Physics 35200 R. Wald
Statistical Mechanics Spring, 2002

Problem Set III

Due in class on WEDNESDAY, April 24

1) Consider a classical, ideal gas of N point particles, confined by a spherical box, so that the total angular momentum, $\vec{J}$, is conserved. In this case, dynamical motion will be confined to a $(6N-4)$-dimensional surface, $\Gamma_{E,\vec{K}}$, in Γ defined by the equations $H = E$ and $\vec{J} = \vec{K}$. If the dynamical evolution is appropriately ergodic on $\Gamma_{E,\vec{K}}$, the time-averaged behavior of the gas will be described by the "modified microcanonical ensemble" $\delta(H-E)\delta^3(\vec{J}-\vec{K})d^{3N}xd^{3N}p$. Parallel the arguments given in class to obtain the most probable distribution function, $f(\vec{x},\vec{p})$, in this case. (You may express your answer in terms of certain constants which you need not evaluate, but you must specify clearly what integrals need to be done to evaluate them.) Show that when $\vec{K} = 0$, we again obtain the Maxwell-Boltzmann distribution.

2) Consider a classical ideal gas of N particles of mass m in a volume V in a state described by the Maxwell-Boltzmann distribution.
(a) Calculate the average magnitude of velocity, $<|\vec{v}|>$, of the particles, and the "root mean square dispersion", $\Delta v = [<v^2> - <|\vec{v}|>^2]^{1/2}$, about $<|\vec{v}|>$.
(b) Calculate the pressure exerted by the gas on a wall resulting from elastic collisions of the gas with the wall.

3) Define the "Schmaxwell-Boltzmann" distribution function of a gas with N particles of mass m and total energy E by

$$F(\vec{x},\vec{p}) = ap^{-2}\exp(-bp^2/2m)$$

where $b = N/2E$ and $a = V^{-1}(N/2\pi)^{3/2}(2mE)^{-1/2}$. If one examines the gas at a "random time", what is the relative probability that the gas will be well described by the Schmaxwell-Boltzmann distribution rather than the Maxwell-Boltzmann distribution?