One Ring to Rule Them All: The Optimization of Traffic Circles

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Summary

Our goal is a model that can account for the dynamics of vehicles in a traffic circle. We mainly focus on the rate of entry into the circle to determine the best way to regulate traffic. We assume that vehicles circulate in a single lane and that only incoming traffic can be regulated (that is, incoming traffic never has the right-of-way).

For our model, the adjustable parameters are the rate of entry into the queue, the rate of entry into the circle (service rate), the maximum capacity of the circle, and the rate of departure from the circle (departure rate). We use a compartment model with the queue and the traffic circle as compartments. Vehicles first enter the queue from the outside world, then enter the traffic circle from the queue, and lastly exit the traffic circle to the outside world. We model both the service rate and the departure rate as dependent on the number of vehicles inside the traffic circle.

In addition, we run computer simulations to have a visual representation of what happens in a traffic circle during different situations. These allow us to examine different cases, such as unequal traffic flow coming from the different queues or some intersections having a higher probability of being a vehicle destination than others. The simulation also implements several life-like effects, such as how vehicles accelerate on an empty road but decelerate when another vehicle is in front of them.

In many cases, we find that a high service rate is the optimal way to maintain traffic flow, signifying that a yield sign for incoming traffic is most effective. However, when the circle becomes more heavily trafficked,

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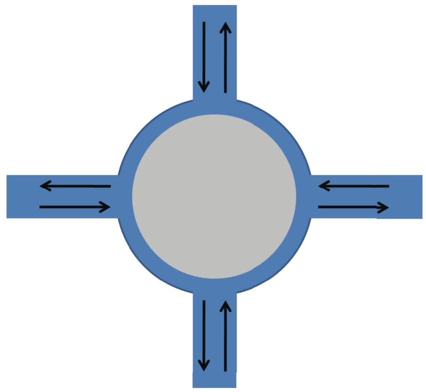

Figure 1. A simple traffic circle. Traffic circles may have more than one lane and may have a different number of intersections.

a lower service rate better accommodates traffic, indicating that a traffic light should be used. Thus, a light should be installed in most circle implementations, with variable timing depending on the expected amount of traffic.

The main advantage of our approach is that the model is simple and allows us to see clearly the dynamics of the system. Also, the computer simulations provide more in-depth information about traffic flow under conditions that the model could not easily show, as well as enabling visual observation of the traffic. Some disadvantages to our approach are that we do not analyze the effects of multiple lanes nor stop lights to control the flow of traffic within the circle. In addition, we have no way of analyzing singular situations, such as vehicles that drive faster or slower than the rest of the traffic circle, or pedestrians.

Introduction

Traffic circles, often called rotaries, are used to control vehicle flow through an intersection. Depending on the goal, a traffic circle may take different forms; **Figure 1** shows a simple model. A circle can have one or more lanes; vehicles that enter a traffic circle can be met by a stop sign, a traffic light, or a yield sign; a circle can have a large or small radius; a circle can confront roads containing different amounts of traffic. These features affect the cost of the circle to build, the congestion that a vehicle confronts as it circles, the travel time of a vehicle in the circle, and the size of the queue of vehicles waiting to enter. Each of these variables could be a metric for evaluating the efficacy a traffic circle.

Our goal is to determine how best to control traffic entering, exiting, and traversing a traffic circle. We take as given the traffic circle capacity, the arrival and departure rates at each of the roads, and the initial number of vehicles circulating in the rotary. *Our metric is the queue length, or buildup, at each of the entering roads.* We try to minimize the queue length by allowing the rate of entry from the queue into the circle to vary. For a vehicle to traverse the rotary efficiently, its time spent in the queue should be minimized.

We make the following assumptions:

- We assume a certain time of day, so that the parameters are constant.
- There is a single lane of circulating traffic (all moving in the same direction).
- Nothing impedes the exit of traffic from the rotary.
- There are no singularities, such as pedestrians trying to cross.
- The circulating speed is constant (i.e., a vehicle does not accelerate or decelerate to enter or exit the rotary).
- Any traffic light in place regulates only traffic coming into the circle.

