Penguin Rescue!

Dunedin, New Zealand (Penguin Press): A daring mission has been mounted to rescue two adorable penguins who had been given up as lost after a spaceship crash. Risking her life with an untested experimental jet pack, a brave Chemistry penguin mixed a witches’ brew of propellant, fueled the pack, and set off across the sky in search of her missing colleagues, who were running out of fish when last heard from.

“We are, like, so grateful for this, like, attempt, and, like, we, like, hope for her, like, success,” stated a jittery CS 5 student. “We like, love our, like, penguins and are, like, so helpless with our, like, homework assignments without, like, their, like, help.”

Further quotes were unavailable due to an unexpected attack from a WRIT 1 instructor.
Mergesort

\[
\text{msort([42, 3, 1, 5, 27, 8, 2, 7])} \\
\text{msort([42, 3, 1, 5])} \quad \text{msort([27, 8, 2, 7])} \\
\text{merge([1, 3, 5, 42], [2, 7, 8, 27])} \\
\]
“Easy” Problems

Sorting a list of $n$ numbers: $[42, 3, 17, 26, \ldots, 100]$

$n \log_2 n$

Multiplying two $n \times n$ matrices:

\[
\begin{pmatrix}
3 & 5 & 2 & 7 \\
1 & 6 & 8 & 9 \\
2 & 4 & 6 & 10 \\
9 & 3 & 2 & 12 \\
\end{pmatrix}
\begin{pmatrix}
1 & 5 & 5 & 4 \\
5 & 12 & 8 & 6 \\
7 & 6 & 1 & 5 \\
9 & 23 & 5 & 8 \\
\end{pmatrix}
= 
\begin{pmatrix}
\end{pmatrix}
\]

“Easy” Problems

“The Shortest Path Problem (i.e. “Google Maps”)”

Edsger Dijkstra

“Easy” Problems

“Polynomial Time” = “Efficient”

$n, n^2, n^3, n^4, n^5, \ldots$

How about something like $n \log_2 n$?

The “class” $P$

“Hard” Problems

Snowplows of Northern Minnesota

Burrsburg

Tundratown

Freezeapolis

Frostbite City

Shiversville

Brute-force? Greed?
“Hard” Problems

The Traveling Salesperson Problem

New York
San Francisco
Claremont

Moscow
Paris

Brute Force? Greed?

The Hamiltonian Path Problem

Athens, GA
Rome, GA
Homer, GA
Damascus, GA
Bethlehem, GA

Those are some peachy names!

William Rowan Hamilton

n² Versus 2ⁿ

The Meder-O-Matic performs 10⁹ operations/sec

<table>
<thead>
<tr>
<th>n</th>
<th>n²</th>
<th>2ⁿ</th>
<th>n!</th>
</tr>
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<tbody>
<tr>
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<td>100</td>
<td>1024</td>
<td>&lt; 1 sec</td>
</tr>
<tr>
<td>30</td>
<td>900</td>
<td>10⁹</td>
<td>1 sec</td>
</tr>
<tr>
<td>50</td>
<td>2500</td>
<td>10¹⁵</td>
<td>11.6 days</td>
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<td>70</td>
<td>4900</td>
<td>10²¹</td>
<td>31,688 years</td>
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<th>2ⁿ</th>
<th>n!</th>
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<td>10⁹³</td>
<td>2ⁿ</td>
<td>n!</td>
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<tr>
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<td>10⁹³</td>
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<tr>
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<td>10¹⁸⁰</td>
<td>10¹⁸⁰</td>
<td>S+1</td>
<td>S</td>
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</table>

Here’s an Idea!

Let’s just buy a computer that’s twice as fast!

Size of largest problem solvable with “old” computer in one hour = S

Size of largest problem solvable with “new” twice-as-fast computer in one hour = 2S
So are there polynomial time algorithms for the Snowplow and Travelling Salesperson, and Hamiltonian Path Problems?!

If a problem is NP-complete, it doesn’t necessarily mean that it can’t be solved in polynomial time. It does mean...

Are There Problems That Are Even Harder Than NP-Complete?

Tens of thousands of other known problems go in this cloud!!