Physics 35200 R. Wald
Statistical Mechanics Spring, 2002

Problem Set II

Due in class on Monday, April 15

1) (a) Consider a classical gas composed of N non-interacting point particles, each of mass m, which is confined to a volume V. Calculate the total volume $\mathcal{V}[\Gamma_E]$ of the "energy shell" Γ_E. You may use the fact (shown in Appendix E of Mazenko) that the "area" of a unit sphere in a d-dimensional space is $2\pi^{d/2}/\Gamma(d/2)$, where Γ denotes the gamma-function (so Γ(n)=(n-1)! when n is an integer; Stirling's formula provides an excellent approximation for $\Gamma(x)$ for large x).

(b) Two boxes each have volume V and each contain a classical gas of N particles with energy E, as in part (a). The boxes are now placed in "thermal contact" with each other, so that they can exchange energy with each other, but not change their volume or number of particles. On account of this energy exchange, the energy, E_1, within the first box now may vary between 0 and 2E. Obtain the probability distribution, $p(E_1)dE_1$, that the energy contained within the first box will be within dE_1 of E_1 if one examines the system at a "random time".

2) (Counts as 2 problems) (a) Consider a non-relativistic particle of mass m in one dimension in a box of size L, with infinite potential walls (i.e. $V = 0$ for $0 < x < L$; $V = \infty$ otherwise). Find its energy eigenstates and eigenvalues.

(b) Now generalize the results of part (a) to the case of N, non-interacting, distinguishable particles in 3-dimensions in a cubic box of size L with infinite potential walls.

(c) For the situation of part (b), derive a formula for the number of quantum states with total energy between E and $E + \Delta E$, where $\Delta E/E << 1$, but ΔE is large enough to include many states. (Hint: Find the average density of states in $3N$-dimensional "k-space" from part (b) and multiply by the appropriate volume.)

(d) For a 1m^3 box of (monatomic) gas at ordinary pressure and room temperature conditions we have $N \sim 10^{25}$ and the total energy is $E = \frac{3}{2}NkT \sim 10^{12}$ ergs. Assume that the mass of each particle is $m \sim 10^{-22}$gm. Use the result of (c) to estimate the number of quantum states with total energy within 1 erg of $E = 10^{12}$ ergs, assuming that the particles in the gas are distinguishable.

(e) For the system of part (d), estimate the probability that a given "single particle energy level" will be occupied by more than one particle, i.e., compare the number of single particle levels in the relevant energy range to the number of particles. (If this probability is small, a boson or fermion gas will behave in the same manner as a gas of distinguishable particles.)