Problem Set VI

Due in class on Monday, May 13

1) A box of total volume V is divided by a partition into 2 equal volumes. In the first volume is a classical monatomic ideal gas with N_1 particles of species 1 at tempertature T_1, and in the second volume is a classical monatomic ideal gas with N_2 particles of species 2 at tempertature T_2. The partition is now removed and the total system is allowed to come to thermal equilibrium. Calculate the change in entropy occurring in this process under the assumptions that (a) species 1 and 2 are distinguishable, and (b) species 1 and 2 are indistinguishable.

2) An ideal monatomic gas of N particles is confined in an insulated cylindrical box of cross-sectional area A by a moveable piston. The piston is attached to a spring of spring constant K. Initially, the gas is at temperature T_0 and the piston is held in place by an external force at the equilibrium position of the spring, $z = z_0$. The external force is then removed, and the piston starts to oscillate under the action of the spring and gas pressure. These oscillations of the piston eventually damp out, and the gas eventually returns to a themal equilibrium state. What is the final temperature, T, of the gas?

3) In the winter, one wishes to maintain one's house at temperature T_1. On account of imperfect thermal insulation of the house, this requires an input of energy, ΔE, each day. The outside air and ground have a huge energy, E_0, but they are at a temperature $T_0 < T_1$. By the arguments given in class, energy cannot flow of its own accord from outside the house to inside the house (although, of course, it does flow the other way). Thus, one will need to purchase some energy, ϵ, from the power company each day. However, show that it is not necessary to purchase the full ΔE of energy, i.e., that it is possible to use a device (known as a "heat pump") which extracts some energy from the outside air or ground. Derive a lower limit on ϵ. Design a heat pump, using a box of an ideal gas, which achieves this limit.