Problem Set V

Due in class on Monday, May 6

1) (Counts as 2 problems) a) Show that for a classical ideal gas (of N indistinguishable particles), the observable entropy, $\mathcal{S}$, associated with a coarse-grained distribution function, $\{n_i\}$, is given by $\mathcal{S} = -k\sum n_i \ln n_i$ (apart from an additive constant depending only upon the "cell size" in γ). Thus, in the "continuum limit", for a distribution function $f(\vec{x}, \vec{p})$ we have $\mathcal{S} = -k\int f \ln f d^3x d^3p$. Evaluate $\mathcal{S}$ for the Maxwell-Boltzmann distribution function and show that for this "most probable" case, $\mathcal{S}$ equals the "thermodynamic entropy", S, defined by $S = k\ln(\Omega/N!)$.

   b) For a quantum ideal gas, write down the formula for $\mathcal{S}$ for an arbitrary coarse-grained distribution $\{n_i\}$ of the particles into energy levels for the cases of (i) distinguishable particles, (ii) bosons, and (iii) fermions. Substitute the "most probable" distribution derived in class for each case to obtain the thermodynamic entropy, S. Pass to the continuum limit to get integral expressions for S in terms of the parameters α and β. (You need not evaluate these integrals in the boson and fermion cases.) For distinguishable particles, compare your result with the result of problem 2(c) of set II.

   c) Show that in all 3 cases of part (b), the temperature, T, of the gas is related to the Lagrange multiplier β by $\beta = 1/kT$.

2) Consider a lattice with N sites. At each site is placed a spin-1/2 particle with magnetic moment μ. A magnetic field B is applied, so the energy of each particle is $-\mu B$ if its spin is "up" and $+\mu B$ if its spin is "down".

   a) Calculate the density of quantum states of the system with total energy E and thereby obtain a formula for the entropy, $S(E, N)$. Does it make any difference whether the particles are treated as distinguishable or as fermions?

   b) Calculate the temperature, T, as a function of E and N. Under what conditions is T negative?