The Models

A Simplified Model

We model the system as being continuous; our approach can be thought of as modeling the vehicle mass dynamics of a traffic circle. The simplest model assumes that the rate of arrival to the back of the entering queue and the rate of departure from the queue into the traffic circle are independent of time. Thus, the rate of change in the length of the queue is

$$\frac{dQ_i}{dt} = a_i - s_i, \tag{1}$$

where Q_i is the number of cars in the queue coming in from the ith road, a_i is the rate of arrival of vehicles into the ith queue, and s_i is the rate of removal, also called the service rate, from the ith queue into the traffic circle.

We introduce the parameter d_i, the rate at which vehicles exit the traffic circle. We let C be the number of vehicles traveling in the circle. Then we model the change in traffic in the rotary by the difference between the influx and outflux of vehicles, where the outflux of vehicles depends on the amount of traffic in the rotary:

$$\frac{dC}{dt} = \sum s_i - C \sum d_i. \tag{2}$$

An Intermediate Model

The model above simplifies the dynamics of a traffic circle. The most-glaring simplifications are that there is no way to indicate that the circle has a maximum capacity and that the flow rate into the traffic circle s_i does not depend on the amount of traffic already circulating. These are both corrected by proposing that the traffic circle has a maximum capacity $C_{\max}$. As the number of vehicles circling approaches this maximum capacity, it should become more difficult for another vehicle to merge into the circle. At the extreme, when the traffic circle is operating at capacity, no more vehicles should be able to be added. Now, the s_i in the previous model can be represented logistically as

$$s_i = r_i \left(1 - \frac{C}{C_{\max}}\right),$$

where r_i is how fast vehicles would join the circle if there were no traffic slowing them down. Thus, the equation governing the rate at which the ith queue length changes becomes

$$\frac{dQ_i}{dt} = a_i - r_i \left(1 - \frac{C}{C_{\max}}\right), \qquad \textbf{(3)}$$

and the equation for the number of vehicles in the traffic circle becomes

$$\frac{dC}{dt} = \sum r_i \left(1 - \frac{C}{C_{\max}}\right) - \sum d_i C. \qquad \textbf{(4)}$$

A Congestion Model

The previous two models still fail to take into account congestion, which alters the circulation speed, which in turn affects the departure rate d_i of vehicles from the circle. Equation **(3)** still holds, but we need to vary d_i. The vehicles will travel faster if there is no congestion, so they will be able to depart at their fastest rate $d_{i,\max}$. When the circle is operating at maximum capacity, the departure rate will decrease to be $d_{i,\min}$. Thus, the number of vehicles present in the circle is affected positively in the same manner as in **(4)**, but the lessening factor changes to the weighted average of the $d_{i,\max}$ and $d_{i,\min}$:

$$\frac{dC}{dt} = \sum r_i \left(1 - \frac{C}{C_{\max}}\right) - C\left[\sum d_{i,\max}\left(1 - \frac{C}{C_{\max}}\right) + \sum d_{i,\min}\left(\frac{C}{C_{\max}}\right)\right]. \qquad \textbf{(5)}$$

Extending the Model Using Computer Simulation

We create a computer simulation in Matlab to account for variables that would be too complicated to use in the mathematical model. The mathematical model does not address the vehicles' speeds while inside the traffic circle, so the computer simulation focuses mostly on areas related to vehicle speed:

- enabling drivers to accelerate to fill gaps in the traffic (with a maximum speed),
- forcing drivers to decelerate to maintain distance between vehicles
- requiring that drivers accelerate and decelerate when entering and exiting the circle,
- giving probabilistic weights to the different directions of travel,
- keeping track of time spent within the traffic circle for each vehicle, and
- giving each intersection a different vehicle introduction rate.

Figure 2 on p. 250 shows an outline of the program flow and design.

Simulation Assumptions

This model makes several key assumptions about the vehicles and the circle:

- All vehicles are the same size, have the same top speed, and accelerate and decelerate at the same rate.
- The circle has four intersections and a single lane of traffic.
- All drivers have the same spatial tolerance.
- There are no pedestrians trying to cross the circle.

Limitations

The assumption of one lane is not a key factor, because we assume that vehicles travel at the same speed. Hence, we do not need to put the slow vehicles in one lane and vehicles passing them in another lane. However, in reality there will indeed be slower vehicles, and vehicles decelerating to exit would offer opportunities for other vehicles to use a different lane to maintain a faster speed. Additionally, we cannot let emergency vehicles through the circle if there is only one lane; for a more detailed discussion of emergency vehicles and traffic circles, see Mundell [n.d.].

