One Ring to Rule Them All:
Optimizing Traffic Circles

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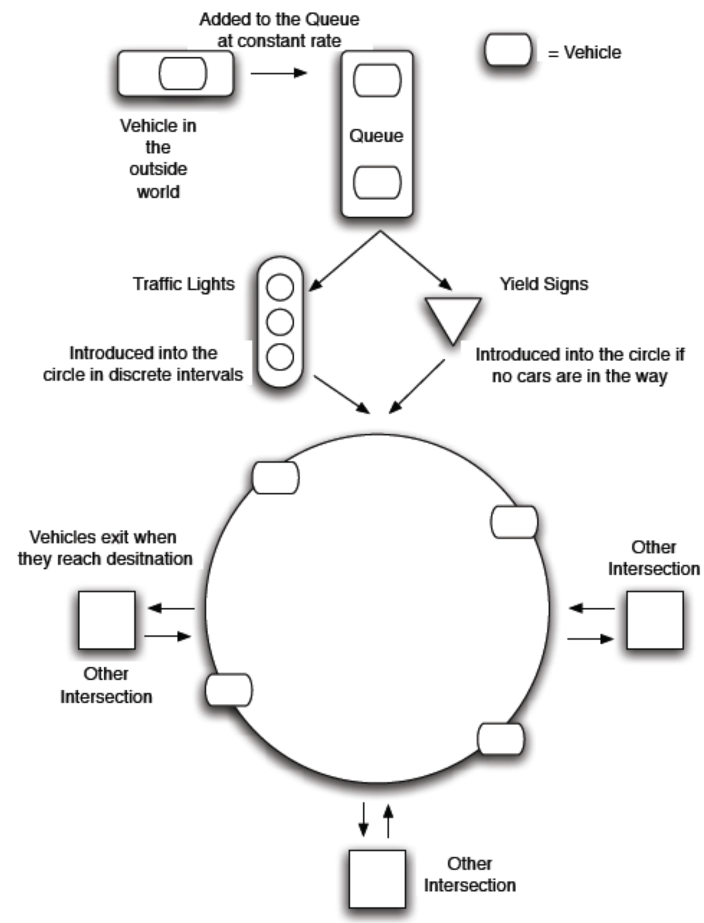
Problem

- "use a model to determine how best to control traffic flow in, around, and out of a circle"
- Implement yield signs or traffic signals to regulate incident traffic

Terminology

- Arrival rate
- Service rate
- Queue flow
- Circle capacity
- Departure rate



The "Best" Traffic Circle

- Possible measures of traffic circle quality
  - Time
    - From arrival to departure
    - To circulate
  - Buildup
    - In the queue
    - Congestion in the circle

Objective

- Our approach: minimize buildup in queue
  - Correlated to minimization arrival until departure time
  - Much of time lost (in our experience) is waiting to enter the circle
  - Impatience: waiting in line isn't any fun

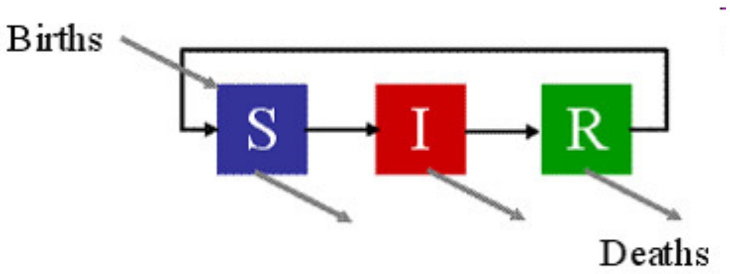
Our Approach

- Mathematical Models
  - Model typical behavior
- Computer Simulation
  - Address some of the limitations of the mathematical model

