Physics 31600 R. Wald
Classical Mechanics Autumn, 2002

Problem Set V

Due in class on Tuesday, November 12

1) A body of mass m is constrained to move only horizontally (i.e., in the x-direction) and is attached to a fixed support by a spring of spring constant k. A particle also of mass m is attached to the first body by a massless rod of length l as shown. A gravitational field is present, so the potential energy of the second particle is mgz. Suppose that, initially, the first body is at the equilibrium position of the spring, but the particle below it is displaced from the vertical by a small angle θ_0 as shown. Solve for the subsequent motion of both bodies.

2) Consider a "double pendulum' with each pendulum of mass m and length l, and with a vertical gravitational force $-mg\hat{z}$ acting on each mass. At $t = 0$ the double pendulum is vertical, but the top mass is given a small "kick", so that it has "velocity" $\dot{\theta}_{10}$. Solve for the subsequent motion of the masses.

3) Consider a particle of mass m moving in a 2-dimensional plane, with a potential, $V = V(r)$, which depends only on the radius r.
(a) Use the symmetry under rotations to reduce the problem to one of one-dimensional motion in r.

(b) Explicitly write down the equation of motion in r and show that a circular orbit exists at every radius at which $dV/dr > 0$.

(c) Obtain the general solution for the radial motion resulting from an arbitrary small perturbation about a circular orbit of radius r_0. (Allow the perturbation to change p_ϕ.) Determine the conditions on V such that this circular orbit is stable.