By not allowing control devices inside the circle, we restrict possible configurations. We also limit the effectiveness of our stoplight model; it prevents vehicles from entering the circle but does not inhibit the movement of vehicles within in the circle.

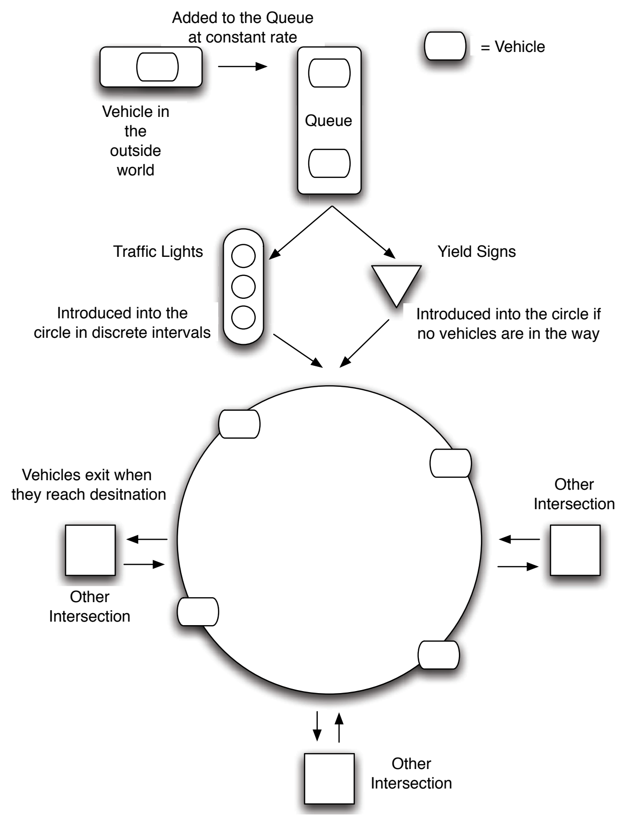

Figure 2. Program flow. Each intersection is modeled as a queue of vehicles with a traffic control device. Vehicles are added to the queue at a constant rate. For a vehicle to leave the queue and enter the traffic circle, the area in the circle must be clear of other vehicles. Additionally, if the queue has a traffic light, the light must be active.

Since we do not allow for different vehicle properties (size, acceleration, top speed, etc.), we cannot model the effects of large trucks, motorcycles, or other nonstandard vehicles (such as large and unwieldy emergency vehicles) on the flow of traffic.

Giving all of the vehicles the same acceleration and top speed, along with forcing all drivers to have the same spatial tolerance, prevents modeling aggressive drivers and their interaction with timid ones. Additionally, since cars in the simulation decelerate before exiting, even if they are already moving slowly, we generate a small proportion of false traffic backups.

Limiting the size and number of intersections of the circle does not really limit our ability to model real-world traffic circles. Since we are mostly looking at driver behavior with the computer simulation, we should see the same behaviors as we scale up the circle and its corresponding traffic.

Analyzing the Models

The Simplest Model

In all of the above models, the rate r_i is indicative of the regulation imposed at the ith intersection. A near-zero r_i indicates that a traffic light is in use; a larger r_i indicates that a yield sign, regulating only the incoming traffic, is in place.

For the simplest model, we can use **(1)** and **(2)** to find explicit formulae for the queue length and the number of vehicles in the rotary by integrating with respect to time:

$$Q_i = [a_i - s_i]\, t + Q_{i0}, \qquad C = \frac{\sum s_i}{\sum d_i} + \left(C_0 - \frac{\sum s_i}{\sum d_i}\right) e^{-\sum d_i t}.$$

Therefore, given the inputs of the system, we can predict the queue length. To minimize the queue length, we solve **(1)** for when the queue length is decreasing ($dQ_i/dt < 0$) and find that the s_i term should be maximized.

