RL: Lecture 20

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Topics

- Temporal-difference Learning
- n-step TD
- Planning

Temporal-difference learning

Bootstrap method (update value estimates from other value estimates)

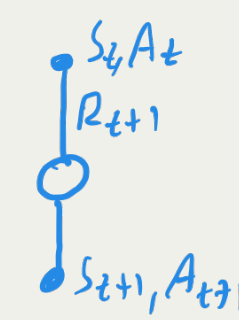
On-policy prediction:

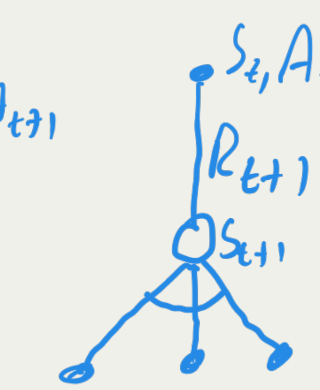
$$V(S) \leftarrow V(S) + \alpha[R + \gamma V(S') - V(s)]$$

Policy control:

Sarsa:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma \qquad - Q(S_t, A_t)]$$





Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma \qquad - Q(S_t, A_t)]$$

Expected Sarsa:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[R_{t+1} + \gamma \qquad - Q(S_t, A_t)]$$

Maximization bias

$$\max_a Q(S', a) \text{ overestimates the max Q value}$$

Solution:
Keep two Qs whose estimates are independent: Q_1, Q_2

Unbiased estimate of max Q value:
$Q_1(S', argmax_a Q_2(S', a))$
or:
$Q_2(S', argmax_a Q_1(S', a))$

Double Q-learning, for estimating $Q_1 \approx Q_2 \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q_1(s, a)$ and $Q_2(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, such that $Q(\mathit{terminal}, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using the policy ε-greedy in $Q_1 + Q_2$
    Take action A, observe R, S'
    With 0.5 probabilility:

$$Q_1(S, A) \leftarrow Q_1(S, A) + \alpha\Big(R + \gamma Q_2\big(S', \operatorname{argmax}_a Q_1(S', a)\big) - Q_1(S, A)\Big)$$

    else:

$$Q_2(S, A) \leftarrow Q_2(S, A) + \alpha\Big(R + \gamma Q_1\big(S', \operatorname{argmax}_a Q_2(S', a)\big) - Q_2(S, A)\Big)$$

    $S \leftarrow S'$
  until S is terminal

n-step TD

n-step TD prediction:

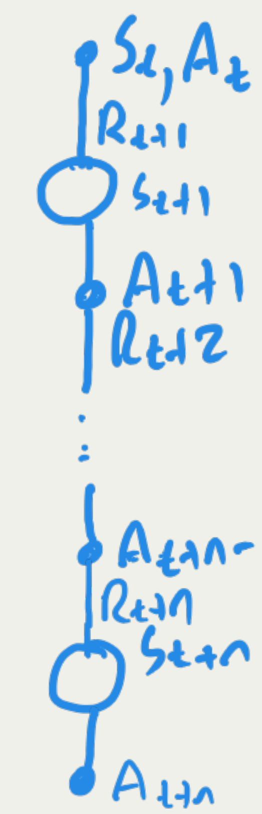
Total reward from t to $t+n$:
$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$
Update function: $V(S_t) \leftarrow V(S_t) + \alpha[G_{t:t+n} - V(S_t)]$

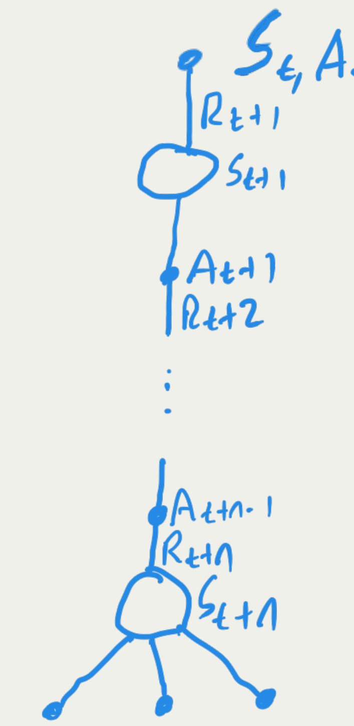
n-step TD for estimating $V \approx v_\pi$

Input: a policy π
Algorithm parameters: step size $\alpha \in (0, 1]$, a positive integer n
Initialize $V(s)$ arbitrarily, for all $s \in \mathcal{S}$
All store and access operations (for S_t and R_t) can take their index mod $n+1$