Mathematical Models

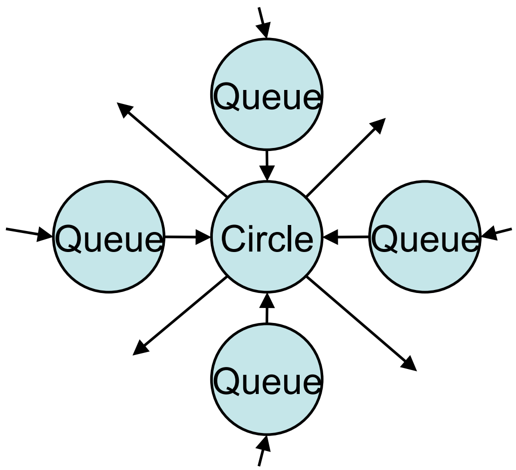
Compartment Models

- Used to describe flows between groups
- Classic example: SIR model (susceptible, infected, recovered)
  - Models infectious diseases



Our Compartment Model

- Groups: cars inside traffic circle, cars in queues
- Flows: from outside world to queue, from queue to circle, from circle to outside world



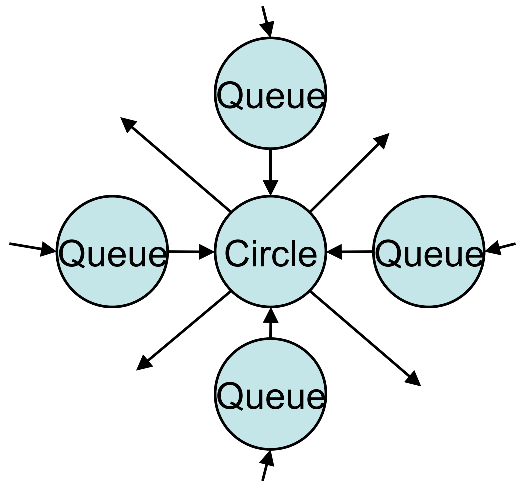
Assumptions and Restrictions

- Cars enter queues at a constant rate
- Cars are modeled continuously rather than discretely
- We cannot model the activity of individual cars
- We cannot model stop lights inside the circle

Basic Model

$$\frac{dQ}{dt} = a - s$$

$$\frac{dC}{dt} = s - d * C$$



Q	# of cars in queue
C	# of cars in circle
a	Rate of cars entering into the queue
s	Rate of cars entering the circle from the queue
d	Rate of cars leaving the circle
t	time

Problem with s and d

- Problem: It is inaccurate to assume that s and d will be constant
  - They are both affected by the number of cars already in the circle
- Solution: make them dependent on C

New Forms of *s* and *d*

$$s = r * (1 - \frac{C}{C_{max}})$$

$$d = d_{max} * (1 - \frac{C}{C_{max}}) + d_{min} * (\frac{C}{C_{max}})$$

C	# of cars in circle
s	Rate of cars entering the circle from the queue
r	Rate of cars entering when the circle is empty
d	Rate of cars leaving the circle
C_{max}	Maximum # of cars allowed in the circle
d_{max}	Speed of cars in the circle when it is completely empty
d_{min}	Speed of cars in the circle when it is completely full

Congestion Model

$$\frac{dQ}{dt} = a - r * (1 - \frac{C}{C_{max}})$$

$$\frac{dC}{dt} = r * (1 - \frac{C}{C_{max}}) - C * (d_{max} * (1 - \frac{C}{C_{max}}) + d_{min} * (\frac{C}{C_{max}}))$$

Q	# of cars in queue
C	# of cars in circle
a	Rate of cars entering into the queue
r	Rate of cars entering when the circle is empty
C_{max}	Maximum # of cars allowed in the circle
d_{max}	Speed of cars in the circle when it is completely empty
d_{min}	Speed of cars in the circle when it is completely full
t	time

Finding Solutions

- Q has an explicit solution which is exponential

$$Q = (a - r * (1 - \frac{C}{C_{\max}})) * t + Q_0$$

- C's equation is too complicated to give us any information

Simplifying C'