Intermediate Model

For the model with a carrying capacity, again we find explicit formulae for the queue length and the number of vehicles in the rotary:

$$Q_i = \left[a_i - r_i\left(1 - \frac{C}{C_{\max}}\right)\right] t + Q_{i0},$$

$$C = \frac{\sum r_i}{\frac{\sum r_i}{C_{\max}} + \sum d_i} + \left(C_0 - \frac{\sum r_i}{\frac{\sum r_i}{C_{\max}} + \sum d_i}\right) e^{-\left(\frac{\sum r_i}{C_{\max}} + \sum d_i\right) t}.$$

We can also solve for where **(3)** is less than zero to find the service rates for which the queue lengths are decreasing:

$$r_i > \frac{a_i}{1 - \frac{C}{C_{\max}}}.$$

Congestion Model

In modeling congestion, the model is too complex to intuit what conditions would minimize the queue length. The differential equation **(5)** is quadratic:

$$\frac{dC}{dt} = AC^2 + BC + D,$$

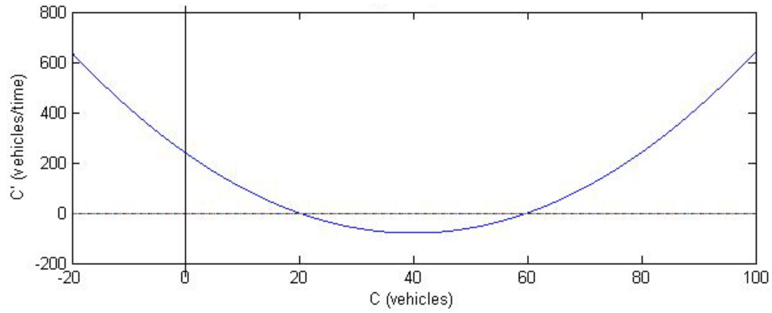

Figure 3. The relationship between dC/dt and C for the congestion model using sample parameters values $r_1 = r_2 = r_3 = r_4 = 60$, $d_{1,\max} = d_{2,\max} = d_{3,\max} = d_{4,\max} = 2$, $d_{1,\min} = d_{2,\min} = d_{3,\min} = d_{4,\min} = 0.5$, and $C_{\max} = 30$.

where

$$A = \frac{\sum d_{i,\max}}{C_{\max}} - \frac{\sum d_{i,\min}}{C_{\max}}, \quad B = -\left(\frac{\sum r_i}{C_{\max}} + \sum d_{i,\max}\right), \quad D = \sum r_i.$$

Since $\sum d_{i,\max} > \sum d_{i,\min}$, it will always be the case that $A > 0$. In addition, $B < 0$ and $D > 0$. This means that the curve for dC/dt is a concave-up quadratic curve with a positive y-intercept and a global minimum at some $C > 0$. Furthermore, for $C = C_{\max}$, we have

$$\frac{dC}{dt} = -\frac{d_{i,\min}}{C_{\max}},$$

which is always negative for $d_{i,\min} > 0$. Thus, the global minimum for the curve must be in the fourth quadrant. **Figure 3** shows an example of such a curve, using sample parameters.

We notice from **Figure 3** that there are two equilibrium points for the differential equation:

$C = \frac{-B-\sqrt{B^2-4AD}}{2A}$ is a stable equilibrium point, and

$C = \frac{-B+\sqrt{B^2-4AD}}{2A}$ is an unstable equilibrium point.

Also, since for $C = C_{\max}$, we have $dC/dt < 0$, the number of vehicles will eventually decrease to an equilibrium value less than $C_{\text{limit}} < C_{\max}$.

Since our metric for how well a traffic circle operates depends on how many vehicles are in the queues, we would like the queue flow $(a_i - s_i)$ to be as small as possible. In other words, we would like s_i to be as large as possible. In the congestion model, the queue flow is given by **(3)**.

Without loss of generality, we analyze queue 1. The equations for each queue differ only by their a_i and r_i, and we keep these the same for each queues in the simulations. Since the only changing variable in **(3)** is C, when $C = C_{\text{limit}}$ the queue length Q_1 will also be at its equilibrium.

Using this fact, we can evaluate whether to use a traffic light or not and how long the light should be red. We compare different values for the service rate constant r_1 and the value of dQ_1/dt at $C = C_{\text{limit}}$. The results can be seen in **Figure 4**, which shows that when r_1 increases, dQ_1/dt decreases.

