

The Ohmic Region of a Lightbulb

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Abstract

While a light bulb does not obey Ohm's law at high currents, there is a theoretical ohmic region at low currents. The resistance of the lightbulb at room temperature, R_0 , was calculated to be $5.13 \pm 0.10 \Omega$, the dominant error arising from the ambient temperature fluctuations around the apparatus. The calculated temperature of the filament at the bulb's maximum rated current was $2540 \pm 40 \text{ K}$, so only $5.2 \pm 0.4\%$ of the light emitted was in the visible spectrum.

While a wire's resistance is linear at constant temperatures, it is not linear when temperature is changing. By measuring the coefficients of the linear and power regions' resistance dependence on current, the lightbulb's resistance, temperature, emitted color, and efficiency can be determined as a function of current. The ability to accurately calculate all these quantities using only two voltmeters is essential for evaluating or designing lightbulbs.

Theory

Metals should behave in accordance to Ohm's law, as long as their temperature is held the same. While this is a good assumption for resistors, which are designed to disperse heat, it's not a good assumption for light bulbs. Filament light bulbs work because they emit blackbody radiation, the phenomenon commonly known when metal glows red hot.

The formula for the temperature coefficient of resistance is

$$R = R_0 (T - T_0)^{0.831} \quad (1)$$

where T_0 is the reference temperature, R_0 is the conductor resistance at the reference temperature, and 0.831 is a temperature coefficient of resistance [1]. While this is often specified in terms of α and β polynomial coefficients for metals, the power fit is more appropriate for tungsten in the expected range of temperatures.

According to research, all filament-based light bulbs are constructed of tungsten. Experimentally determined α and β are $4.82 \cdot 10^{-3} \text{ K}^{-1}$ and $6.76 \cdot 10^{-7} \text{ K}^{-2}$ respectively

[2].

Assuming the bulb reaches its steady state instantaneously¹, we know that the power entering the bulb has to equal the power leaving the bulb. The net power entering the bulb can be defined by

$$P_{net\ in} = IV \quad (2)$$

where I is the current through the lightbulb and V is the voltage drop across the lightbulb. This power directly heats the filament through a process known as Joule heating. This heat is dissipated through conduction, convection, and radiation. Lets assume that the bulb is in a near-vacuum² so conduction and convection are limited and most energy is lost through blackbody radiation. According to the Stefan-Boltzmann Law,

$$P_{out} \propto T^4 \quad (3)$$

where T is temperature difference in K. This temperature difference is relative to the ambient temperature because blackbody radiation is absorbed as well as emitted.

The relation between current and voltage for the lightbulb under the large-current approximation, given by

$$V = m_1 + m_2 \cdot I^{5/3} \quad (4)$$

where V is voltage in mV, I is current in mA, m_1 is the vertical offset due to the linear region of resistance in mV, and m_2 is the power coefficient relating resistance to current in $\frac{V}{A^{5/3}}$.

If we experimentally determine R_0 , we can set Eq. (1) equal to the derivative of Eq. (4), $\frac{dV}{dI}$ and solve for temperature to calculate the temperature of the filament at any current.

¹This is a good assumption because incandescent lightbulbs turn on and off nearly without delay

²There are two types of bulbs - vacuum and gas-filled. Vacuum bulbs have the advantage of no conduction, but an inert gas slows down the filaments evaporation, letting it run longer and more efficiently at a higher temperature. The break-even point is $6 - 10 \frac{W}{cm}$ of filament [3]. Since, on an order of magnitude estimation, the bulb's wattage to filament length is far less than the break-even point, it's not unreasonable to assume there is a near-vacuum inside our bulbs.

The resulting equation is

$$T = 1.53 \cdot T_0 \left(\frac{m_2 I^{\frac{2}{3}}}{R_0} \right)^{0.8312} \quad (5)$$

From temperature, we can calculate statistics using blackbody radiation curves, such as percentage of light emitted as visible light, infrared, and UV.

Experiment

The setup for the experiment is shown in Figure 1. By varying the output voltage from V_{in} , the voltage across the lightbulb can be determined directly through V_2 , and the current through the lightbulb can be calculated by dividing the voltage across V_1 by the resistance of R . By calculating the current via a voltmeter instead of an ammeter, we can measure the current with increased accuracy. Data is collected by varying V_{in} across its range from 0V to 6V. A variable resistor was placed in series with the rest of the circuit so V_{in} can be more finely adjusted.