- We can put C' into quadratic form

$$\frac{dC}{dt} = A * C^2 + B * C + D$$

- where

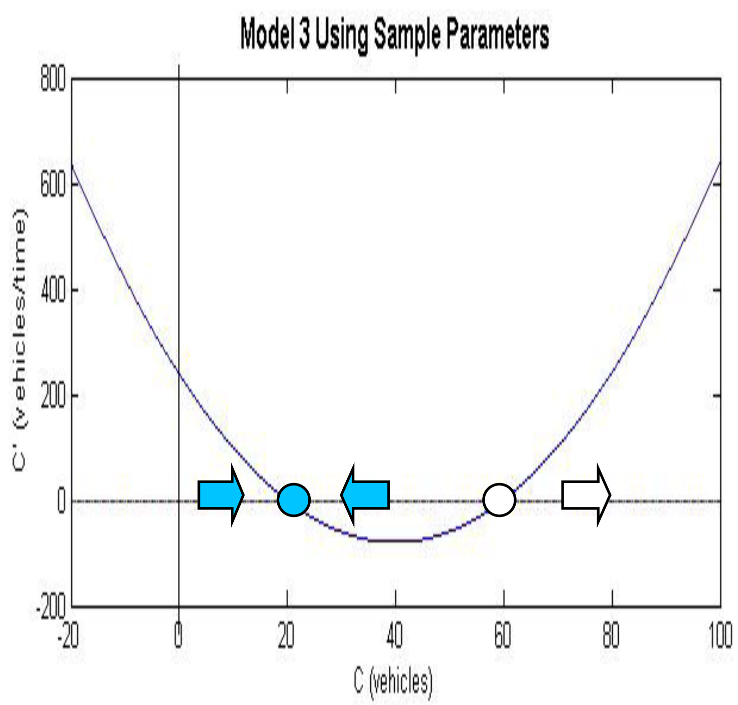
$$A = \frac{d_{\max} - d_{\min}}{C_{\max}}$$

$$B = -(\frac{r}{C_{\max}} + d_{\max})$$

$$D = r$$

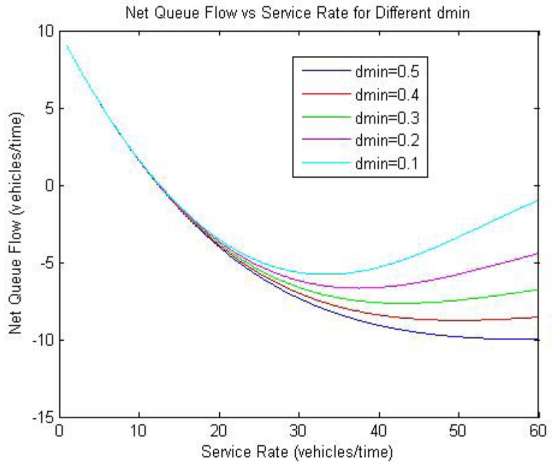
Looking at C Graphically

- A>0, B<0, and D>0 which gives this form of graph
- $C' < 0$ for $C = C_{max}$ so any solution will reach a critical C value

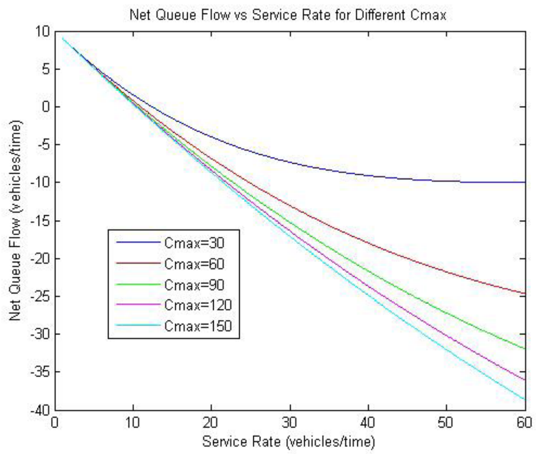


Varying Parameters

- Different values of d_{min} give us different optimal values of r



- Flow is improved for larger values of C_{max}



Matlab Simulation

Drivers

- Assured clear distance
  - Entering the circle
  - Acceleration into open space
  - Braking for slow drivers
- Acceleration when entering circle
- Braking for exits