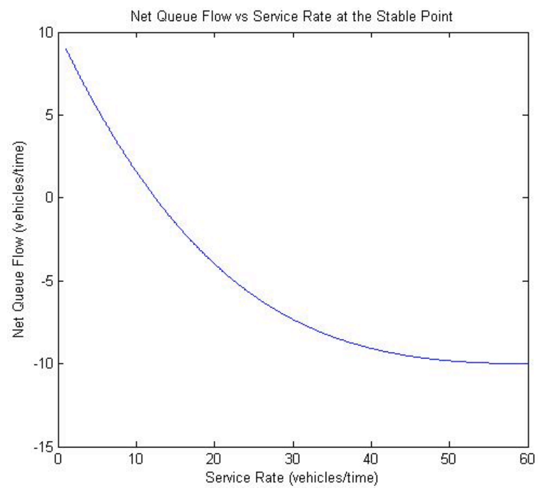

Figure 4. The relationship between r_1 and dC/dt for the congestion model with $C = C_{\text{limit}}$. The parameter values are $d_{1,\max} = d_{2,\max} = d_{3,\max} = d_{4,\max} = 2$, $d_{1,\min} = d_{2,\min} = d_{3,\min} = d_{4,\min} = 0.5$, $C_{\max} = 30$, and r_1 changed from 1 to 60.

A real-life situation is congestion of the traffic circle. Decreasing $d_{1,\min}$ would cause vehicles to exit the circle more slowly when there is more congestion. Using lower departure rates to approximate slower vehicle speeds inside the circle, we can examine what happens for decreasing values of $d_{1,\min}$. The results are shown in **Figure 5**. For values of $d_{1,\min} < 0.5$, the smallest value for dQ_1/dt is not at $r_1 = 60$ but at a smaller value.

Another situation that the congestion model can approximate is additional lanes. A crude approximation is that each lane adds $C_{\max}$ to the capacity. **Figure 6** shows the results of plotting r_1 versus the $C_{\max}$ for different numbers of lanes. As in the previous plots, the correlation is negative.

Simulation Results

An interesting effect that we see in our simulation is the buildup of vehicles in front of each exit. As vehicles decelerate to exit, they force vehicles behind them to decelerate to maintain a safe distance. This buildup creates a longer queue at the intersection before the exit, since the buildup prevents those vehicles from entering the circle. In **Figure 7**, we see a large number of vehicles in the fourth queue and a buildup in the fourth quadrant.

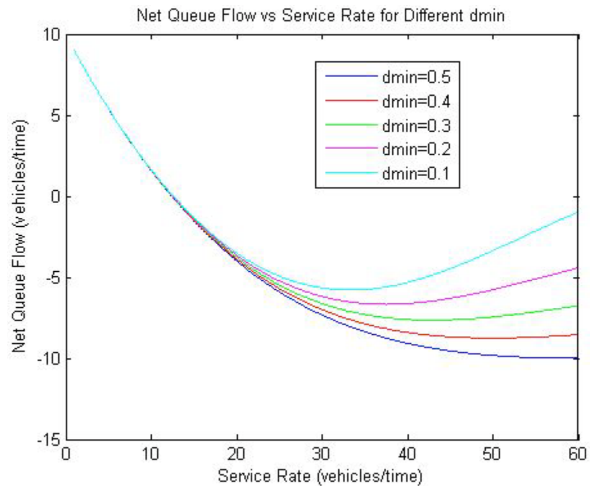

Figure 5. The relationship between r_1 and dC/dt for the congestion model with $C = C_{\text{limit}}$, with parameter values $d_{1,\max} = d_{2,\max} = d_{3,\max} = d_{4,\max} = 2$, and $C_{\max} = 30$. The values of r_1 range from 1 to 60 for different values of $d_{1,\min}$.

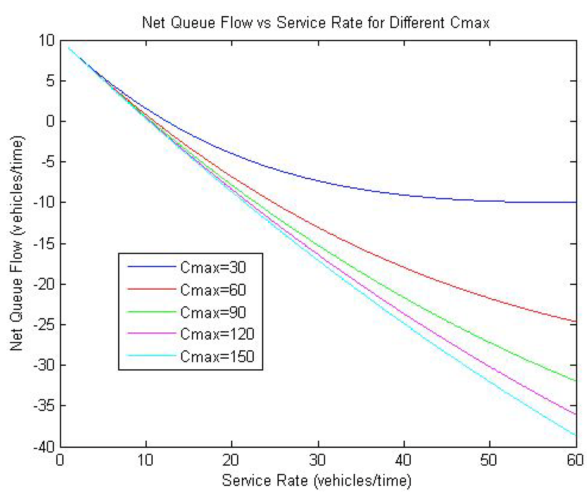

Figure 6. The relationship between r_1 and dC/dt for the congestion model with $C = C_{\text{limit}}$. The parameter values are $d_{1,\max} = d_{2,\max} = d_{3,\max} = d_{4,\max} = 2$, $d_{1,\min} = d_{2,\min} = d_{3,\min} = d_{4,\min} = 0.5$, $C_{\max} = 30$, and r_1 changed from 1 to 60.

