

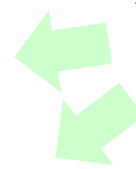
left-shift by 1

11

110

3 << 1

6



What does *left-shifting* do to the **value** of a #?

left-shift by 2

11

1100

3 << 2

12

right-shift by 1

101010

10101

42 >> 1

21



What does *right-shifting* do to the **value** of a #?

42 >> 2 ?

Being bit-wise

You **don't** need to convert to binary for these three...

7 << 1

left-shift

5 << 4

170 >> 2

right-shift

Try these for a bit...

14: 1110

9: 1001

You **do** need to use binary for these two!

14 | 9
or

14 & 9
and

In processors **shifts**, **ands**, **ors**, **adds**, and **subtractions** are *very fast*, whereas **multiplying**, **dividing**, and **mod**, which are relatively **slow**.

Given this, what is a way to compute these expressions using *only fast* operations, maybe in combination?

$N // 4$

$N * 7$

$N * 17$

$N \% 16$

extra fleek!

Being bit-wise

You **don't** need to convert to binary for these three...

7 << 1 → 14
left-shift

5 << 4 → 80

170 >> 2 → 42
right-shift

Try these for a bit...

14: 1110
9: 1001

You **do** need to use binary for these two!

14 | 9 → 15
or 1111

14 & 9 → 8
and 1000

In processors **shifts**, **ands**, **ors**, **adds**, and **subtractions** are *very fast*, whereas **multiplying**, **dividing**, and **mod**, which are relatively **slow**.

Given this, what is a way to compute these expressions using *only fast* operations, maybe in combination?

- N // 4
- N * 7
- N * 17
- N % 16
extra fleek!

Let's first look at **why** you'd bother ... !?

Intel x86 processor instructions and their speeds (2016)

In processors **shift, and, or, add, and subtract** are ***much faster*** than **multiply, divide, and mod**, which are ***relatively slow***.

Table C-16. General Purpose Instruction:

Instruction	Latency ¹	Throughput
	first time in a row	rest of times (in a row)
CPUID	0F_3H	0F_3H
ADC/SBB reg, reg	8	3
ADC/SBB reg, imm	8	2
ADD/SUB	1	0.5
AND/OR/XOR	1	0.5
BSF/BSR	16	2
BSWAP	1	0.5
BTC/BTR/BTS	8-9	1
CLI		
CMP/TEST	1	0.5
DEC/INC	1	0.5
IMUL r32	10	1
IDIV MOD is the same	66-80	30

Intel® 64 and IA-32 Architectures



Old Microsoft *systems-interview* question, #42:

42. Give a fast way to multiply a number by 7.

43. How would you check for finding out where to find...

Intel x86 processor instructions and their speeds (2014)

In processors **shift**, **and**, **or**, **add**, and **subtract** are *much faster* than **multiply**, **divide**, and **mod**, which are *relatively slow*.

Table C-16. General Purpose Instruction:

Instruction	Latency ¹	Throughput
CPUID	0F_3H	0F_3H
ADC/SBB reg, reg	8	3
ADC/SBB reg, imm	8	2
ADD/SUB	1	0.5
AND/OR/XOR	1	0.5
BSF/BSR	16	2
BSWAP	1	0.5
BTC/BTR/BTS	8-9	1
CLI		
CMP/TEST	1	0.5
DEC/INC	1	0.5
IMUL r32	10	1
IDIV MOD is the same	66-80	30

Given this, what is a way to compute these statements using combinations from only the *fast* operations above?

$$N // 4 \rightarrow N \gg$$

$$N * 7 \rightarrow$$

$$N * 17 \rightarrow$$

$$N \% 16 \rightarrow$$

Insight: Ancient Egyptian Multiplication

Ancient Egyptian multiplication

From Wikipedia, the free encyclopedia



Next time?

Insight: Ancient Egyptian Multiplication

$$21 \times 6 == 126$$

$$21 \quad 6$$

AEM/RPM algorithm

Write the factors in two columns.

Repeatedly **halve** the LEFT and **double** the RIGHT. (toss remainders...)

Pull out the RIGHT values where the LEFT values are ***odd***.

Sum those values for the answer!

Why does this work?

$$11 \times 15 == 165$$

$$11 \quad 15$$

← Try it here

or RPM...



Здравствулте!
Американские
Студенты

Buddy, can
you spare
an eye?



*When traveling,
always insist on
binary
accommodations ... !*

See you at lab
– *in just a **bit!***

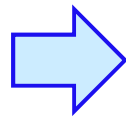
This room is a 10!



Insight: Multiplication in binary w/o bits!

21 6
10 12
 5 24
 2 48
 1 96

Decimal



 110₆
x 10101₂₁

 110₆ 6
 0000₁₂
 11000₂₄ 24
 000000₄₈
+ 1100000₉₆

 1111110₁₂₆ + 96

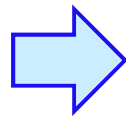
 126

Binary

Insight: Multiplication in binary w/o bits!

11 15
5 30
2 60
1 120

Decimal



1111₁₅
x 1011₁₁

1111₁₅ 15
11110₃₀ 30
000000₆₀
+ 1111000₁₂₀

10100101₁₆₅

15
30
+ 120

165

Binary