Driver Limitations

- Single type of vehicle
- Single type of driver

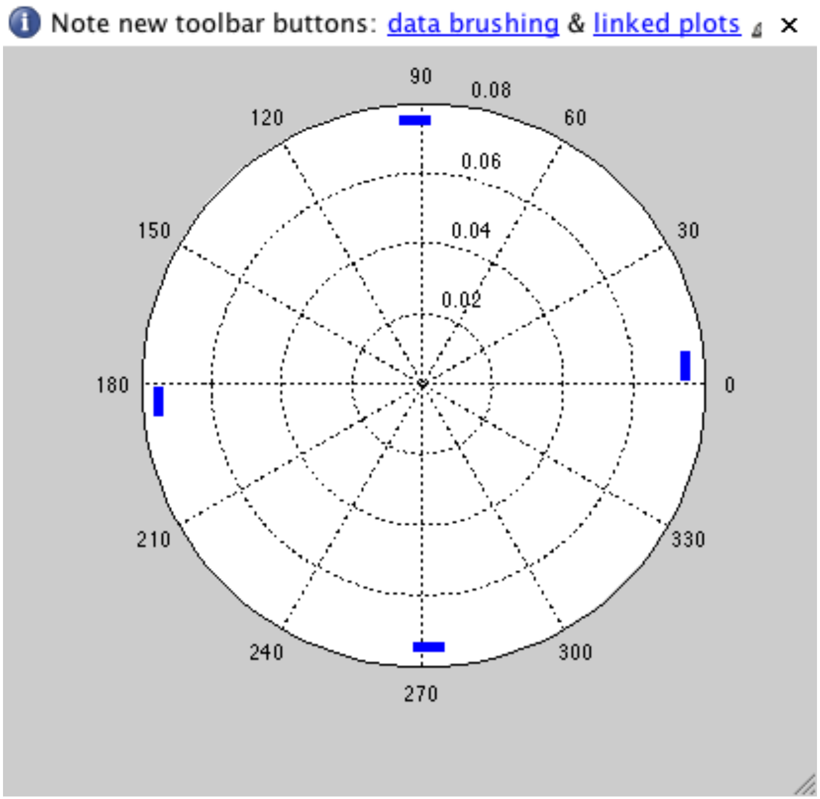
Traffic

- Four roads meet at a traffic circle
- Queues of traffic at each intersection
  - Different arrival rates
  - Traffic control devices
    - Yield and Highway Stoplight
- Different destinations
  - Randomized
  - Dependent on initial intersection

Traffic Limitations

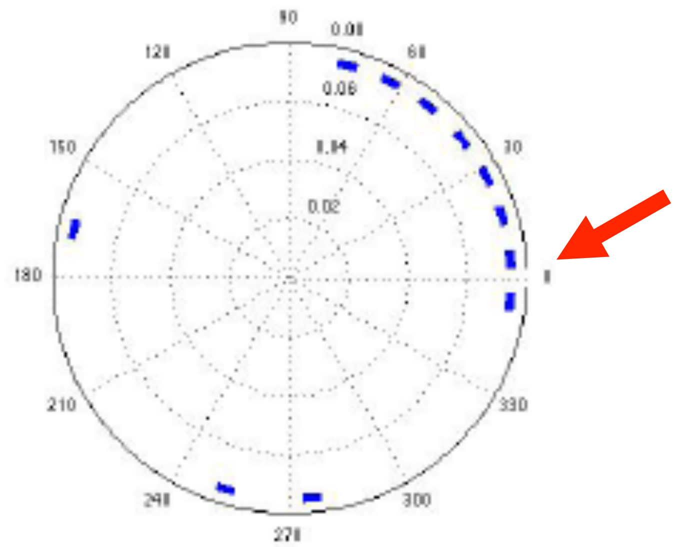
- Four road setup
  - Equally spaced
- Fixed size of the circle
  - Single lane
- Traffic control devices
  - None allowed inside circle

