Physics 31600 | R. Wald
Classical Mechanics | Autumn, 2002

Problem Set VI

Due in class on Tuesday, November 26
(Note that this problem set is due one week after the midterm.)

1) Consider a space M with coordinates y^μ and define the Poisson-bracket-like map on functions on M by

$$W(f,g) = \sum_{\mu,\nu} W^{\mu\nu} \frac{\partial f}{\partial y^\mu} \frac{\partial g}{\partial y^\nu}$$

where $W^{\mu\nu}$ is any antisymmetric matrix (i.e., $W^{\nu\mu} = -W^{\mu\nu}$) whose components are constant (i.e., independent of y^μ). Show that W satisfies the Jacobi identity, i.e.,

$$W(W(f,g),h) + W(W(h,f),g) + W(W(g,h),f) = 0$$

2) Let $f(q,p;t)$ and $g(q,p;t)$ be (possibly time-dependent) observables on a $2n$-dimensional phase space with Hamiltonian $H(q,p;t)$.
(a) Suppose that both f and g are constants of motion, i.e., $df/dt = dg/dt = 0$. Show that $\Omega(f,g)$ also is a constant of motion.
(b) Suppose that H itself is a constant of motion and $f(q,p;t)$ is a constant of motion. Show that $\frac{\partial^n f}{\partial t^n}$ also is a constant of motion.

3) Consider a particle of mass m in ordinary 3-dimensional space. Let L_x, L_y, L_z denote the usual Cartesian components of angular momentum of the particle, viewed as functions on its 6-dimensional phase space.
(a) Show that $\Omega(L_x, L_y) = L_z$. (Thus, according to problem 2, if L_x and L_y are constants of motion, then so is L_z.)
(b) Suppose that an observable $f(\vec{x},\vec{p})$ depends on $\vec{x}$ and $\vec{p}$ only in the scalar combinations $\vec{x}\cdot\vec{x}$, $\vec{x}\cdot\vec{p}$, and $\vec{p}\cdot\vec{p}$, i.e., suppose that f can be written in the form

$$f(\vec{x},\vec{p}) = h(\vec{x}\cdot\vec{x}, \vec{x}\cdot\vec{p}, \vec{p}\cdot\vec{p})$$

for some function h. Show that $\Omega(f, L_i) = 0$.