Loop for each episode:
    Initialize and store $S_0 \neq$ terminal
    $T \leftarrow \infty$
    Loop for $t = 0, 1, 2, \ldots$:
    | If $t < T$, then:
    | Take an action according to $\pi(\cdot|S_t)$
    | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
    | If S_{t+1} is terminal, then $T \leftarrow t+1$
    | $\tau \leftarrow t - n + 1$ (τ is the time whose state's estimate is being updated)
    | If $\tau \geq 0$:
    | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n,T)} \gamma^{i-\tau-1} R_i$
    | If $\tau + n < T$, then: $G \leftarrow G + \gamma^n V(S_{\tau+n})$ $(G_{\tau:\tau+n})$
    | $V(S_\tau) \leftarrow V(S_\tau) + \alpha\left[G - V(S_\tau)\right]$
    Until $\tau = T - 1$

ep TD control





$G_{t:t+n}$: Total reward from time t to $t+n$

Sarsa:

$= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n \underline{\qquad\qquad\qquad\qquad}$

Expected Sarsa:

$= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n \underline{\qquad\qquad\qquad\qquad}$

function for both: $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[G_{t:t+n} - Q(S_t, A_t))]$

n-step Sarsa for estimating $Q \approx q_*$ or q_π

Initialize $Q(s, a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}$
Initialize π to be ε-greedy with respect to Q, or to a fixed given policy
Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$, a positive integer n
All store and access operations (for S_t, A_t, and R_t) can take their index mod $n + 1$

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Loop for each episode:
   Initialize and store S_0 ≠ terminal
   Select and store an action A_0 ~ π(·|S_0)
   T ← ∞
   Loop for t = 0, 1, 2, . . . :
   |  If t < T, then:
   |      Take action A_t
   |      Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
   |      If S_{t+1} is terminal, then:
   |          T ← t + 1
   |      else:
   |          Select and store an action A_{t+1} ~ π(·|S_{t+1})
   |  τ ← t − n + 1    (τ is the time whose estimate is being updated)
   |  If τ ≥ 0:
   |      G ← Σ_{i=τ+1}^{min(τ+n,T)} γ^{i−τ−1} R_i
   |      If τ + n < T, then G ← G + γ^n Q(S_{τ+n}, A_{τ+n})                (G_{τ:τ+n})
   |      Q(S_τ, A_τ) ← Q(S_τ, A_τ) + α [G − Q(S_τ, A_τ)]
   |      If π is being learned, then ensure that π(·|S_τ) is ε-greedy wrt Q
   Until τ = T − 1
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n-step Off-policy prediction

$$\rho_{t:h} = \prod_{k=t}^{min(h,T-1)} \frac{\pi(A_k|S_k)}{b(A_k|S_k)}$$

$$V(S_t) \leftarrow V(S_t) + \alpha\rho_{t:t+n-1}[G_{t:t+n} - V(S_t)]$$

n-step Off-Policy control

Sarsa:

$$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n Q(S_{t+n}, A_{t+n})$$
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \rho_{t+1:t+n} [G_{t:t+n} - Q(S_t, A_t)]$$

Expected Sarsa:

$$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n \sum_a \pi(a|S_{t+n}) Q(S_{t+n}, a)$$
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \rho_{t+1:t+n-1} [G_{t:t+n} - Q(S_t, A_t)]$$

n-step TD: Off-policy Per-decision Methods: State value

Without control variate:

$$G_{t:h} = \rho_t(R_{t+1} + \gamma G_{t+1:h})$$

With control variate:

$$G_{t:h} = \rho_t(R_{t+1} + \gamma G_{t+1:h}) + (1 - \rho_t)V(S_t)$$

In both cases:

$G_{h:h} = V(S_h)$

$V(S_t) \leftarrow V(S_t) + \alpha[G_{t:t+n} - V(S_t)]$

n-step TD: Off-policy Per-decision Methods: Action value

Without control variate:

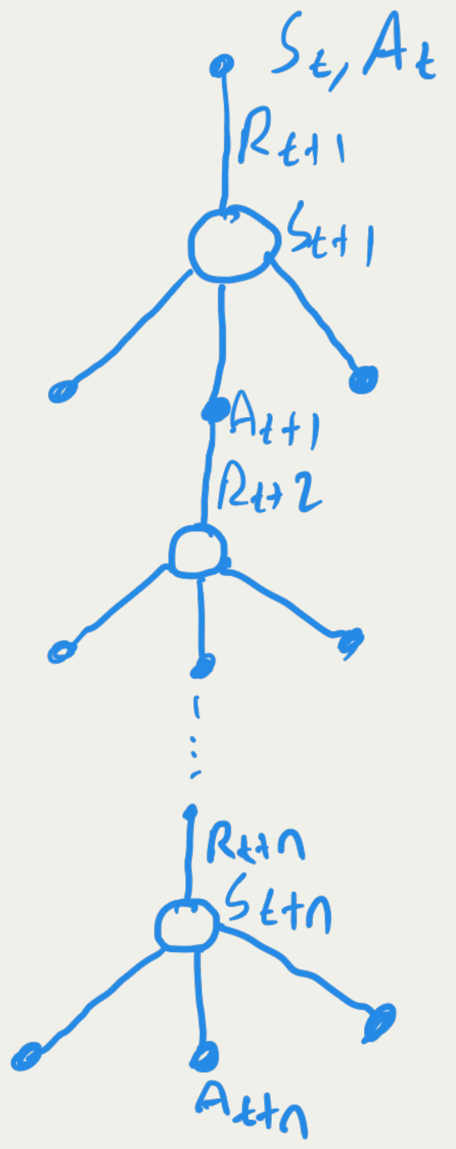
$$G_{t:h} = R_{t+1} + \gamma \rho_{t+1} G_{t+1:h}$$

With control variate:

$$G_{t:h} = R_{t+1} + \gamma \rho_{t+1} (G_{t+1:h} - Q(S_{t+1}, A_{t+1})) + \gamma \bar{V}(S_{t+1})$$

where $\bar{V}(s) = \sum_a \pi(a|s) Q(s, a)$

Off-Policy learning without importance sampling: n-step Tree Backup



n-step:

$$G_{t:t+n} = R_{t+1} + \gamma \sum_{a \neq A_{t+1}} \pi(a|S_{t+1}) Q(S_{t+1}, a) + \gamma \pi(A_{t+1}, S_{t+1}) G_{t+1:t+n}$$

n-step $Q(\sigma)$

A blend between tree-backup and per-decision importance sampling

Types of Models

- Distributional model
- Sample model

What is planning?