Choosing a value for R involves balancing a trade-off between a high value for R , resulting in very precise current data but reduced precision measuring the voltage across the lightbulb, and a low value for R , which results in the opposite. An ideal setup would use an R that equalized percentage error for current and voltage. Four different values of R were used to try to find a decent compromise for R : 1000 Ω , 500 Ω , 100 Ω , and 5 Ω . Using a resistance of 5 Ω for R works well, but the prevalence of voltage error over current error suggests an better value would have been closer to 2 Ω .

Because the bulb's resistance depends on temperature, a large amount of error was created by random temperature fluctuations. Changing the temperature by half a degree

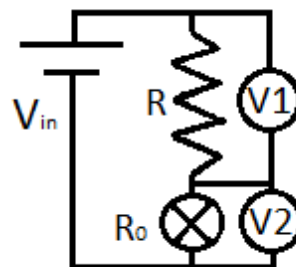


Figure 1: Device used to measure current and voltage across a lightbulb. Resistor R is used with V_1 to obtain an accurate measurement of the current through the circuit through ohm's law. For this experiment, R was 5 Ω . Different data points were taken by varying V_{in} between 0V and 6V.

Kelvin changes the resistance of the bulb by as much as 25%. Because this error was far greater than the error in the voltmeters, two voltage and current data points were taken at every position about a half an hour apart. Uncertainty in the data was then estimated by adding in quadrature the instrument error with the standard deviation between these two points.

Results

Figure 2 shows the data of the voltage across the lightbulb's dependence on current. R_0 , the slope of the left graph, is equal to $5.13 \pm 0.10 \Omega$ and m_2 , the coefficient representing how much the bulb changes with current, is $1.456 \pm .002 \frac{V}{A^{5/3}}$.

Both plots fit their expected trend lines and appear to fit well. The linear region has a reduced χ^2 of 0.05, and the power region has a reduced χ^2 of 1.81. This has to do with how error was estimated: in the power region, random error due to temperature fluctuations

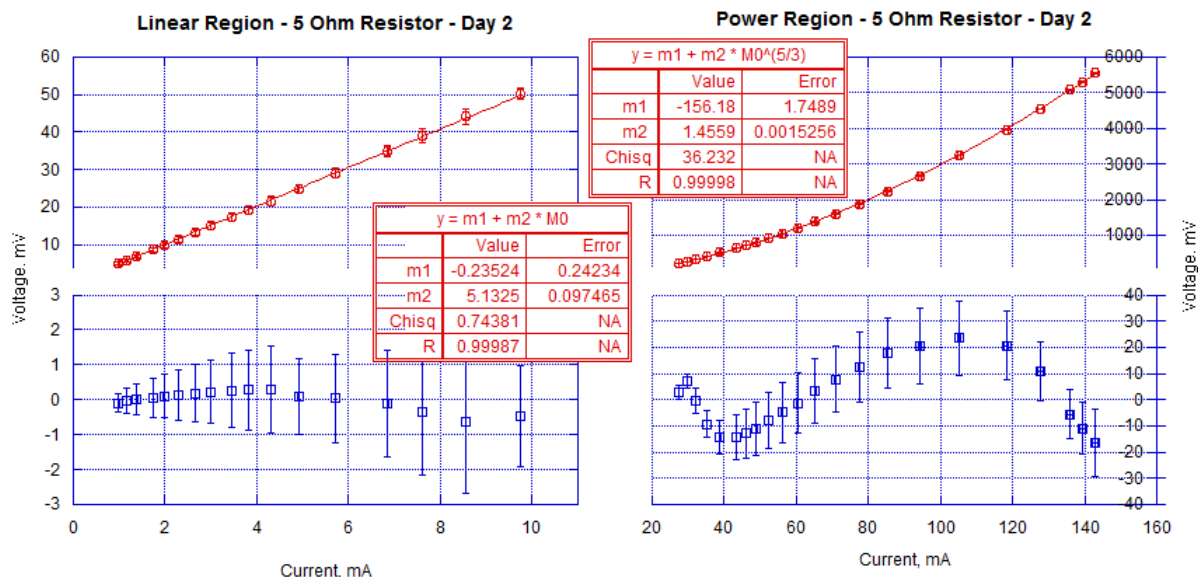


Figure 2: Data from both the linear (ohmic) and power regions of voltage across the lightbulb versus current through the light bulb in red, and residuals in blue. The slope of the left graph, m_2 , is equal to R_0 , the resistance of the bulb at room temperature. On the right, m_2 represents the resistance's dependence on current. The sinusoidal residuals on both plots warrant further analysis.

were the principle cause of error, but in the linear region, principle error was due to the measuring equipment. The voltmeter likely did not produce as much error as the box listed. The linear plot contains the origin, another indication of good fit. The parameters of the power fit are similar to those seen when the same experiment was performed the first week.