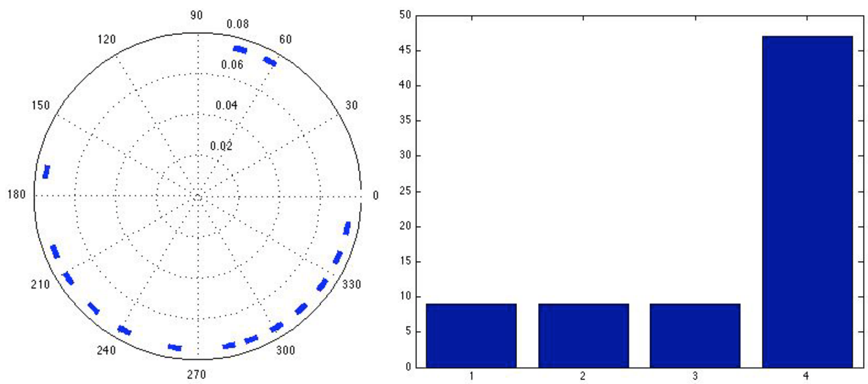

Figure 7. Vehicles build up before the first intersection as vehicles slow down to exit. Additionally, the queue at the fourth intersection is quite long, because vehicles cannot enter the traffic circle.

Another interesting element of real life that the simulation shows is the bunching and expanding effect that vehicles experience. Because vehicles can decelerate more quickly than they accelerate, the vehicles bunch up behind a slow moving vehicle, then expand again as that vehicle accelerates into the free space ahead. **Figure 8** shows an example of this compaction.

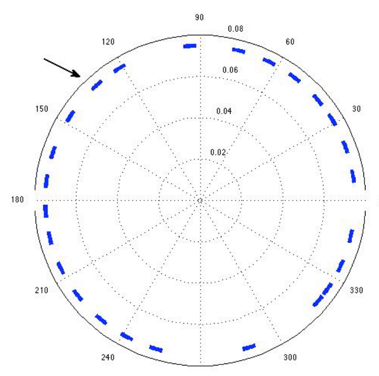

Figure 8. The arrow in the second quadrant points out a real-life effect, bunching, which happens because drivers decelerate faster than they accelerate.

We test several rotary and vehicle setups to explore optimal circle design:

- A single intersection with high arrival and service rates creates a large traffic buildup in the quadrant immediately following it, even though the vehicles have random destinations. **Figure 9** shows the buildup in quadrant 1 when the first intersection (at angle 0) has a high arrival and service rate. However, queue 1 is not appreciably longer than the others.

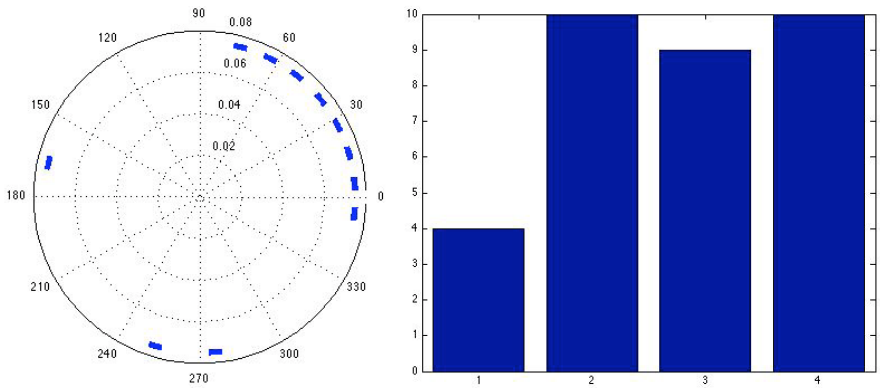

Figure 9. The first intersection has both high arrival and service rates, which creates a traffic buildup before the next intersection. However, the queue for the first intersection does not increase, since there is limited traffic coming from the intersection behind it.

