Problem Set VII

Due in class on WEDNESDAY, May 29

1) a) Calculate the chemical potential, μ, of a classical, monatomic ideal gas of particles of mass, m, as a function of m, its temperature, T, number of particles, N, and volume V. How is the formula for μ modified if the single particle energy is "offset" from zero by ϵ_0, i.e. if

$$H = \sum_i^N \left\{ \frac{|\vec{p}_i|^2}{2m} + \epsilon_0 \right\}$$

b) Suppose a dynamical system contains 3 species of particles, labeled A,B,C. Suppose that the chemical reaction $A + B \rightleftharpoons C$ can occur. Show that a necessary condition for thermal equilibrium of the system is $\mu_A + \mu_B = \mu_C$.
c) Suppose each of the particle species A, B, C of part (b) can be treated as a monatomic, ideal gas. (For example, A could represent free electrons, B could represent free protons, and C could represent neutral hydrogen atoms.) Suppose further, that the total energy is $E = E_A + E_B + E_C - \epsilon N_C$. Here, E_A, E_B, and E_C are the usual kinetic energies of the three gases, whereas ϵ is the binding energy of the C-particle (so the energy of the C-particle is "offset" from zero). Assume, for simplicity, that $N_A = N_B$. The system is in thermal equilibrium at temperature T. Calculate the ratio N_A^2/N_C as a function of T, ϵ, and the masses of the particles.

2) Suppose we have a dynamical system in contact with a "heat bath" but now, in addition to being able to exchange energy with the heat bath, suppose the system can "exchange volume" with the bath, i.e. the wall separating the systems is mobile. (Physically, this corresponds to "holding our system at fixed pressure".) Show that the probability that our system is in a given macroscopic state (with respect to a coarse-grained observable $\mathcal{O}$) is proportional to $\exp(-\mathcal{G}/kT)$, with $\mathcal{G} \equiv E - T_0 \mathcal{S}_\mathcal{O} + P_0 V$, where T_0 and P_0 are the temperature and pressure of the heat bath, E and V are the energy and volume of our system, and $\mathcal{S}_\mathcal{O}$ is its observable entropy with respect to $\mathcal{O}$ for the given value of E and V.

3) A cubic box of side L containing a classical gas of N particles of mass m is placed in a uniform gravitational field of strength (i.e., acceleration) g. (The box is oriented so that the gravitational field is normal to a pair of faces.) The gas is in thermal equilibrium at temperature T.
a) Calculate the number density distribution, $n(\vec{x})$, of the gas.
b) Calculate the heat capacity (at constant volume) of the gas.

4) One wishes to measure the mass, m, of a classical body by weighing it. The body is hung on a vertical spring of spring constant K in a uniform gravitational field, g, and the mass of the body is then determined from the formula $m = -Kz/g$, where z denotes the displacement of the spring from its equilibrium position. Suppose K and g are known exactly and that z can be measured with perfect accuracy, so that the only source of error is the buffeting of the mass by air molecules at temperature T. Suppose that only one reading of z is taken. What uncertainty, Δm, should be assigned to the measurement of m?

5) a) Consider a classical spinning particle with magnetic moment $\vec{\mu} = \mu_0 \vec{S}$. A magnetic field $\vec{B}$ is applied, resulting in an interaction energy $E = -\vec{\mu} \cdot \vec{B}$. The particle is now placed in contact with a heat bath at temperature T. Find the probability distribution $p(\theta)d\theta$ for finding the angle between $\vec{S}$ and $\vec{B}$ to be within $d\theta$ of θ. If N such particles are present, what is the expected magnetization?
b) Now consider a quantum mechanical particle of spin s, with $\vec{\mu}_0 = \mu_0 \vec{S}$. Then S_z can take any of the $(2s+1)$ values, $-s, -s+1, \ldots, s-1, s$. Again a magnetic field $\vec{B}$ (in the z-direction) is applied and the particle is placed in contact with a heat bath at temperature T. Calculate the probability $p(S_z)$ of each of the allowed values of S_z. If N such (distinguishable) particles are present, what is the expected magnetization? Compare your answers to these questions to the classical case in the limit of large s.