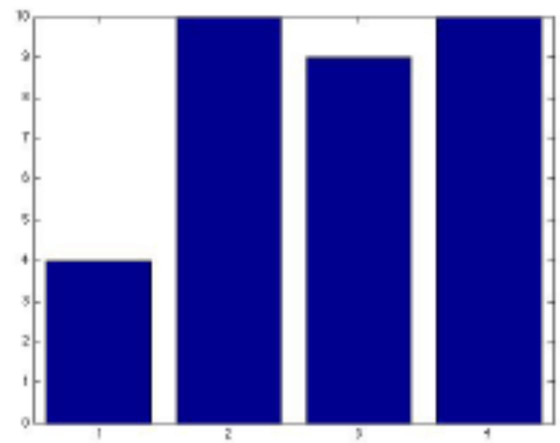
Simulation Demonstration



Simulation Results

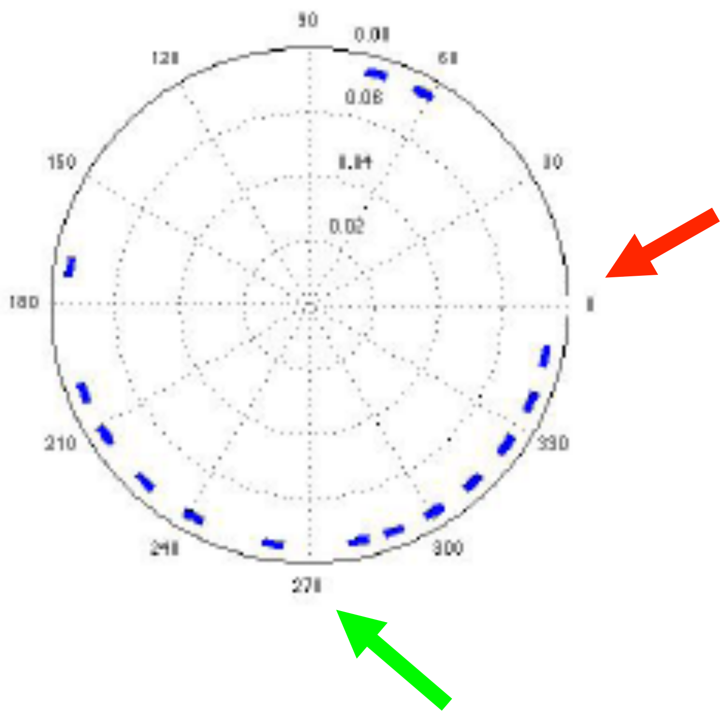
- Single intersection with higher traffic

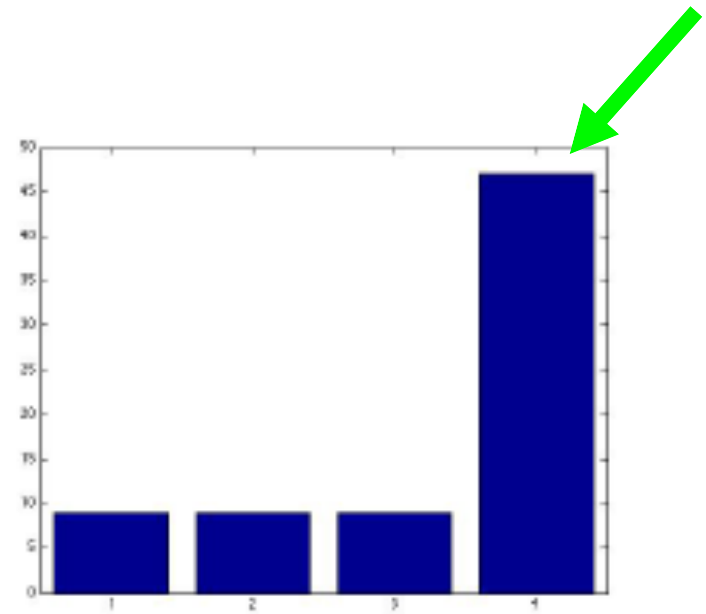




Popular Intersection Results

- Large buildup in previous intersection





Results (cont.)

- Different queues have high service (enter circle) and high arrival (arrive at intersection) rates
- Mostly random

Conclusions

- Most of the time, letting vehicles enter the circle as quickly as possible is optimal.

- During periods of high traffic, slowing the rate of entry into the circle helps prevent congestion.

Conclusions (cont.)

- If any single road has high traffic, its vehicles should be given preference in entering the circle. This will help prevent a large queue.
- Traffic often builds up in front of each intersection as cars exit, so a separate exit lane could help keep traffic moving.

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Questions