Assume we are doing linear regression, \( \hat{y} = a_1 x + a_0 \). Our loss function for a single instance is \( L(x, y) = (\hat{y} - y)^2 \) Assume \( a_0 = 1.5 \), \( a_1 = 0.5 \), and \( (x, y) = (3, 5) \).

1. What is \( \hat{y} \)?

**Solution:**

\[
\hat{y} = a_1 x + a_0 \\
= 3 \times 0.5 + 1.5 = 3
\]

2. What is the loss?

**Solution:**

\[
L(3, 5) = (\hat{y} - y)^2 \\
= (3 - 5)^2 \\
= 4
\]

3. In which direction should \( a_0 \) change to decrease the loss? \( a_1 \)?

**Solution:**

\( a_0 \) should increase (which will increase \( \hat{y} \)) and decrease \( \hat{y} - y \). \( a_1 \) should increase (which will increase \( \hat{y} \)) and decrease \( \hat{y} - y \).

4. What are the new values of \( a_0 \) and \( a_1 \) if the learning rate is 0.01?
Solution:

\[ a_1 = a_1 - \lambda \frac{\partial L}{\partial a_1} \]
\[ = a_1 - \lambda \frac{\partial L}{\partial \dot{y}} \frac{\partial \dot{y}}{\partial a_1} \]
\[ = a_1 - \lambda (\dot{y} - y) a_1 \]
\[ = 0.5 - .001 \times 2(3 - 5)0.5 \]
\[ = 0.5 + .002 \]
\[ = 0.5002 \]

\[ a_0 = a_0 - \lambda \frac{\partial L}{\partial a_0} \]
\[ = a_0 - \lambda \frac{\partial L}{\partial \dot{y}} \frac{\partial \dot{y}}{\partial a_0} \]
\[ = a_0 - \lambda (\dot{y} - y)1 \]
\[ = 1.5 - .001 \times 2(3 - 5)1 \]
\[ = 1.5004 \]