- One intersection having a much higher chance of being a destination creates the expected buildup in front of the likely exit (**Figure 10**). However, it also creates a substantial buildup in front of the previous exit and a severe increase in that intersection's queue as vehicles are prevented from entering the circle. The buildup in the adjacent road must be taken into account when constructing a traffic circle at a high-volume intersection.
- If one intersection has a high service rate and the standard arrival rate, and another intersection has a high arrival rate and standard service rate, the traffic distribution is mostly random, with a slight tendency towards backups in the quadrant following the intersection with high service rate. We expect this result, since the intersection with high service rate can add only as many vehicles as in its queue, which is limited by its low arrival rate. Also, the intersection with high arrival rate and low service rate has a much longer queue than the other intersections, entirely as expected.

Conclusion

We model the dynamics of a traffic circle to determine how best to regulate traffic into the circle. As shown in **Figure 6** on p. 256, increased capacity decreases the queue flow, which leads to a decrease in queue length. This result indicates that a multiple-lane traffic circle might better accommodate more cars by decreasing the length of the queue in which they wait. However, as shown in the same figure, the marginal utility of increasing the maximum capacity does decrease. When applying a cost function (with cost proportional to the space that the circle occupies), there would exist an

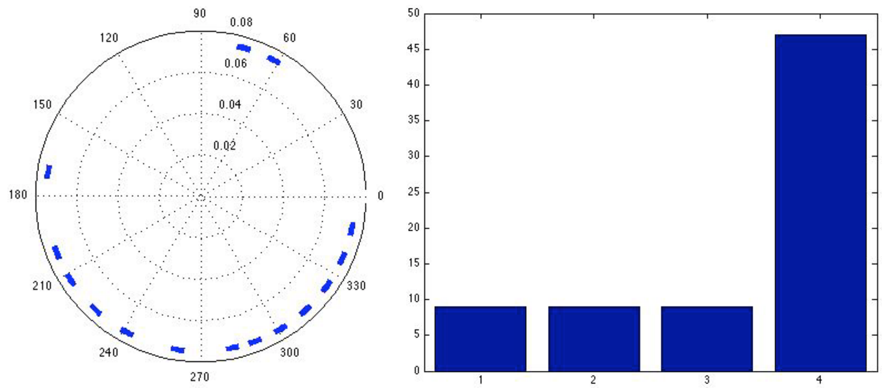

Figure 10. The first intersection has a higher probability of being chosen as a destination. This creates a buildup in front of that intersection and a smaller buildup in front of the previous intersection. It also creates a very large increase in the queue of the previous intersection since those vehicles cannot enter the full circle.

optimum size of the traffic circle.

Although the simpler models indicate that letting vehicles into the rotary as fast as possible would be optimum, analysis of the congestion model shows that if $d_{i,\min}$ is sufficiently small, then the highest service rate is no longer optimal. The implication of this result is that traffic lights could make travel through the rotary more efficient. When many vehicles use the traffic circle, such as during the morning and evening commutes, there could be enough vehicles so that the C_{limit} is reached. In this case, using traffic lights would help ease congestion. However, the duration of the red light should be adjusted according to the $d_{i,\min}$ for the specific traffic circle.

In addition to the mathematical models, we create a computer simulation that tracks individual vehicles' progress through the traffic circle, and their effect on other vehicles. Our simulation shows several traffic effects that can be observed in real life, namely a buildup of vehicles in front of the exits and vehicles bunching together and expanding apart as drivers brake and accelerate. We also test several traffic circle configurations.

Recommendations

Based on both our mathematical and computer models, we recommend:

- **Yield signs should be the standard traffic control device.** Most of the time, letting vehicles enter the circle as quickly as possible is optimal.
- **For a high-traffic rotary, traffic lights should be used.** With high traffic, slowing the rate of entry into the circle helps prevent congestion.

- **If any single road has high traffic, its vehicles should be given preference in entering the circle.** Doing so helps prevent a large queue.
- **Introduce separate exit lanes.** Traffic can build up in front of each intersection as cars exit, so a separate exit lane could help keep traffic moving.

References

Mundell, Jim. n.d. Constructing and maintaining traffic calming devices. Seattle Department of Transportation. http://www.seattle.gov/Transportation/docs/TORONTO4.pdf.

Aaron Abromowitz, Andrea Levy, and Russell Melick.