Problem Set IV

Due in class on Monday, April 29

1) (Counts as 2 problems) Consider a single, non-relativistic quantum mechanical particle in a box of volume V (with infinite potential walls) as in problem 2 of set II. As you found there, the states can be labeled by integers (n_x, n_y, n_z).

   a) Write down the formula for the energy, ϵ, of the particle as a function of V and (n_x, n_y, n_z). Now suppose that the volume V is changed by moving the walls of the box very slowly. In this case, (n_x, n_y, n_z) will remain fixed. Find the rate of change in energy, ϵ, with respect to volume, V, in this process, and thereby obtain a formula for the pressure, P, exerted by a single particle.

   b) Now use the distribution function $n(\epsilon)$ derived in class to obtain the total pressure, P, exerted by a gas of N particles (with η spin states) as function of the quantities α and β for the cases in which particles are (i) distinguishable, (ii) bosons, and (iii) fermions. (You need not evaluate the integrals in the expression for P in cases (ii) and (iii).) Show that in all three cases, we have $PV = 2E/3$.

   c) For the case of fermions, show that when $\beta \to \infty$ (i.e., $T \to 0$) with N held fixed, the quantity $\mu \equiv -\alpha/\beta$ goes to a finite limit. Evaluate μ at $T = 0$ and use the result of part (b) to obtain the pressure exerted by a non-relativistic fermion gas at $T = 0$.

2) In class we derived criteria under which boson and fermion ideal gases behave classically. Significant departures from classical behavior for these gases occur at low temperatures and high densities on account of the "statistics" of indistinguishable particles. However, as also mentioned in class, a quantum ideal gas of distinguishable particles also will depart from classical behavior on account of the discreteness of energy levels. For a quantum gas of N distinguishable particles of mass m in a volume V, estimate the temperature at which non-classical behavior would become manifest. Evaluate this temperature for the values of m and N/V typical of a gas at standard room temperature and pressure.