However, the sinusoidal residuals are cause for surprise and a little alarm. While they are very small in both graphs, they certainly are non-random. Air-conditioning-based temperature fluctuations would have similar frequency, given each data point took roughly the same amount of time to take. It's possible that the voltmeters make periodic small fluctuations depending on the amount of current passing through them.

Using these coefficients, the determined temperature of the filament was 2540 ± 40 K. Since the temperature of a large household bulb is around 3100 K, this value is not unreasonable. The blackbody spectrum for a bulb of perfect emissivity is shown in Figure 3. The intensity of visible light emitted is small relative to the amount of infrared light emitted. In fact, using Planck's Law [4], it can be determined that only $5.2 \pm 0.4\%$ of the light emitted

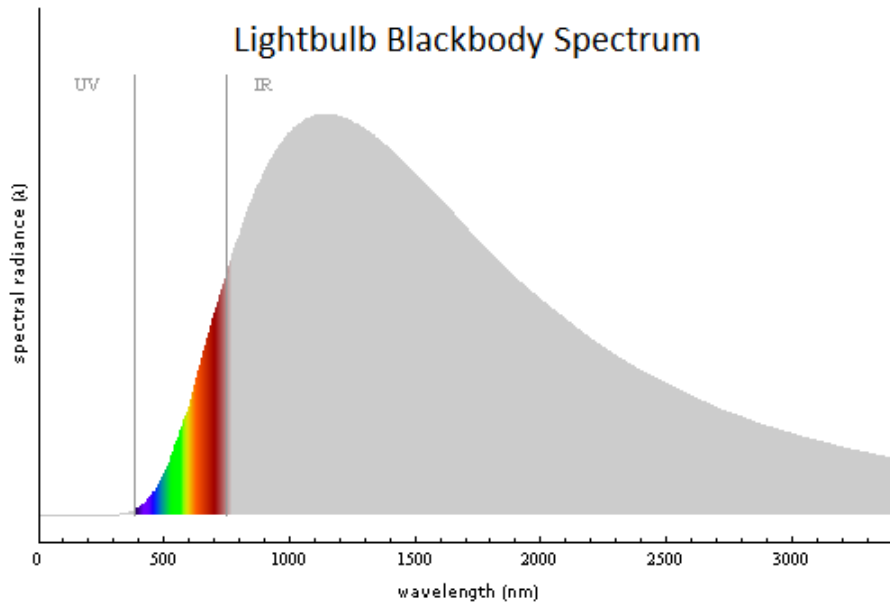


Figure 3: Blackbody emission spectrum for bulb at 2540 ± 40 K with the X axis as wavelength in nm and the Y axis as spectral radiance, the intensity of the light emitted, in $\frac{W}{sr \cdot m^2}$. While some of the energy is radiated in the form of visible light, most of it is radiated as heat in the IR section to the right of the visible portion.

is visible! Nearly all of the remaining $94.8 \pm 0.5\%$ is infrared radiation, and $0.04 \pm 0.01\%$ of the radiation is in the ultraviolet spectrum. These uncertainties are approximate because Planck's law has no simple closed form, so the uncertainty was calculated using the difference between calculations performed on perfect blackbodies at 2540 and 2580 K. While a household incandescent is hotter and emits balanced amounts of all colors that combine to white, this small lightbulb is cooler; the weighted center of its visible spectrum, around yellow or orange, explains the bright-yellow color of the light emitted by the bulb.

Conclusion

While these values are good estimates, nearly all of the error in the temperature was due to error in R_0 , improving the precision in this data would yield the most benefit. Controlling the ambient temperature is necessary so outside fluctuations do not effect the experiment. In addition, using a smaller resistor than 5Ω , such as 2Ω or lower, could redistribute error more equally between current and voltage and yield more precise results.

Overall, this experiment demonstrated the existence of an ohmic region in lightbulbs at currents of under 10 mA, determined approximations of the filament's resistance under different currents, and produced an estimated temperature of the filament and its corresponding emissions spectrum. All of these are necessary to understand the physics of how a lightbulb runs and the environmental impacts of its efficiency and emissions.

References

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