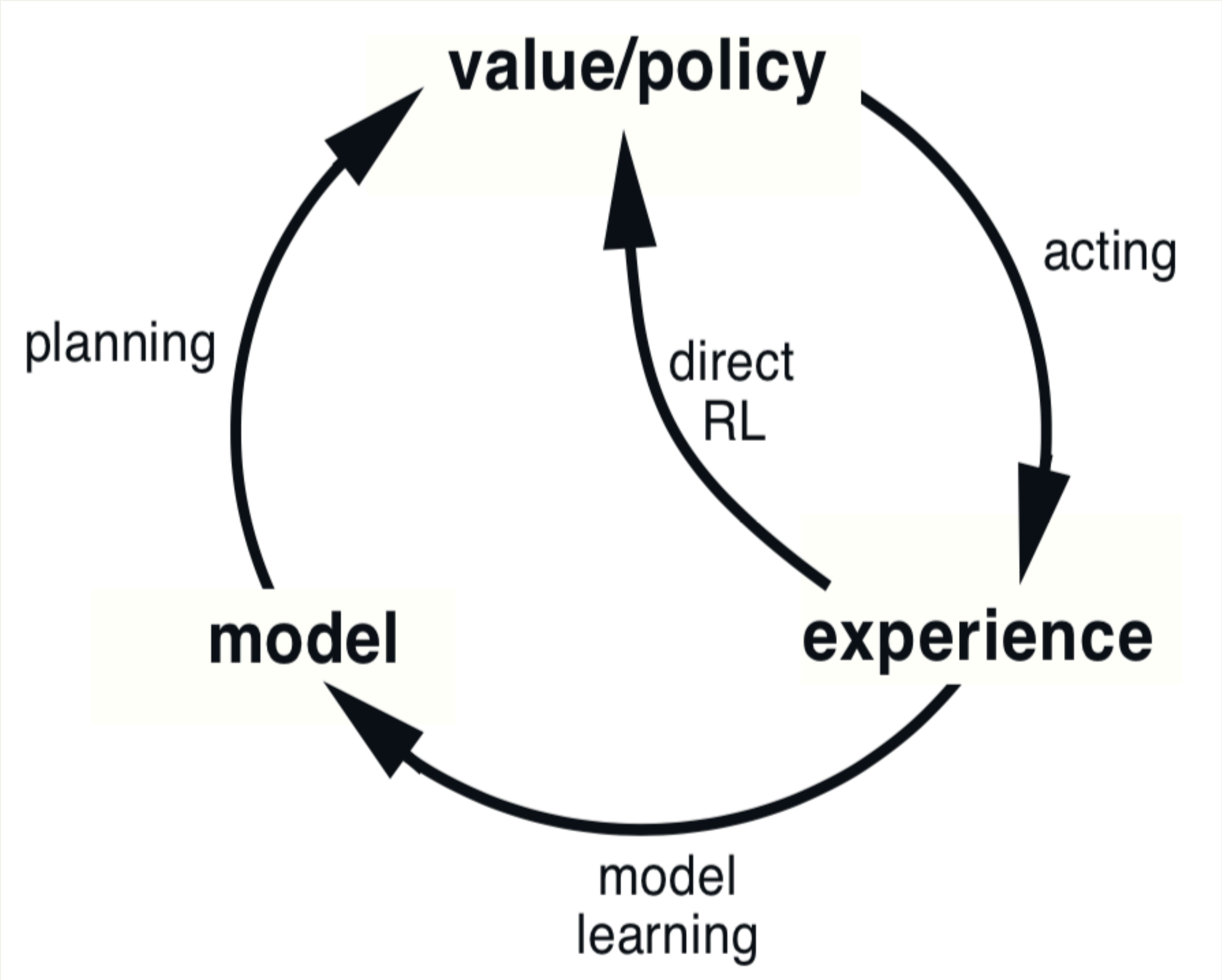
model ->(planning) -> policy

Random-sample one-step tabular Q-planning

Loop forever:
1. Select a state, S, and an action A at random
2. Send S, A to a sample model and obtain a sample next reward R, and a sample next state, S'
3. Apply one-step tabular Q-learning to S, A, R, S'

$$Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \mathbf{max}_a Q(S', a) - Q(S, A)]$$

Online planning



Tabular Dyna-Q Overview

- Direct RL method: one-step tabular Q-learning
- Model-learning method:
  - Assumes environment is deterministic
    - Table-based
    - Given $A_t, S_t \rightarrow R_{t+1}, S_{t+1}$, stores model[($S_t$, A_t)] = (R_{t+1}, S_{t+1})

Tabular Dyna-Q Algorithm

Initialize $Q(s, a)$ and $Model(s, a)$ for all a, s
Loop forever:
1. $S \leftarrow$ current (nonterminal) state
2. $A \leftarrow \epsilon - \text{greedy}(S, Q)$
3. Take action A; observe resultant reward R and state S'
4. $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \text{max}_a Q(S', a) - Q(S, A)]$
5. $\text{Model}(S, A) \leftarrow (R, S')$ (assumes deterministic envirnoment)
6. Loop repeat n times
   $S \leftarrow$ random previously observed state
   $A \leftarrow$ random action previously taken in S
   $R, S' \leftarrow \text{Model}(S, A)$
   $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \text{max}_a Q(S', a) - Q(S, A)]$

Dyna-Q+: Dyna-Q + heuristics for encouraging

- Provide an implicit reward to exploring stale transitions

$$Q(S, A) \leftarrow Q(S, A) + \alpha[R + k\sqrt{\tau(S, A)} + \gamma \mathbf{max}_a Q(S', a) - Q(S, A)]$$

- Allow actions that had never been tried from a state to be considered in planning (initial model was that such an action led back to the same state with a reward of 0)

Sampling

- Uniform
- Prioritized Sweeping
- Trajectory

Monte Carlo Tree Search (MCTS)

Improves the decision that would be made by the rollout policy (or at least no worse)

Repeat while time remaining starting with current state:
1. Selection: Select a leaf node in the expanded tree
2. Expansion: Expand a child of the leaf node
3. Simulation: Follow rollout-policy from expanded node to simulate complete episode
4. Backup: Backup action values to nodes in the tree

Selection: choose a leaf node in the MCTS tree

Traverse MCTS tree until reach a node with unexpanded children.

Use Upper Confidence Bound for Trees (UCT) to decide most promising child.

$q(v)$: Total simulation reward for node v
$n(v)$: Total number of visits (simulation backups) for node v

$$UCT(v) = \frac{q(v)}{n(v)} + c\sqrt{\frac{\log n(v.parent)}{n(v)}}$$

Expansion: expand selected node

If selected node is not terminal, choose an untried action and and create a new MCTS node for the state that generates

Simulation: rollout starting at expanded node

Monte Carlo simulation using rollout policy until terminal state is reached.

Record total reward.

Backup: backup action values to nodes in the MCTS tree

Update $n(v)$ and $q(v)$ for each node v in the MCTS tree from simulation node up to root.

Note: If two-player competitive game, adjust reward to reflect who made the move.

For example, if reward is +1 (player 1 won):

- For v reached from player 1 move, increment $q(v)$
- For v reached from player 2 move, decrement $q(v)$

Selecting a final action

Probably don't want exploration term in UCT

- Child with highest $\frac{q(v)}{n(v)}$, or
- Child of root with highest $N(v)$—it's the one that was explored the most so must have been most promising overall.